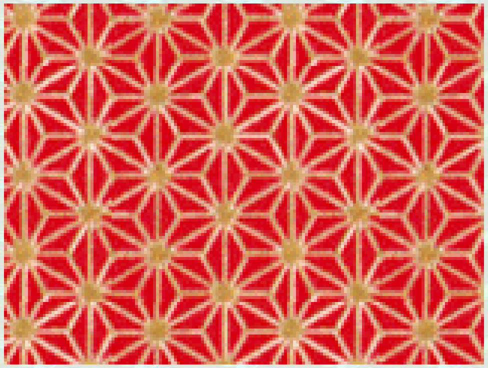


Yanxi Liu

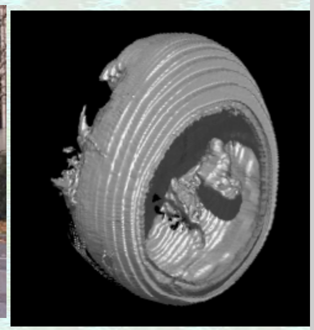
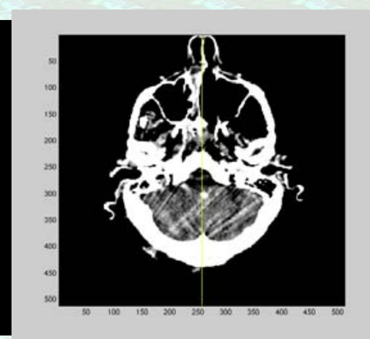
yanxi@cse.psu.edu

CSE and EE of PSU

affiliated UPMC, CMU

Machine Learning for Computational Regularity and Saliency



Acknowledgement

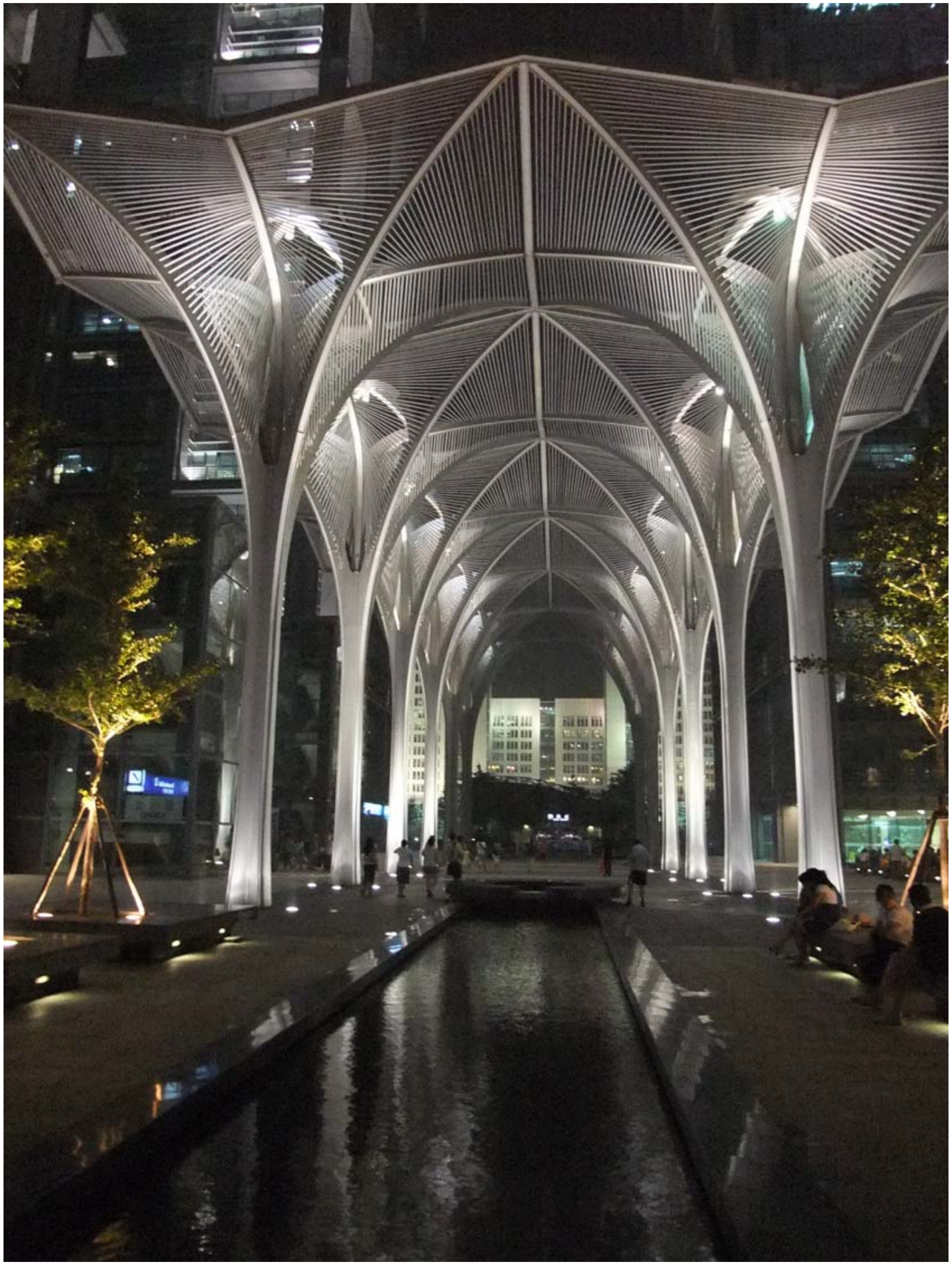
- **Collaborators**: J. Becker, C. Bregler, **K. Brocklehurst**, **O. Carmichael, E. Chastain,** K. Cheng (MD/PhD), J. Cohn, R.T. Collins, F. Dellaert, A. Efros, **J. Hays, S. Kashyap, V. Kwatra**, N. Lazar, S. Lee, **M. Leordeanu, W.C. Lin**, O.L. Lopez (MD), C. Meltzer (MD), **S. Mitra,** D. Nguyen(MD), **M. Park**, W. Rothfus (MD), K. Schmidt, **L. Teverovskiy, Y. Tsin …**
- **Support**: NSF, NIH/NCI/NIBIB, NLM, Microsoft Research, PA Health Department, Gift from Northrop Grumman Corporation, Gift from Alzheimer's Disease Research Center of University of Pittsburgh Medical Center, PSU Grace Woodward grant, PSU CTSA initiative award, Google Research …
- **Inspired by**: R. Popplestone, D. Schashneider, M. Senechal, H.S.M. Coxeter, D. Crowe, M. Leyton, T. Kanade, …

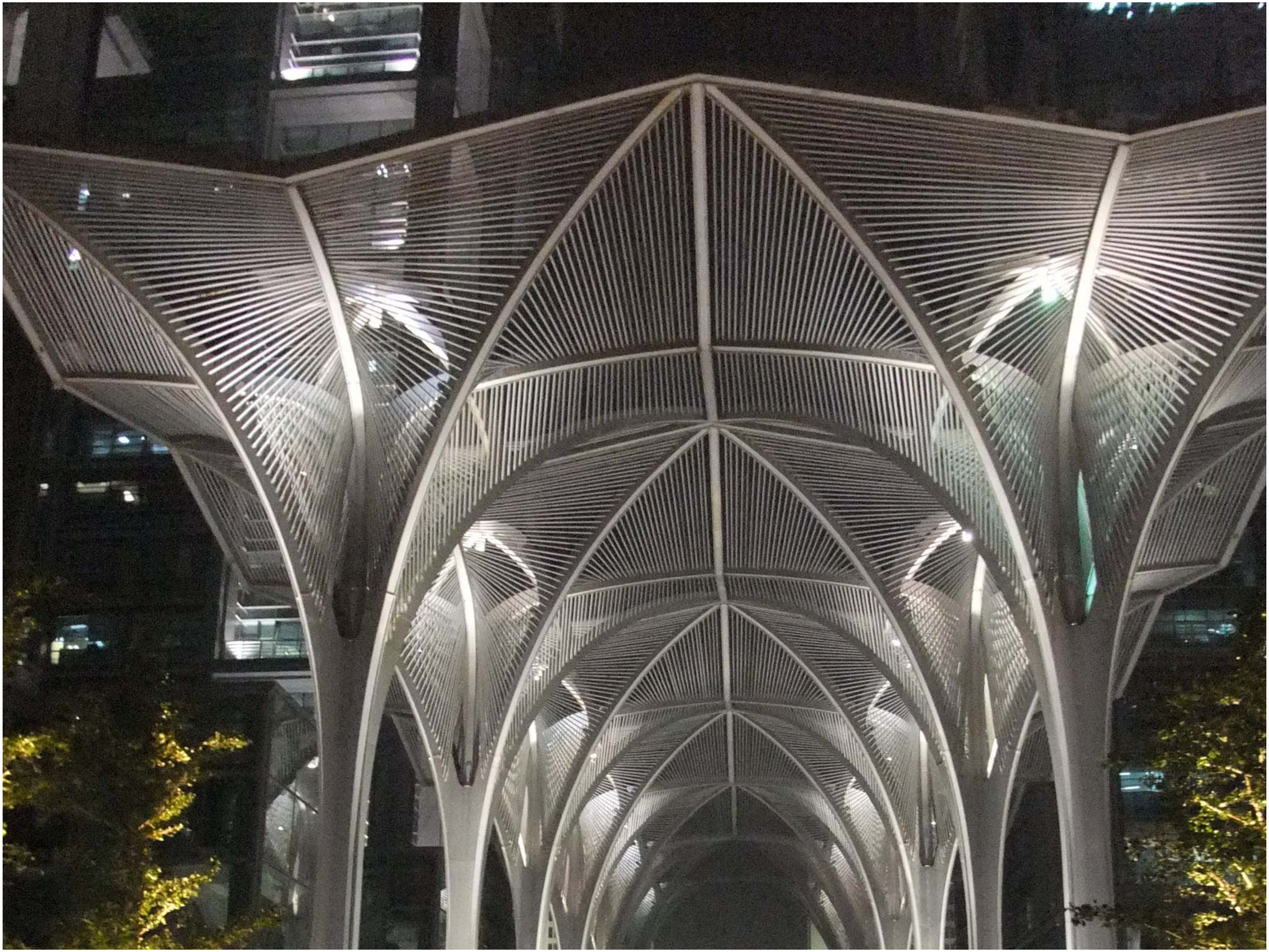
Real world regularity/symmetry,

where are they?

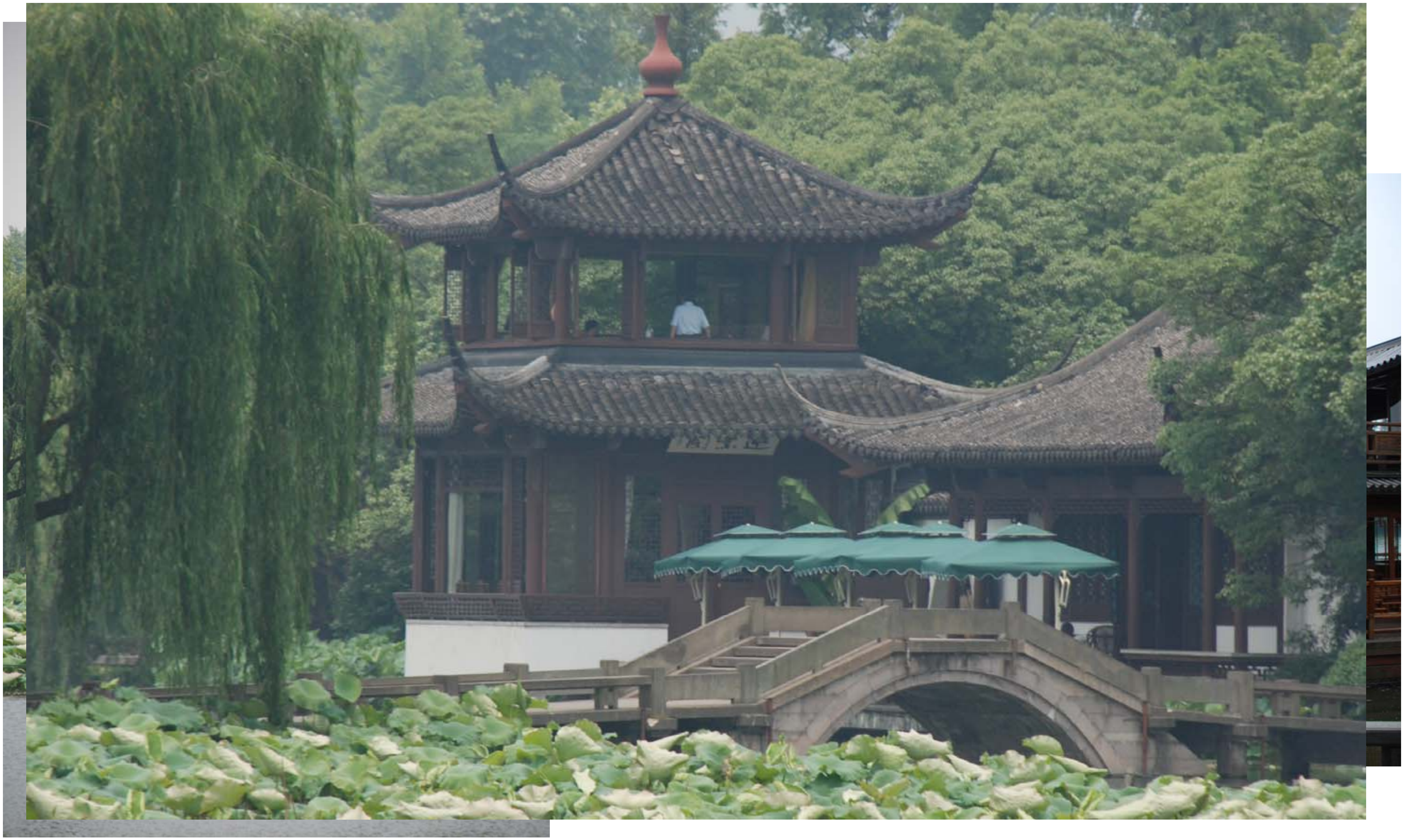
Beijing (July 2009)

- 水立方





Hang Zhou (May 2009)



NIPS 2008, Vancouver, Canada

ECCV 2008 Marseille, France

Anckorage, Alaska (CVPR 2008)

Tokyo, Japan (ACCV 2007)

Graz, Austria (ECCV 2006)

Real world regularity/symmetry,

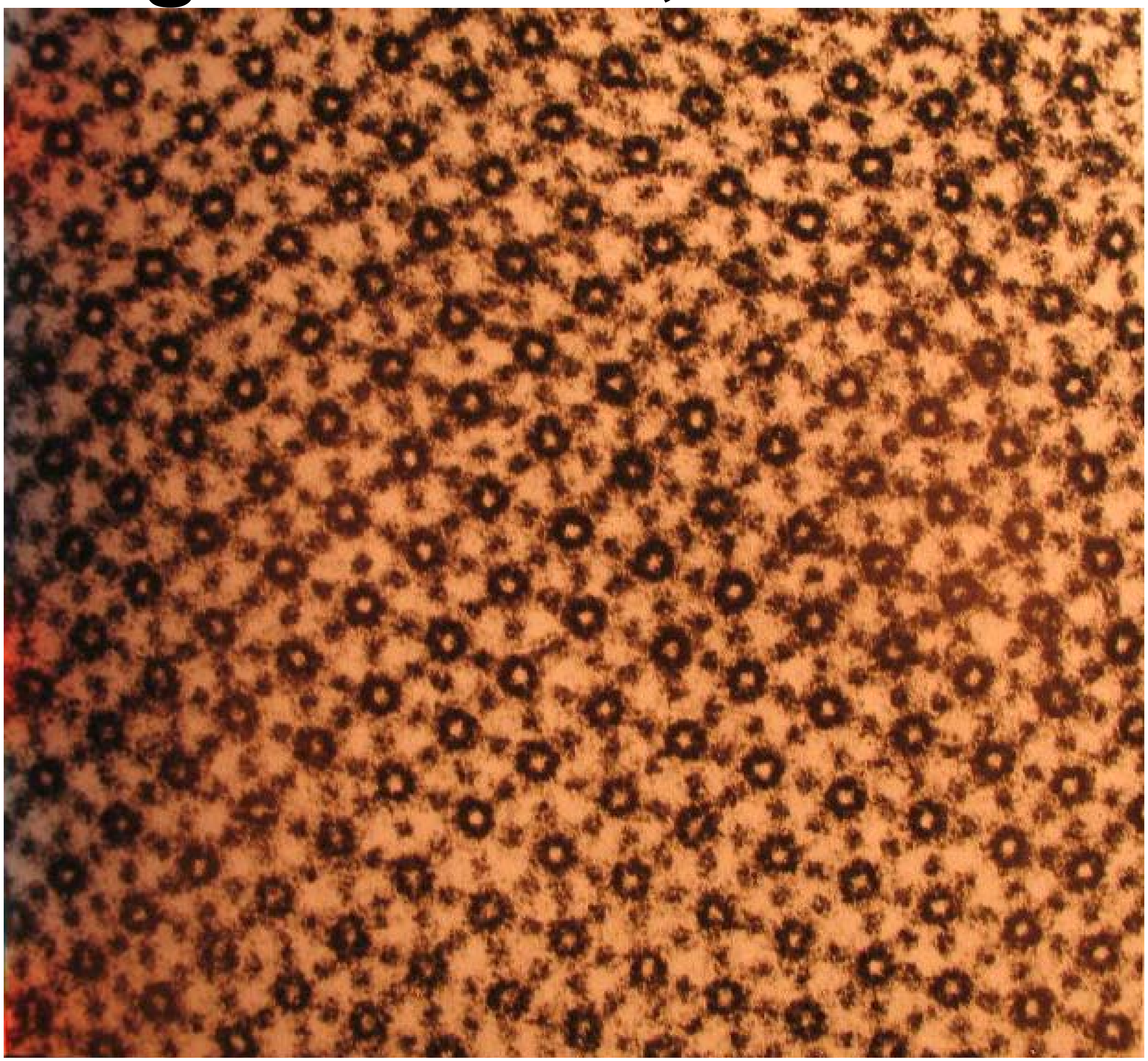
where are they?

Hint:

(1) look around this room … and

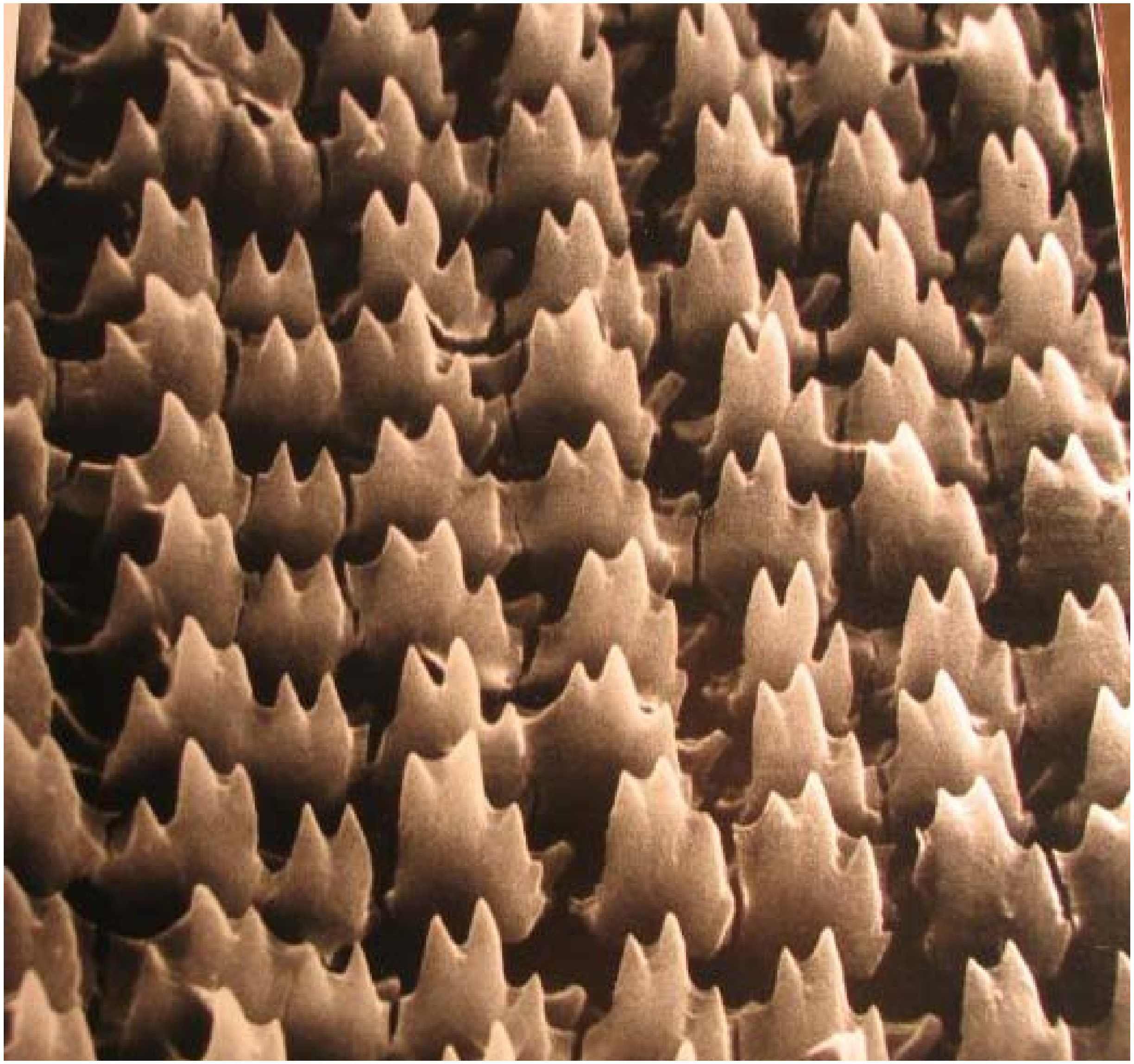
(2) look at each other

Skeletal Muscle magnified 800,000 times



?

Snails Teeth

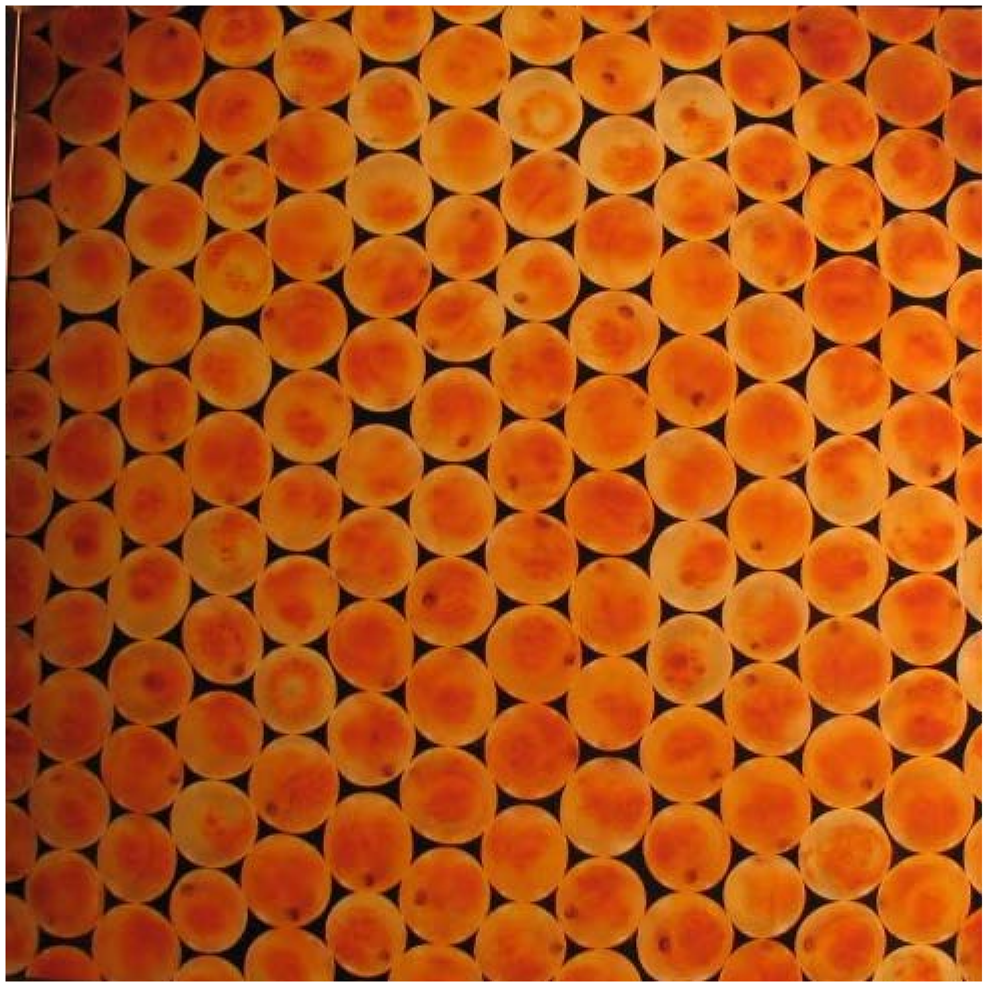


?

Iguana Skin

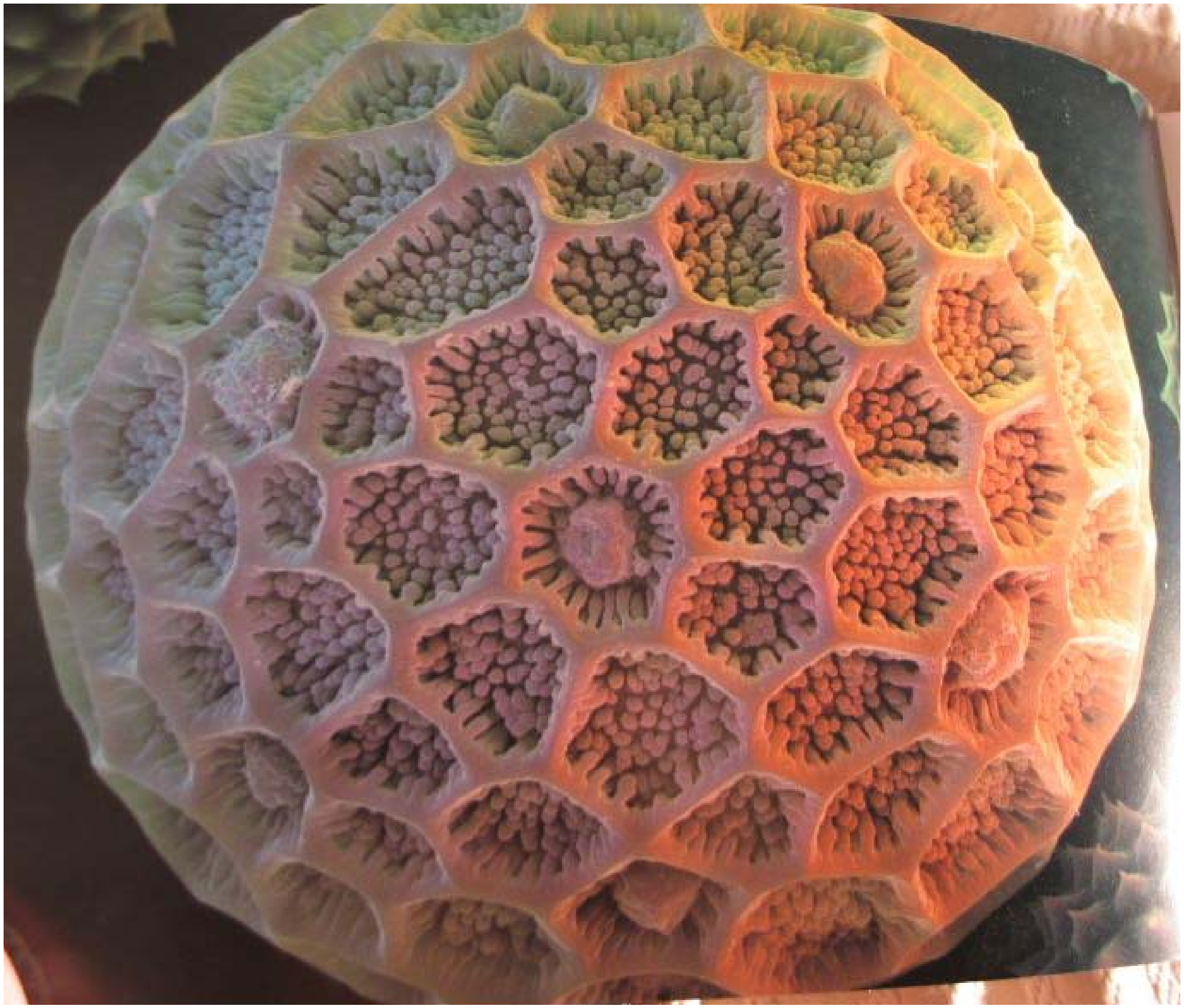
?

Rainbow trout eggs



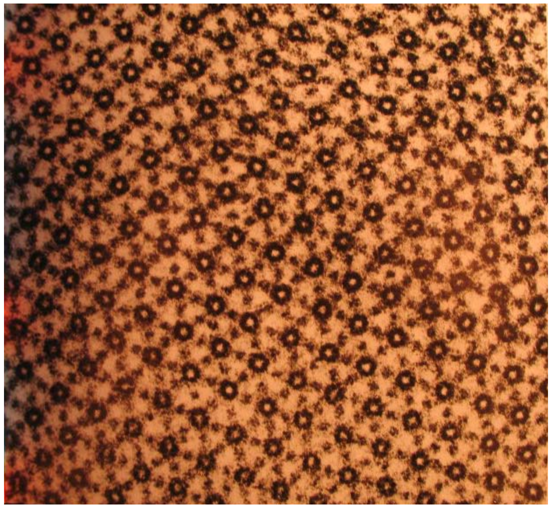
?

Knotweed Pollen

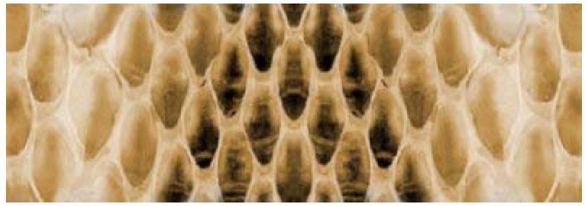


?

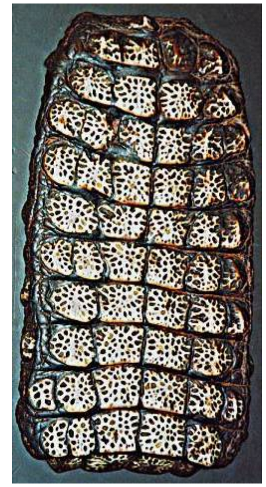
Regular patterns are indeed ubiquitous , ...



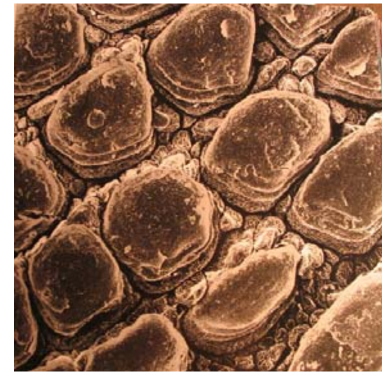
Skeletal Muscle magnified 800,000 times



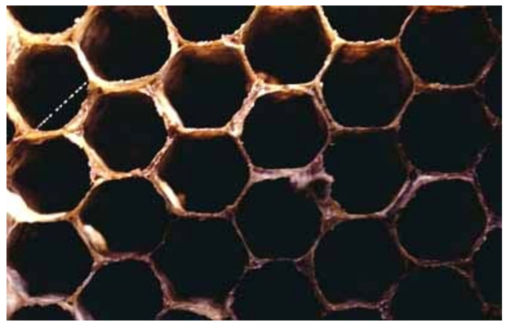
Snake Skin



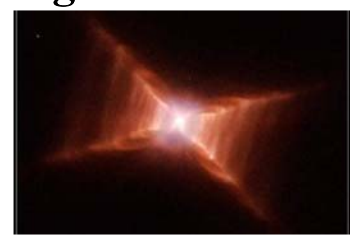
Turtle shell



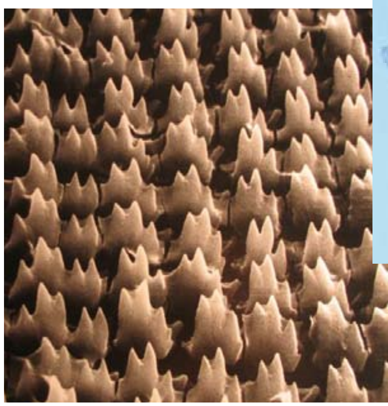
Iguanda Skin



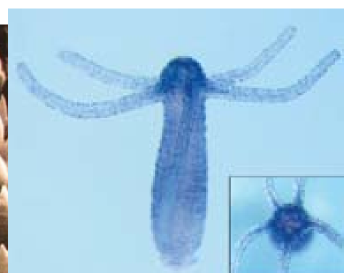
Beehive



Nebulas HD 44179

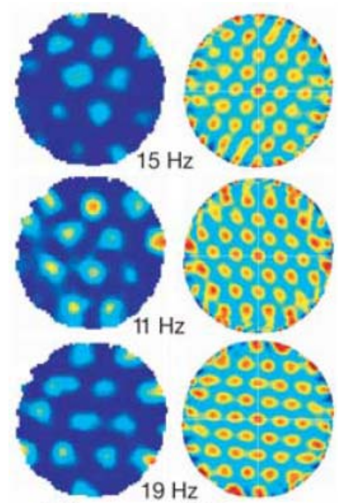


Snail’s Teeth



'Opsin' genes found in aquatic relatives of corals, jellyfish, sea anemones

Rainbow trout eggs



Firing field of grid Cells in rats brain

Knotweed Pollen

Firing Fields of Grid Cells in rats brains

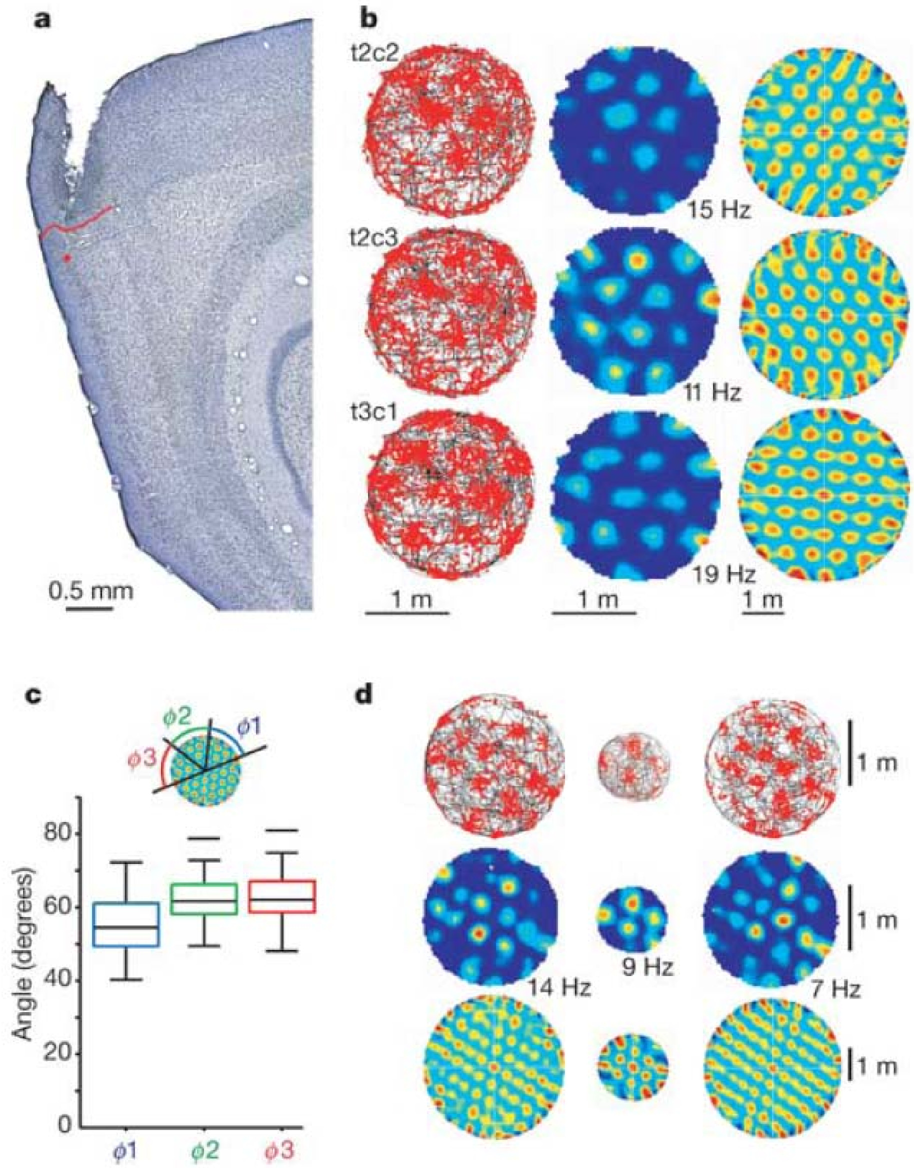


Figure 1 | Firing fields of grid cells have a repetitive triangular structure.

Nature **436**, 801-806 (11 August 2005) doi:10.1038/nature03721

Microstructure of a spatial map in the entorhinal cortex

Torkel Hafting, Marianne Fyhn, Sturla Molden, May-Britt Moser and Edvard I. Moser

An underlying relation between appearance regularity and symmetry

Regular → Symmetry

What is a <u>symmetry</u> anyways?

"Starting from the somewhat vague notion of symmetry = harmony of proportions, … rise to the general idea …that of invariance of a configuration of elements under a group of automorphic transformations."

--- Hermann Weyl

Symmetry, Princeton, 1952

Definition of **Symmetry**

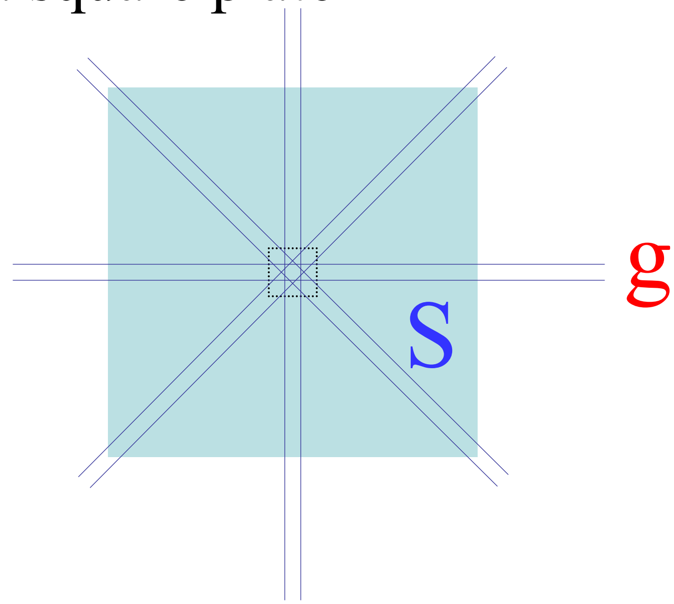
If **g** is a **distance preserving** transformation in n-dimensional Euclidean space R^n , and S is a subset of R^n , then **g** is a **symmetry** of S iff

$$g(S) = \{g(s) | \text{ for all } s \text{ in } S\} = S.$$

Note:
A symmetry is referring to a transformation !

Definition of **Symmetry**

a square plate in R^2



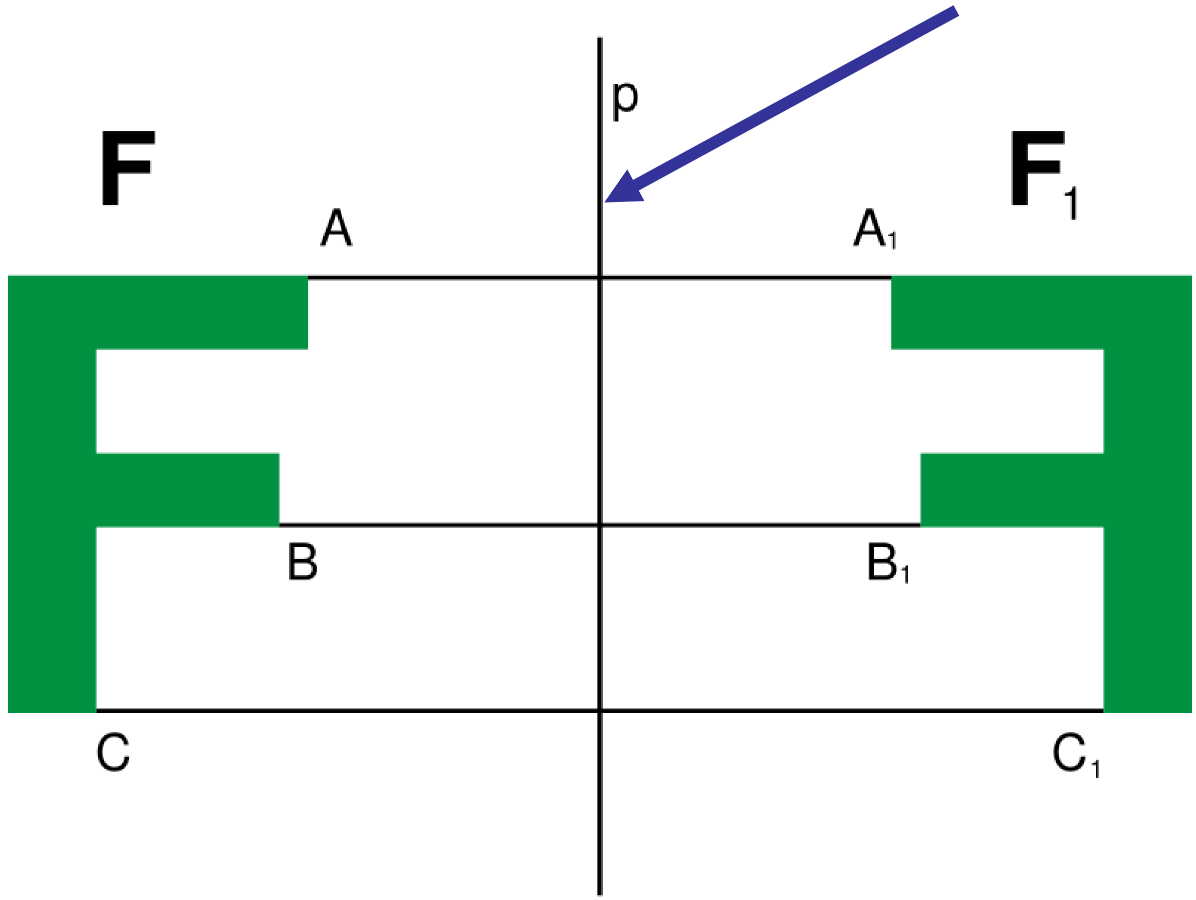
An example:

Reflection about this axis

4-fold rotation around this center

TYPES of PRIMITIVE SYMMETRIES g (2D Euclidean Space)

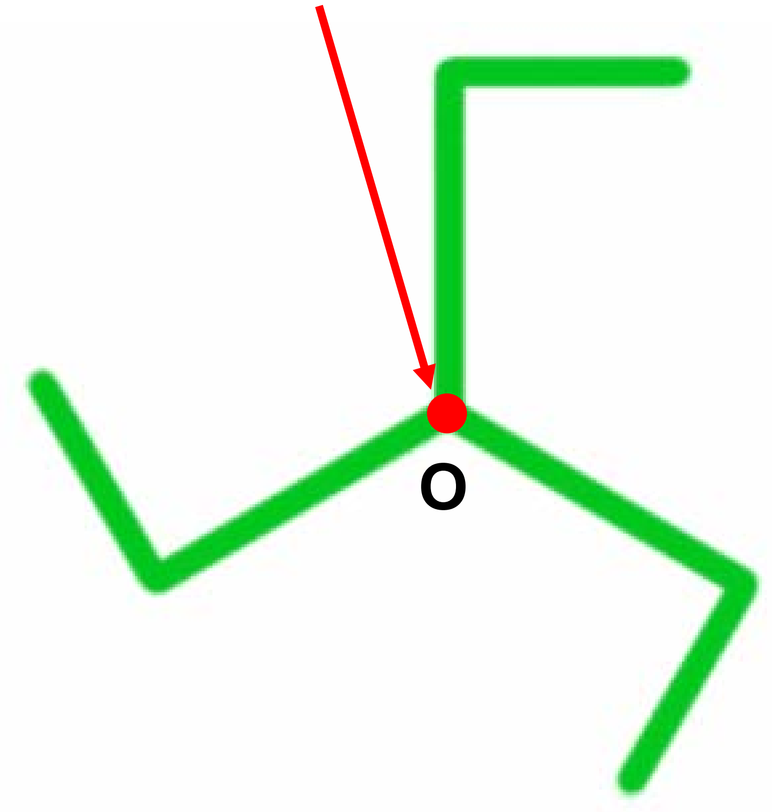
Reflection
invariance = the reflection axis p



With respect to an axis of reflection symmetry

Rotation

invariance: the center of rotation O

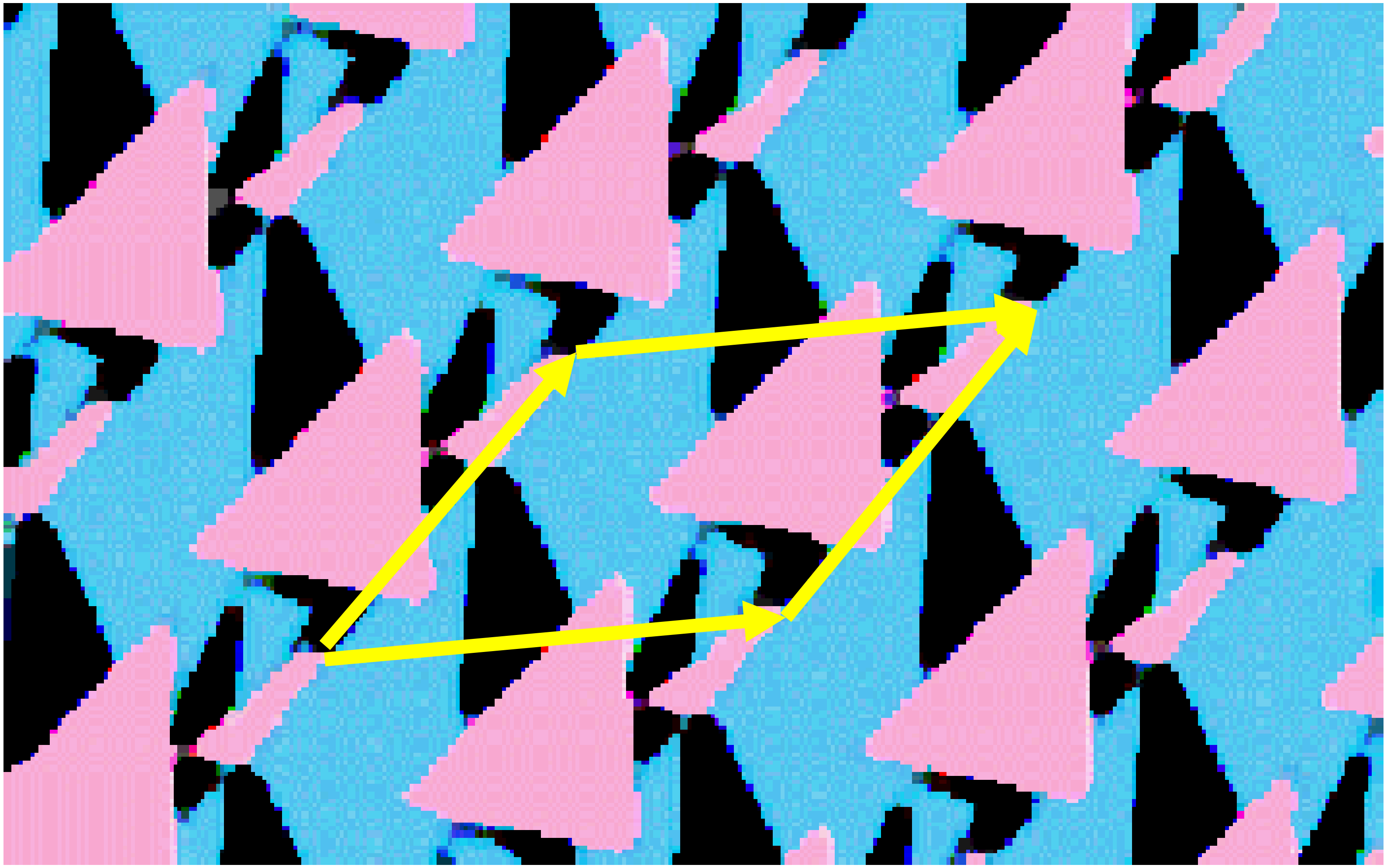


N-fold rotational symmetry:

Rotational symmetry of order *n*, also called *n*-fold rotational symmetry, or discrete rotational symmetry of the nth order, with respect to a particular point (in 2D) or axis (in 3D) means that rotation by an angle of 360°/n does not change the object.

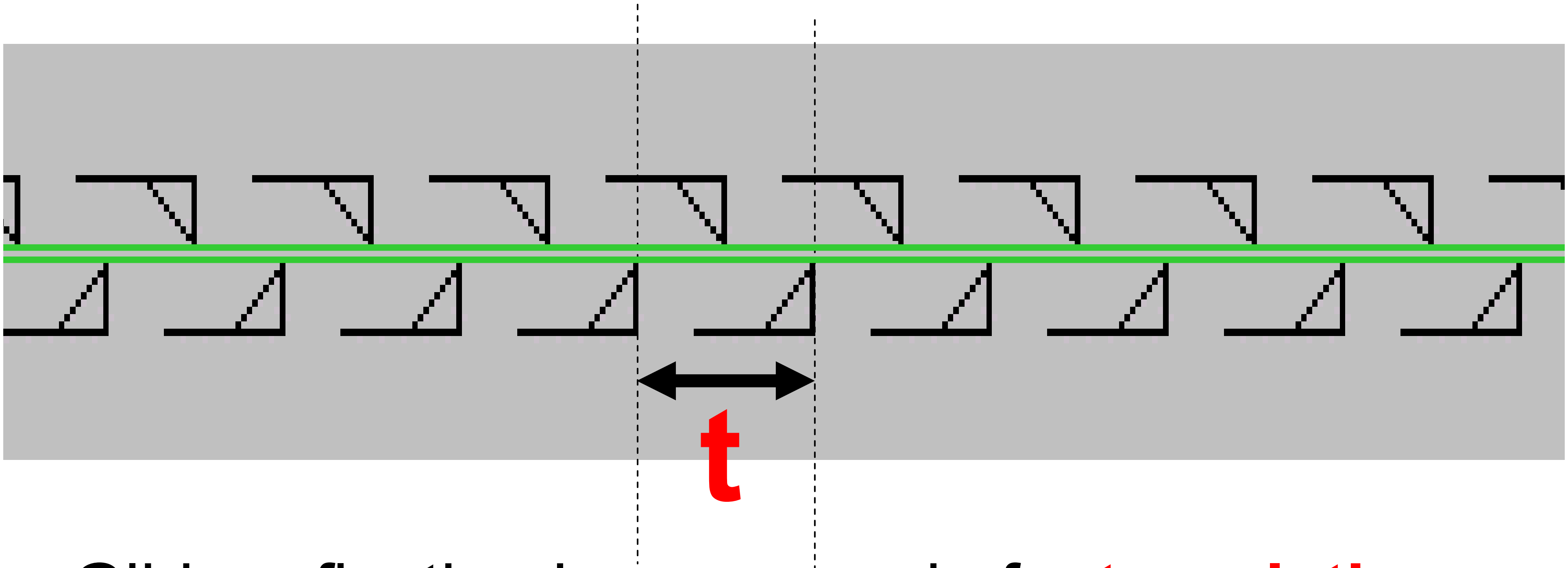
Translation

Invariance = invariant points = **none** (**but g(S) = S**)



Glide-reflection

Invariance = invariant points: **none**



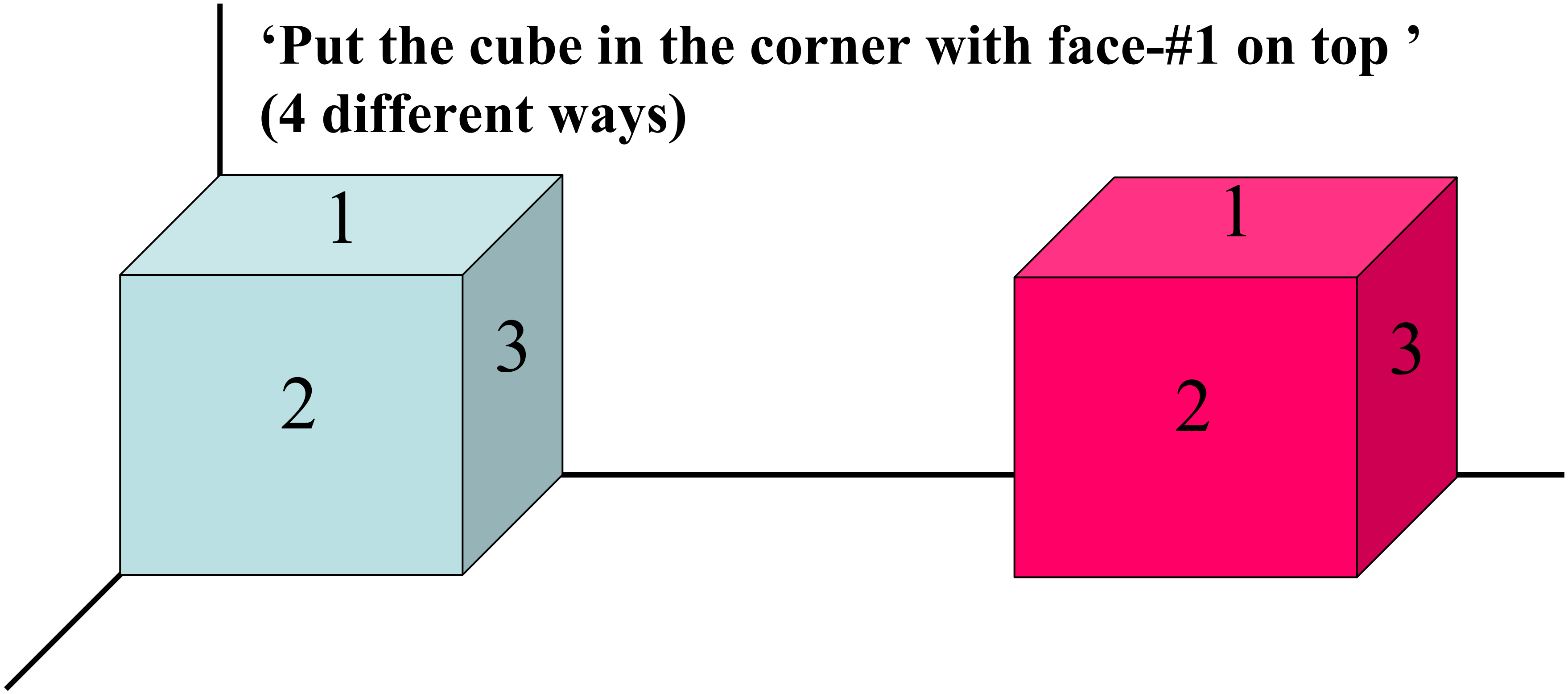
Glide reflection is composed of **a translation** that is ½ of the smallest translation symmetry **t** and **a reflection r** w.r.t. a reflection axis along the direction of the translation

Why symmetry is relevant to computational science?

Example:

- Intelligent robotic assembly planning and execution (Ultimate goal: build a robot that understands symmetry, my Ph.D. thesis)
- …

The Problem Complete and unambiguous assembly task specifications for robots can be tedious and error-prone!



A quiz: 'Put that cube in the corner !' how *many different ways*? What about put the cube by the wall?

Computation: automatic reasoning of relative motions among contacting solids given high level commands

Why symmetry is relevant to computational science?

Examples:

- Intelligent robotic assembly planning and execution (Ultimate goal: build a robot that understands symmetry, my Ph.D. thesis)

- **Texture regularity analysis and manipulation**
- …

Texture Synthesis
(image-based)

An algorithm can take a sample of texture and **generate an unlimited amount of image data**, which though not look exactly like the original, will be perceived by humans to be the **same** texture.

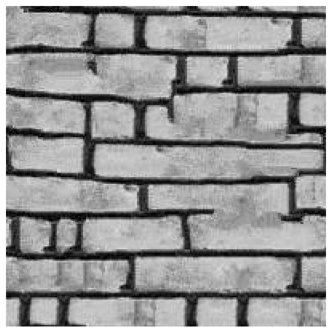
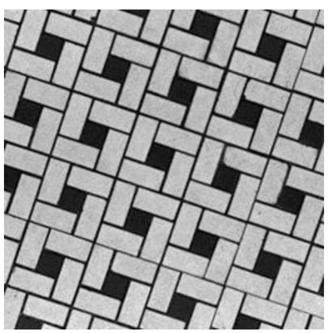
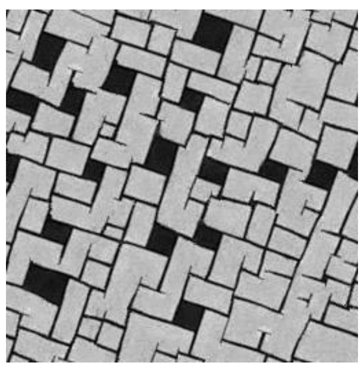
input

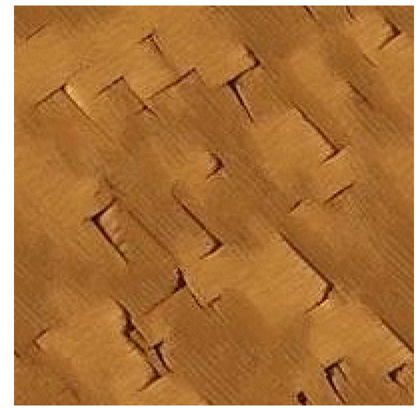
input

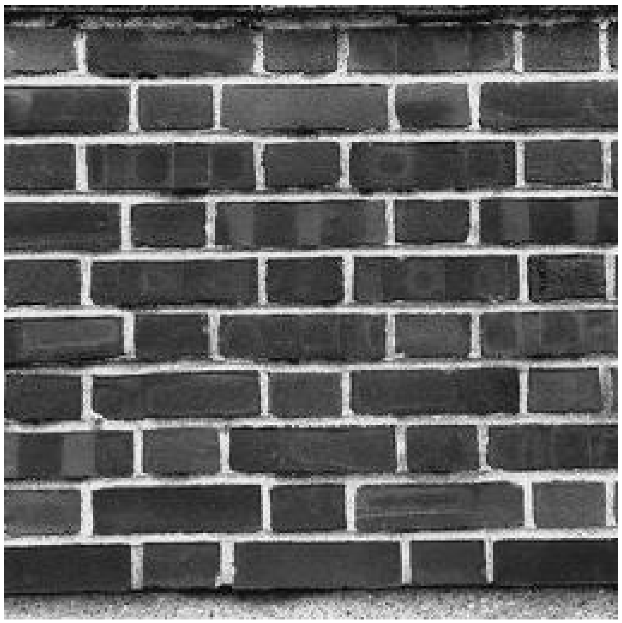
Efros&Freeman 2001 SIGGRAPH

Texture Synthesis for near-Regular Patterns

(pre-2001 published results)



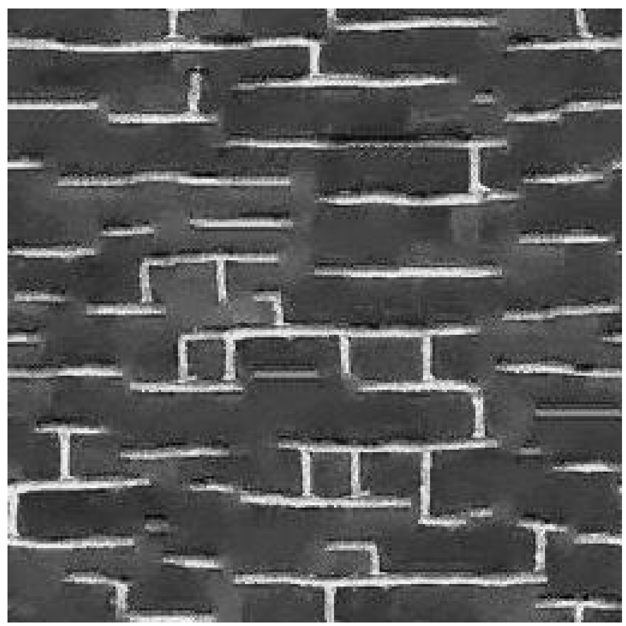


input image

Image Quilting
Efros&Freeman 2001

Portilla & Simoncelli

Xu, Guo & Shum



Wei & Levoy

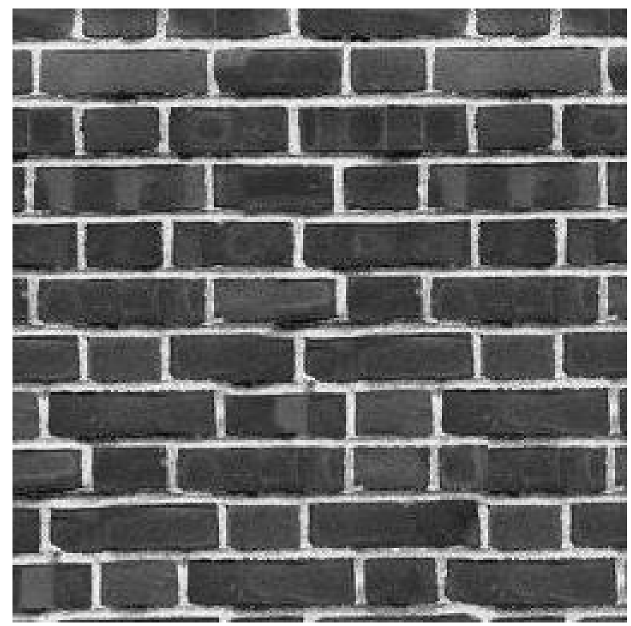
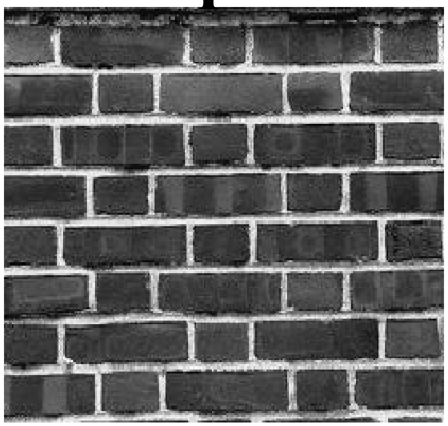


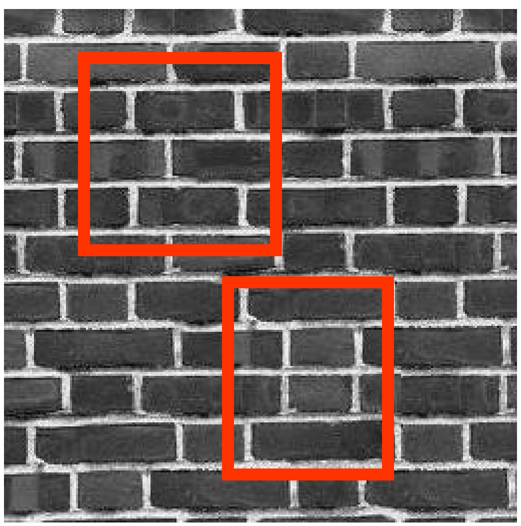
Image Quilting

Example of Near-Regular Texture synthesis results:

input

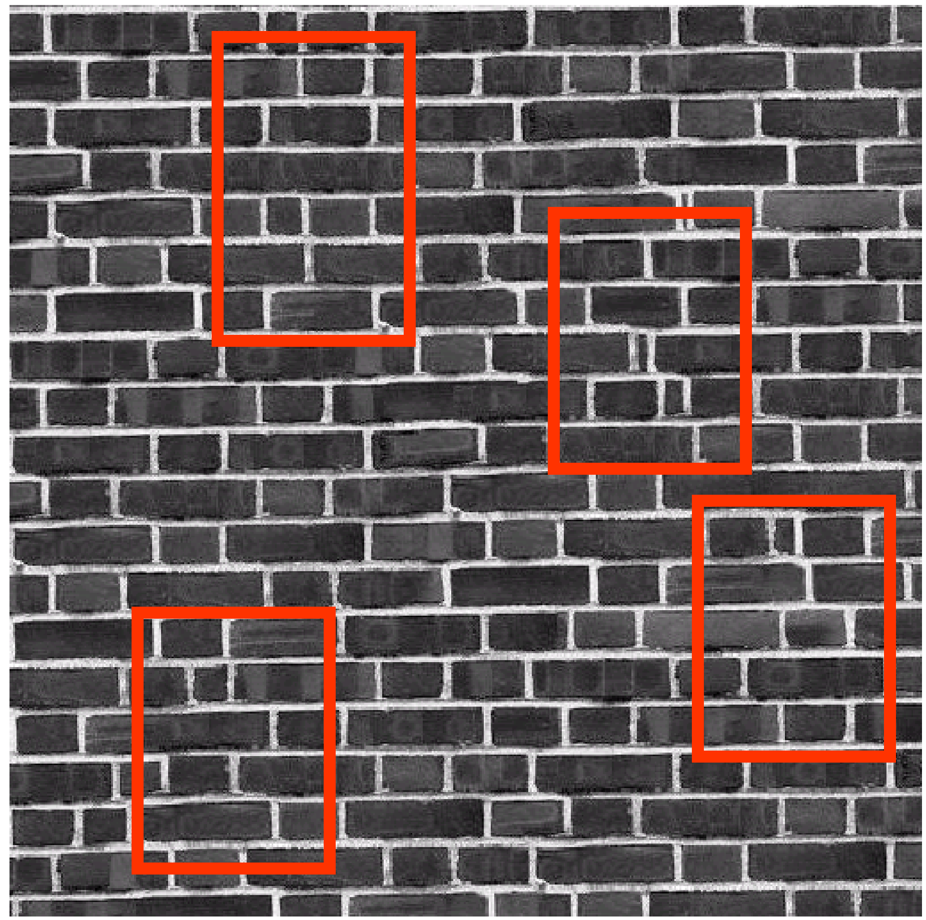


output



Efros/Freeman (2001)

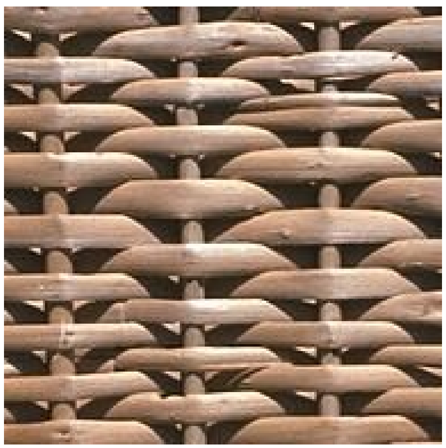
output



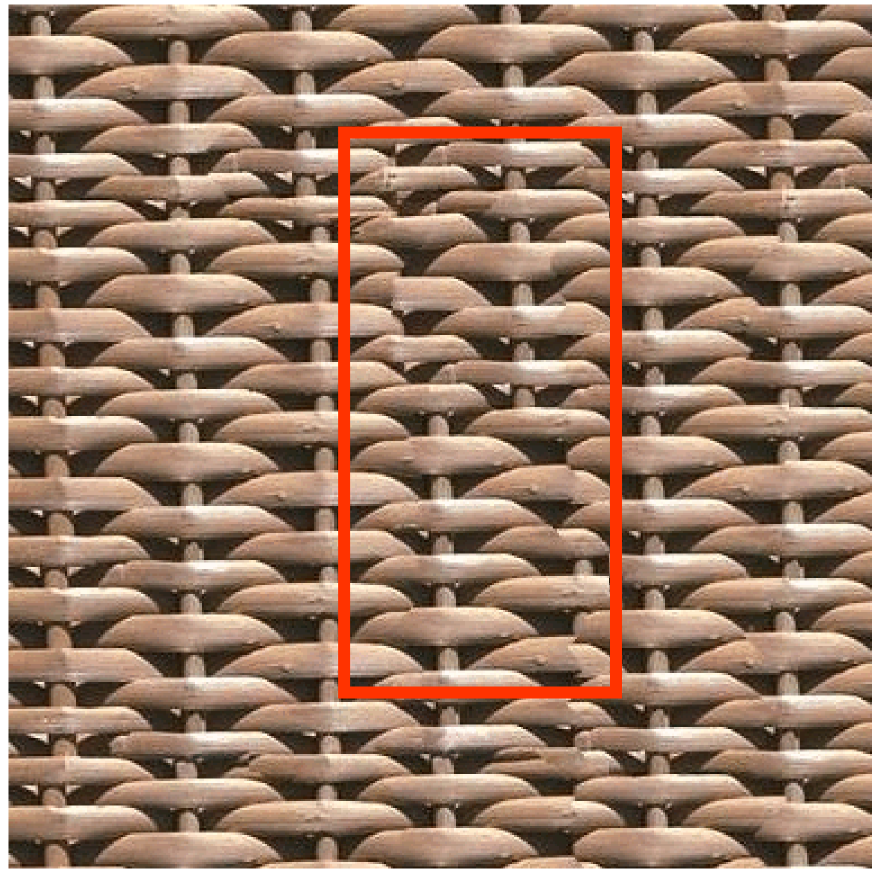
Special thanks to Kwatra et al Graph-cut (2003)

Example of NRT synthesis results:

input

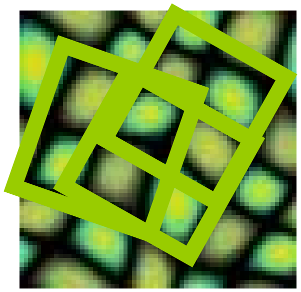


output

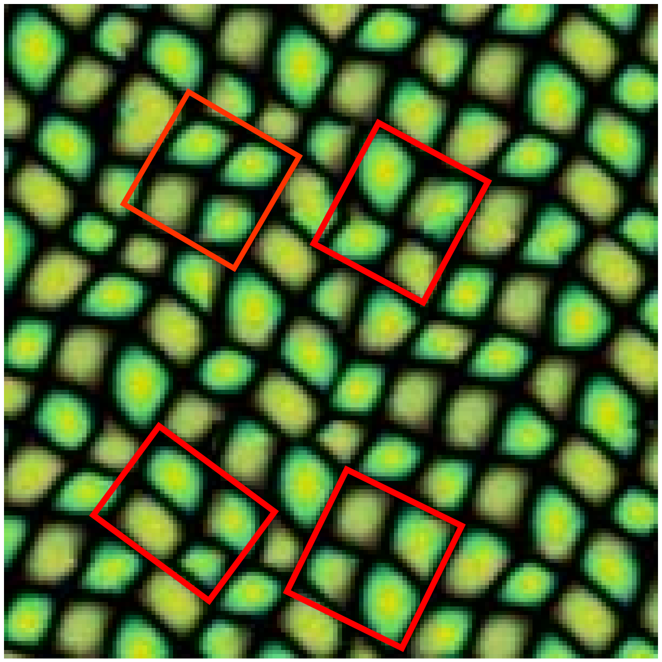


Efros and Freeman 2001
Image Quilting

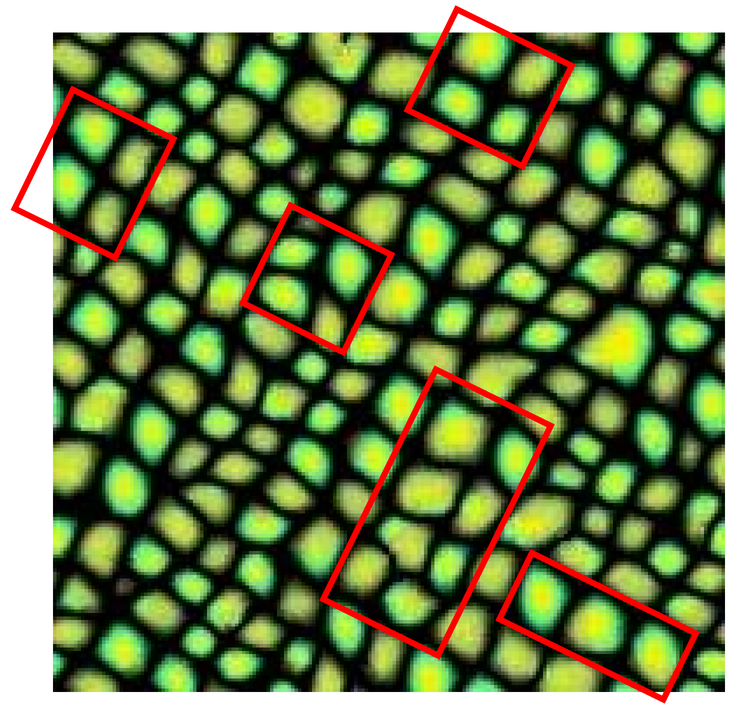
Example of NRT synthesis results:



Input texture



Efros & Freeman
SIGGRAPH'01



Wei & Levoy
SIGGRAPH'00

So, what is the problem?

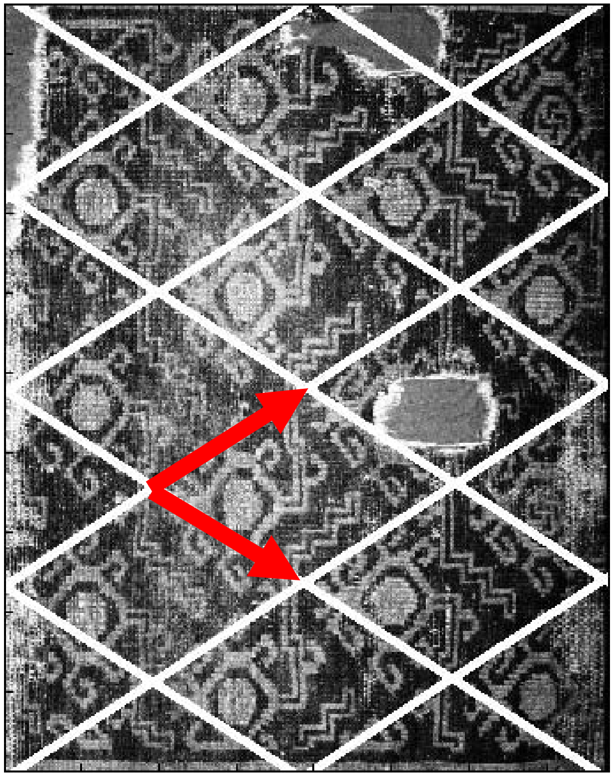
"Determining precisely **what are the patches** for a given texture and **how they are put together** is still **an open problem** … Let us define the … (patch)… to be a **square block** of user-specified size …"

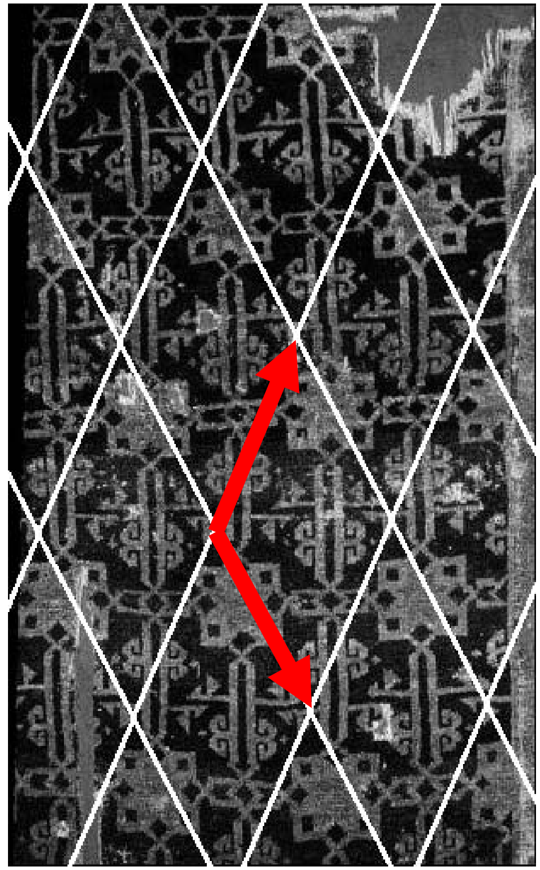
Efros and Freeman

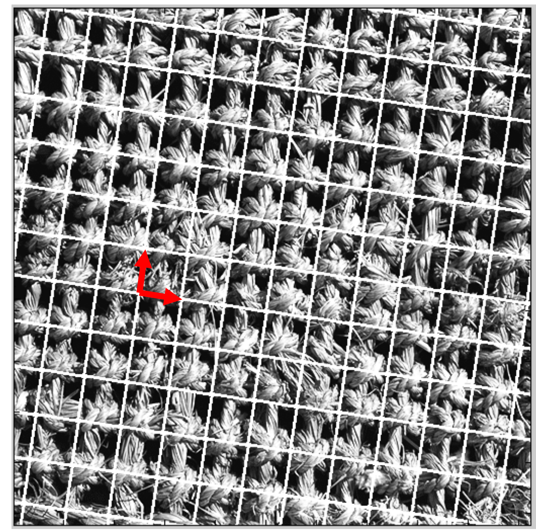
Image Quilting for Texture Synthesis and Transfer

SIGGRAPH 2001

Patches/windows differ in shape, size, and orientations:







Automatically Detected Tiles (patches) from Near-Regular Texture Patterns

(**Liu/Collins CVPR 2000, Liu, Collins and Tsin PAMI March 2004)**

Previous work on texture synthesis is mostly local method (Markov), thus lacking an understanding of the non-local structure of the texture, in particular the fundamental generating region --- the texel!

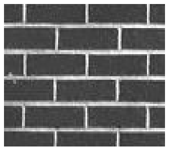
Though texel is a local region its existence is a non-local property of the texture (IJCV 2004, Liu et al)

There have been several Myths around regularities in texture synthesis …

Myth #1 – the choice of the window size in your synthesis algorithm is trivial (NOT)

"All you need is to pick a LARGE window (patch) to preserve input texture regularity"

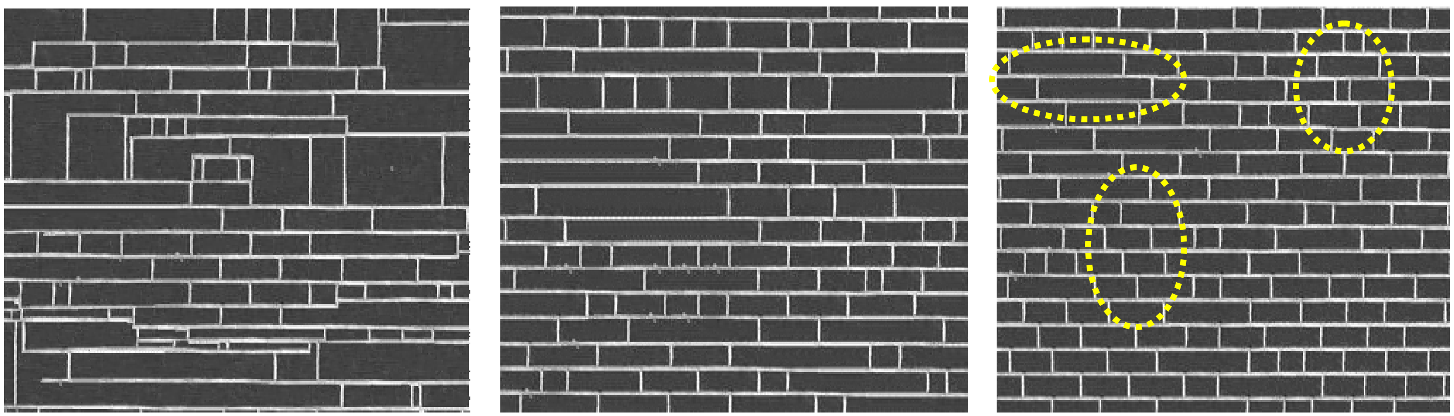
Will the input texture's regularity be preserved in the synthesized texture if we enlarge the window size?



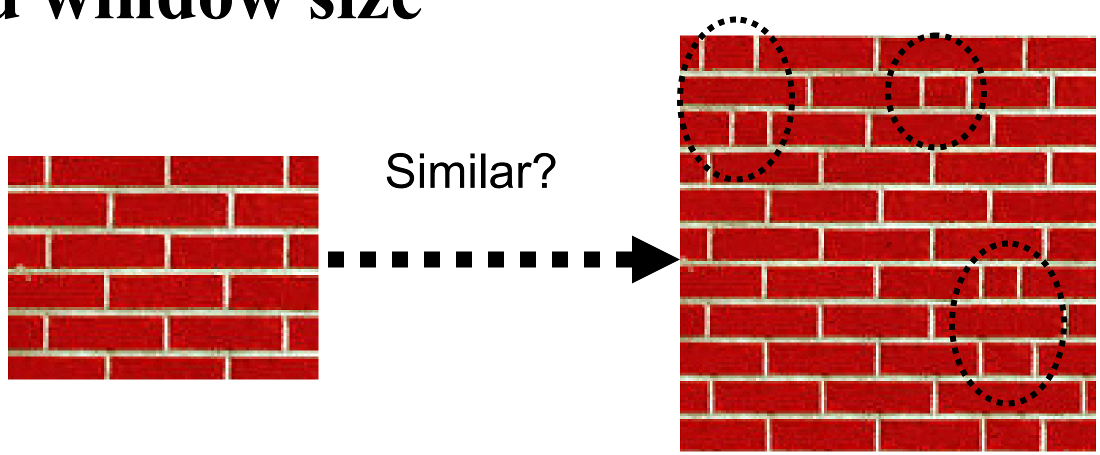
(Efros and Leung 1999)

NO.

Perils #1!
Whose regularity ?

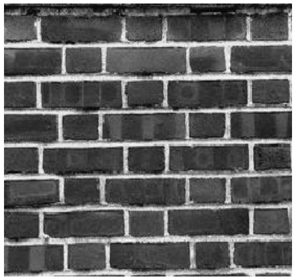


Increased window size →



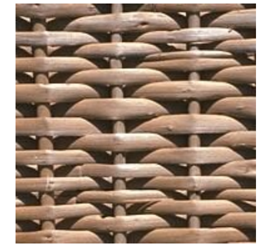
Myth #2 – the generating rule in your synthesis algorithm

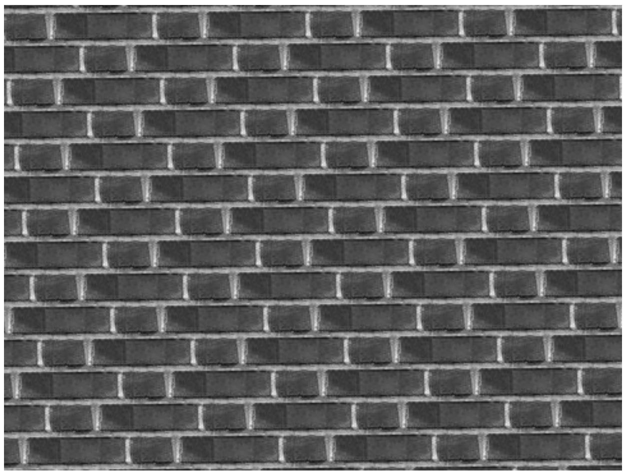
"All you need to do is to tile the chosen 'block' to demonstrate the input texture regularity"



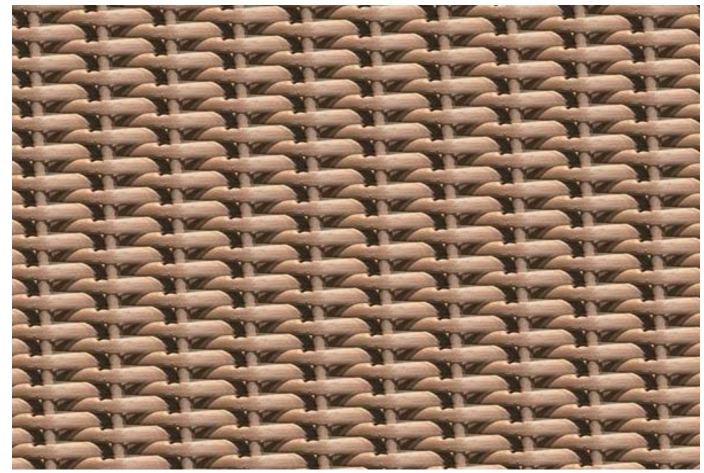
The Promise and Perils of Near-regular Textures

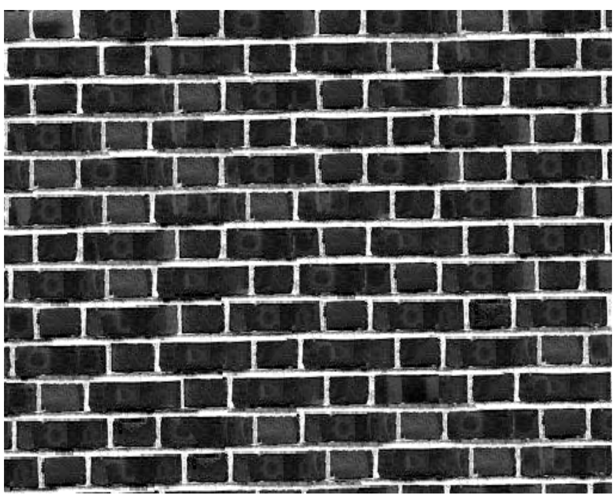
Liu, Lin and Tsin, International Journal of Computer Vision
Vol. 62, No. 1-2, April, 2005, pp. 145 - 159. (accepted 6/04)



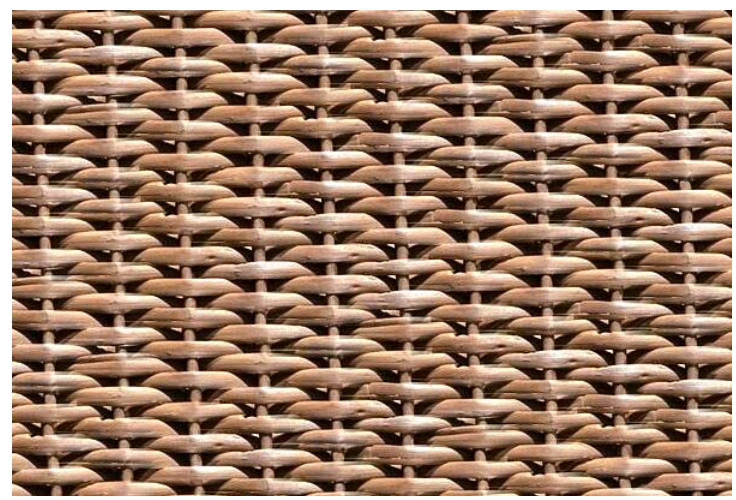


Naïve Tiling





Our result



Myth #3 – texture perception by human observers

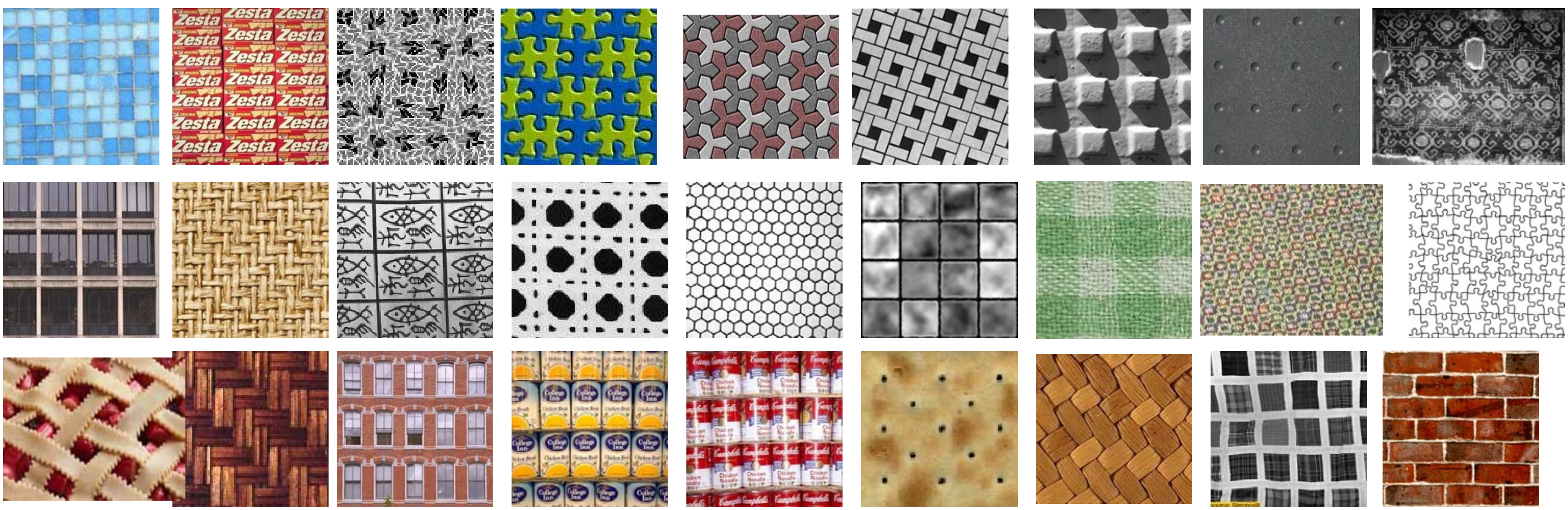
“Human observers can not tell the difference”

Quantitative Evaluation of Texture Synthesis Algorithms by Human Users

• **Quantitative Evaluation on Near Regular Texture Synthesis**

Lin, Hays, Wu, Kwatra, and Liu. CVPR '06 pp. 427 - 434. 2006

Algorithms	**Graph cut**	**Near-regular texture synthesis**	**Regularized patch-based**	**Patch-based**
mean score	3.0	3.4	3.1	2.2
standard deviation	1.11	0.75	0.86	1.24
standard error of the mean	0.054	0.036	0.052	0.075
ANOVA compare against the NRTS	$F(1,858)=54.18$ $p < 0.001$	N/A	$F(1,698)=27.37$ $p < 0.001$	$F(1,698)=315.95$ $p < 0.001$



How to respect the regularity in the input texture computationally???

- To truly understand

  real world regularity →

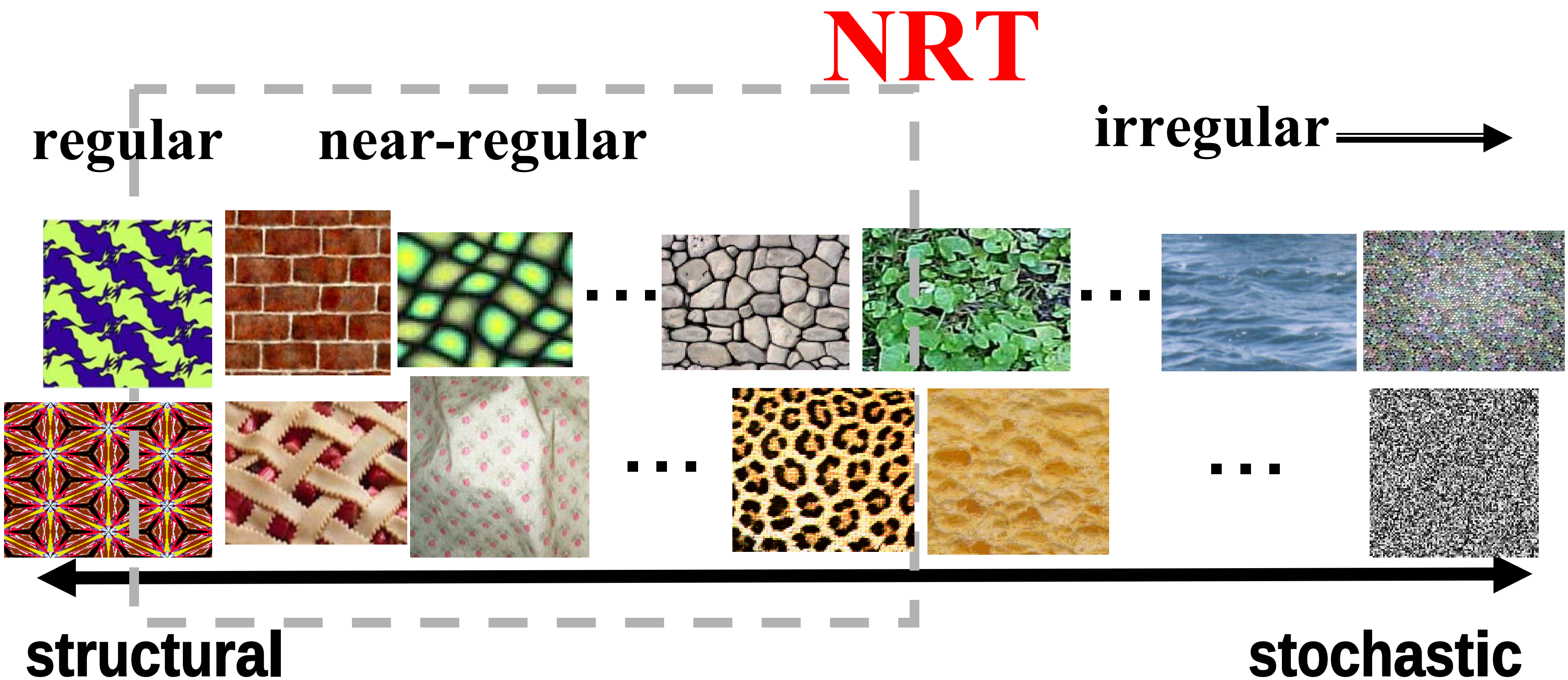
  real world symmetry →

  mathematical symmetry groups

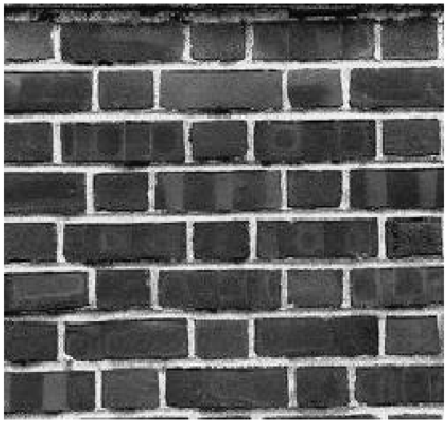
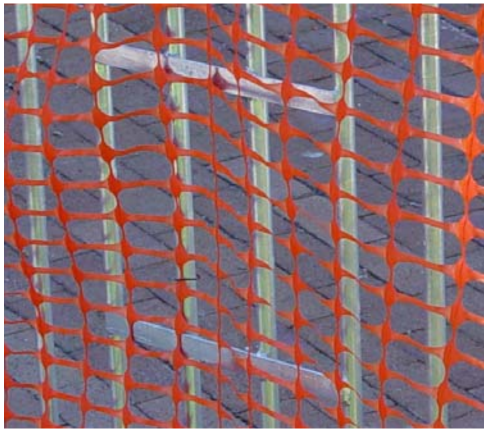
  \+

  statistics/probability theory

A Texture Spectrum in terms of texture REGULARITY



Near Regular Textures (NRT)

NRTs have a **<u>un-deniable tendency towards regularity</u> or symmetry, even though the regularity is often imperfectly presented and intertwined with stochastic signals and random noise.**

Liu, Lin, Hays **Near Regular Texture Analysis and Manipulation**

***ACM Transactions on Graphics (SIGGRAPH)*, Vol. 23, No. 3, August, 2004, pp. 368 - 376.**

The Promise and Perils of Near Regular Textures

by Liu, Tsin and Lin International Journal of Computer Vision (IJCV) , Vol. 62, No. 1-2, April, 2005, pp. 145 - 159.

When the texture geometry deviates from regularity …

Show a short movie from

Near-Regular Texture Analysis and Manipulation

Liu,Lin,Hays SIGGRAPH 2004


FOX
Money
PARTY

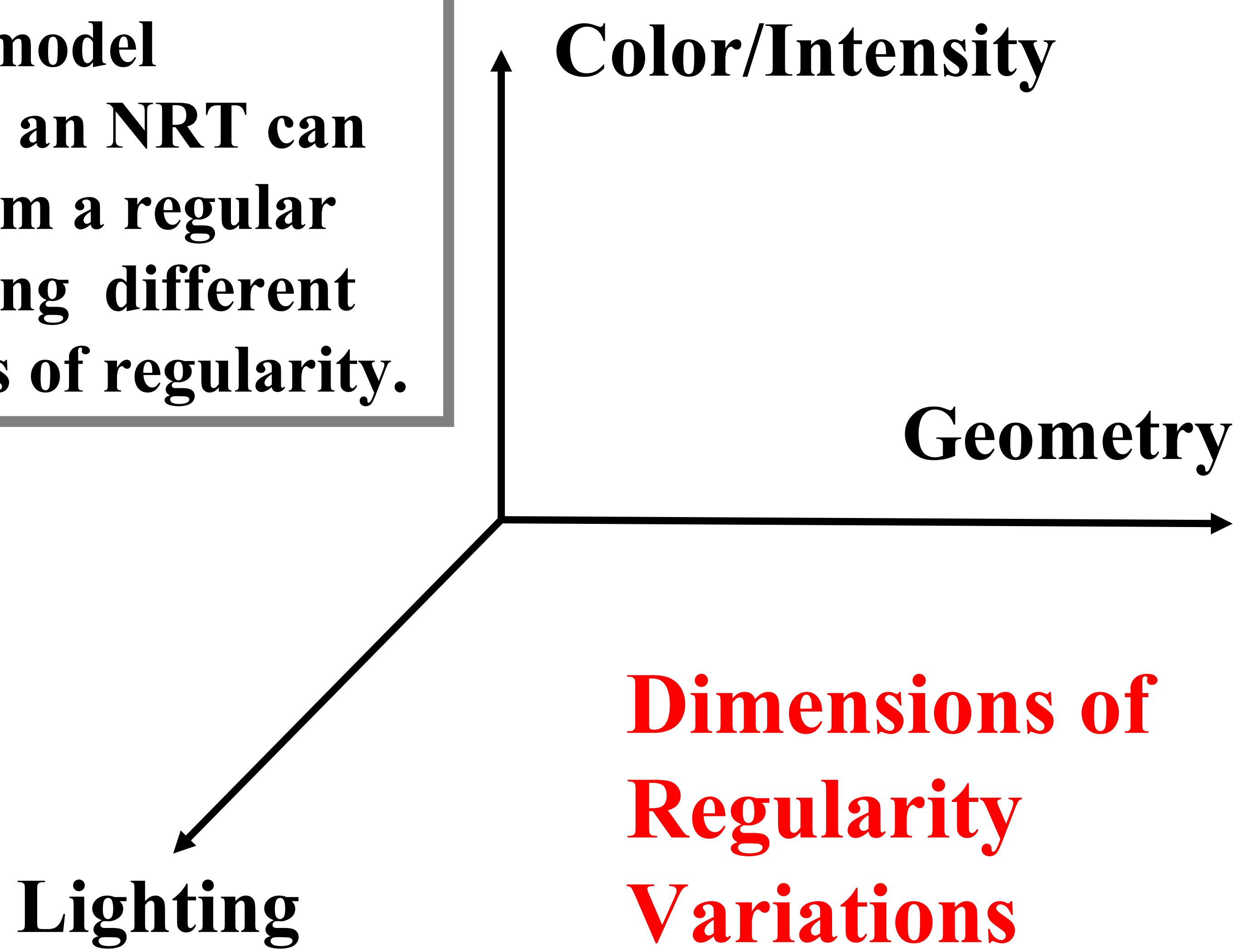
Two Key Insights

#1: Dynamic nature: A Near-Regular Texture can be viewed as a deformation from some Regular Texture

Regular ⇄ Irregular

Two Key Insights

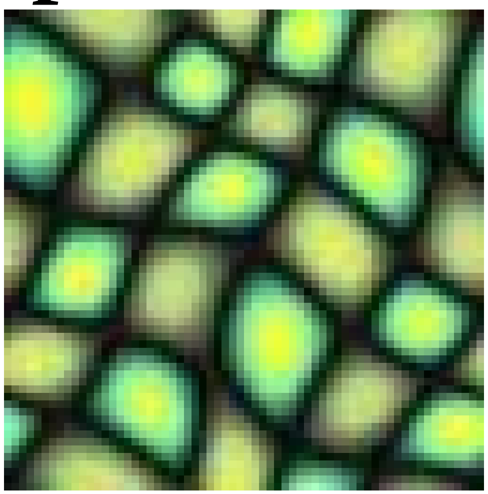
#2: Multi-model variations: an NRT can deviate from a regular texture along different dimensions of regularity.



Dimensions of Regularity Variations

Conceptually,

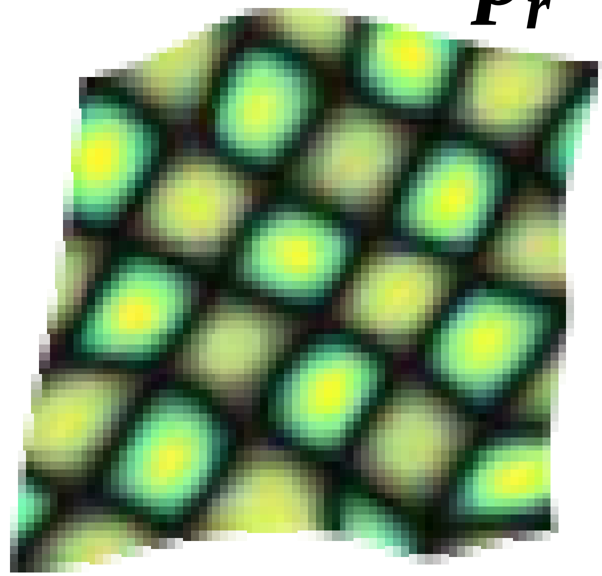
p



Near-Regular Texture

$= d($

p_r



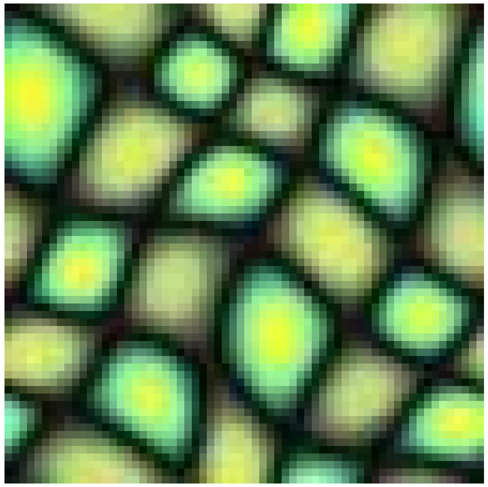
Regular Texture

$)$

proposed in SIGGRAPH 2004, Liu, Lin, Hays

Computationally,

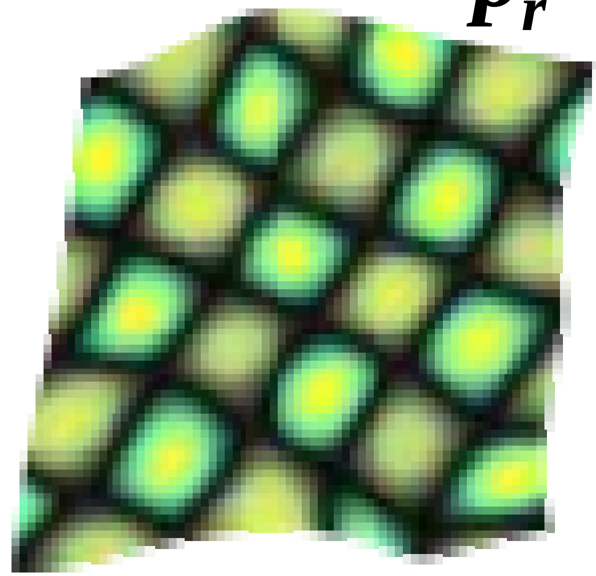
p



Near-Regular Texture

$= d($

p_r



$)$

Regular Texture

where d is <u>deformation field</u> and

$$d = d_{geometry} \times d_{lighting} \times d_{color}$$

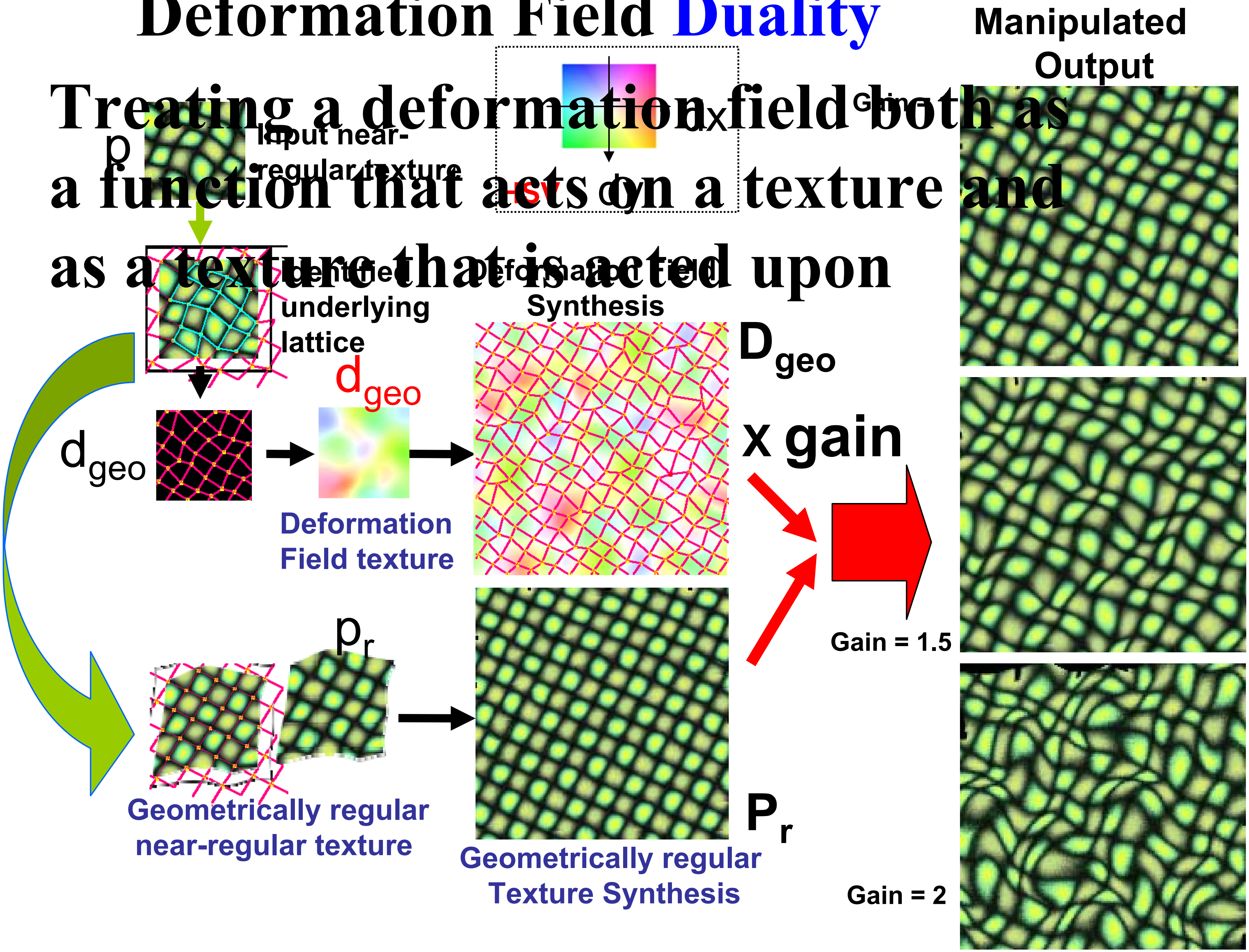
↑ ↑

functional composition


Six FOX
Money
PARTY

Deformation Field Duality

Treating a deformation field both as a function that acts on a texture and as a texture that is acted upon

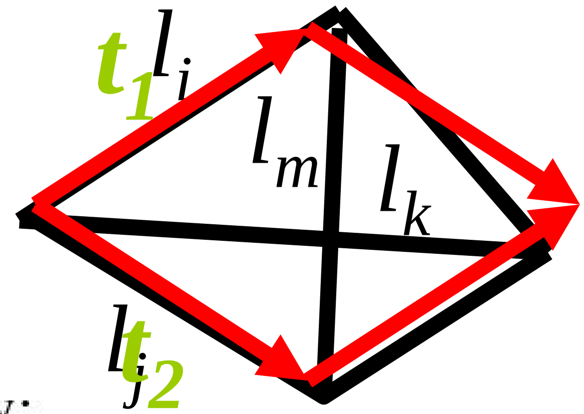


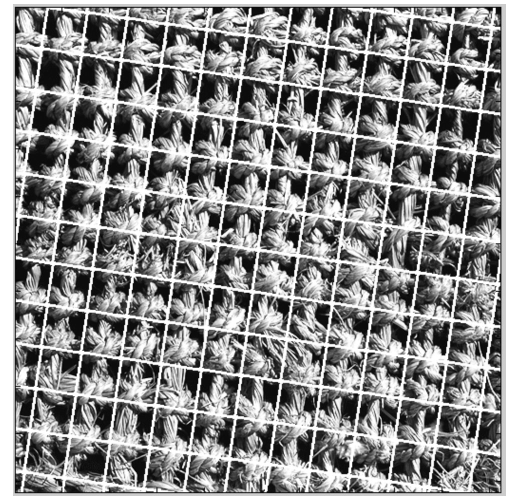
SIGGRAPH04 Movie

Quantified Regularities

Near-regular Texture Analysis and Manipulation
SIGGRAPH 2004
Liu, Lin and Hays

A Pair of Texture Regularity Measures





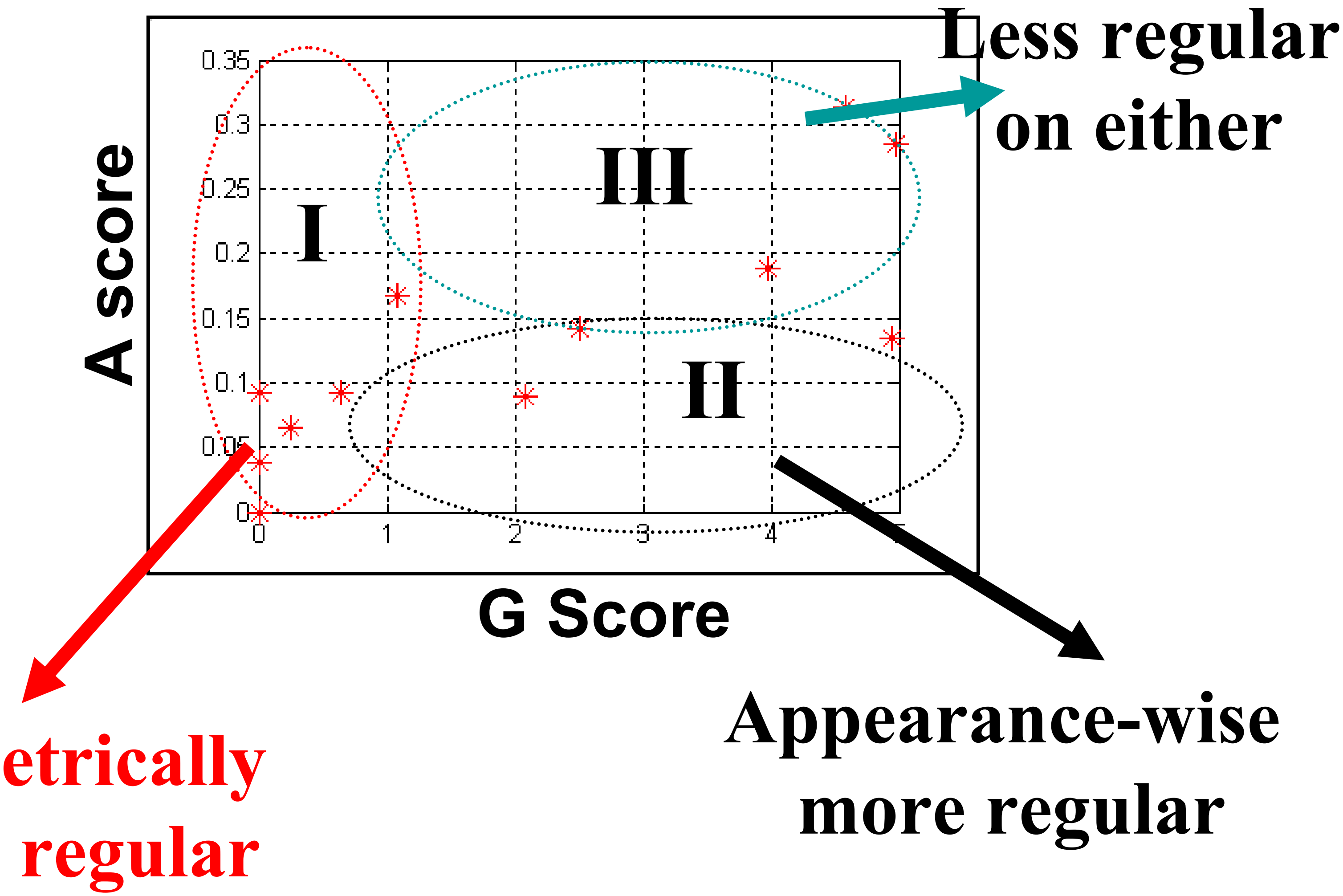
Geometric regularity:

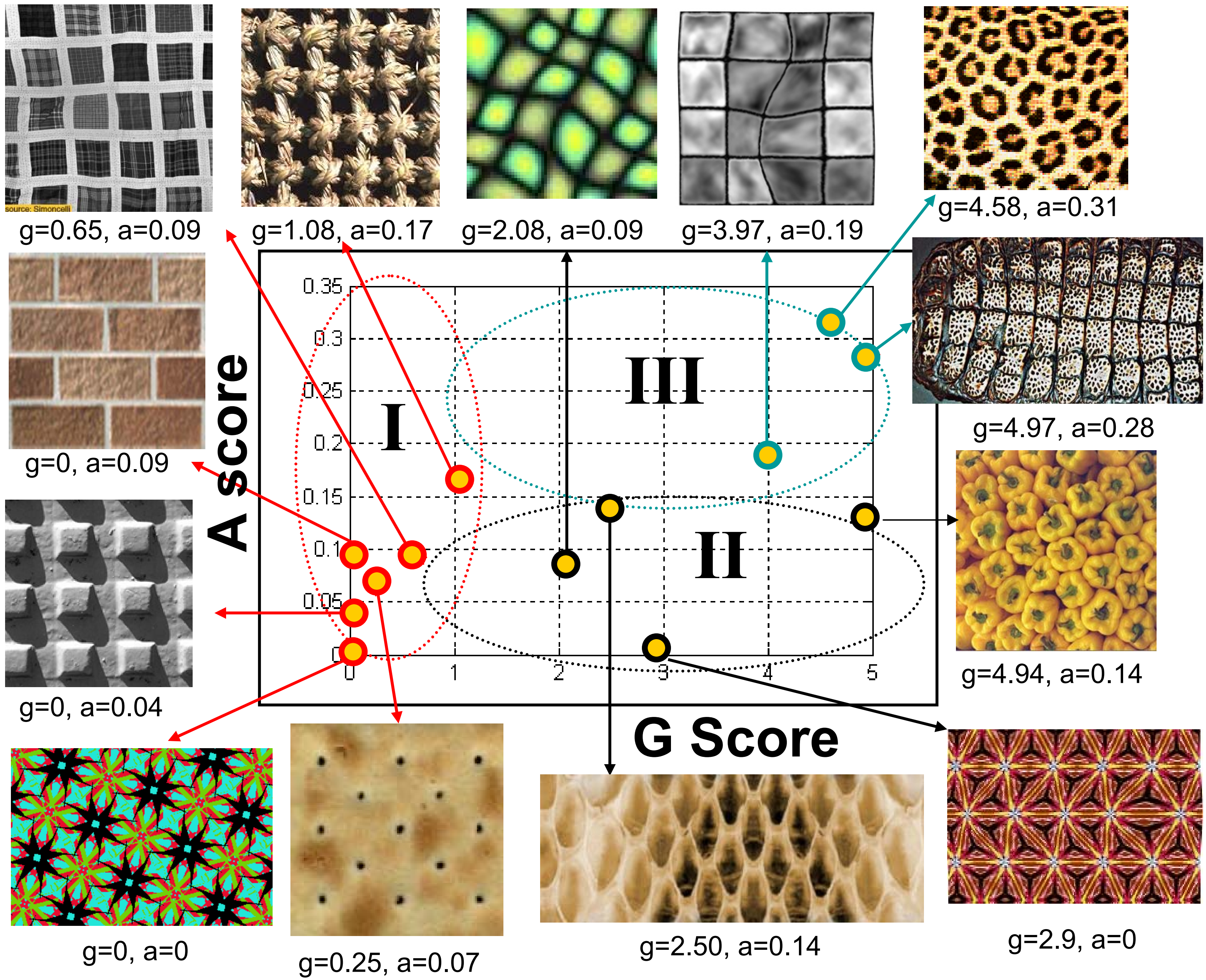
$$G = \sum_{i=1}^{N_i} \frac{(l_i - \|\vec{t_1}\|)^2}{\|\vec{t_1}\|^2} + \sum_{j=1}^{N_j} \frac{(l_j - \|\vec{t_2}\|)^2}{\|\vec{t_2}\|^2} + \sum_{k=1}^{N_k} \frac{(l_k - \|\vec{t_1} + \vec{t_2}\|)^2}{\|\vec{t_1} + \vec{t_2}\|^2} + \sum_{m=1}^{N_m} \frac{(l_m - \|\vec{t_1} - \vec{t_2}\|)^2}{\|\vec{t_1} - \vec{t_2}\|^2}$$

Appearance regularity: $A = \frac{1}{m}\sum_{i=1}^{m} std([T_1(i), T_2(i), ..., T_n(i)])$
where T_i is a reshaped tile (a column vector) of a Type I texture, m is the number of pixels within a tile, and n is the number of tiles.

Three Types of Near-Regular Textures

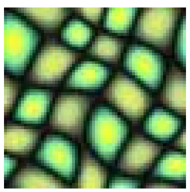
Quantified regularity



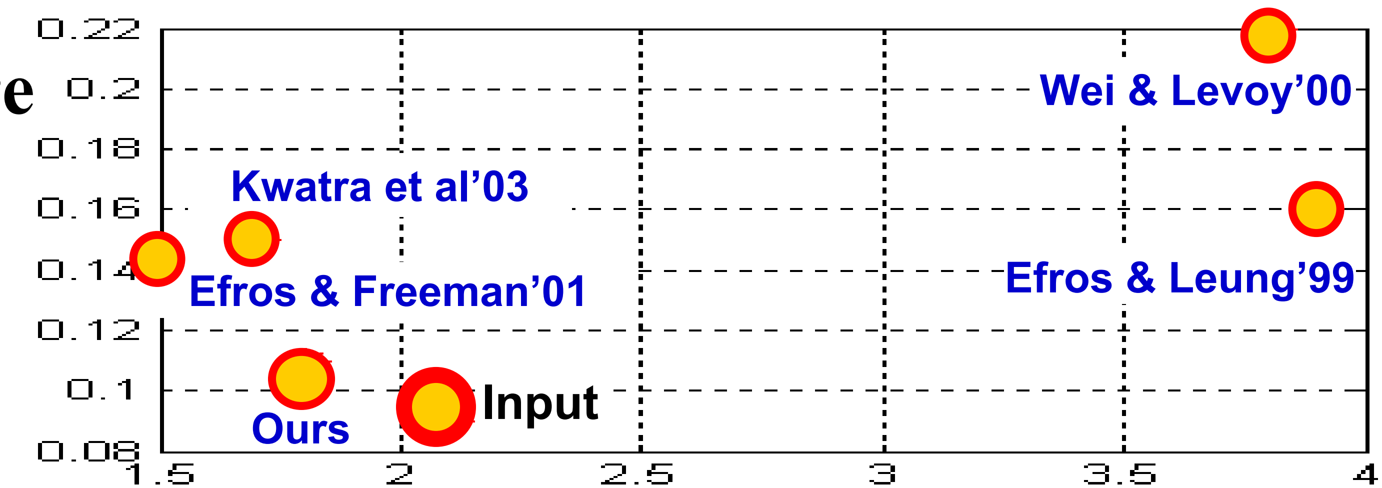


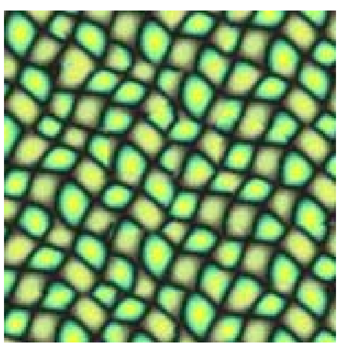
source: Simoncelli
g=0.65, a=0.09
g=1.08, a=0.17
g=2.08, a=0.09
g=3.97, a=0.19
g=4.58, a=0.31
g=0, a=0.09
g=4.97, a=0.28
A score
I
II
III
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
g=4.94, a=0.14
g=0, a=0.04
G Score
g=0, a=0
g=0.25, a=0.07
g=2.50, a=0.14
g=2.9, a=0

Now we can measure Texture Synthesis Results in G-A Space

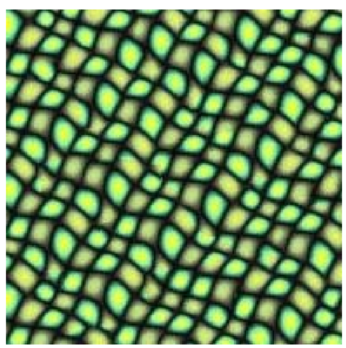


Input Texture
G=2.1, A=0.09

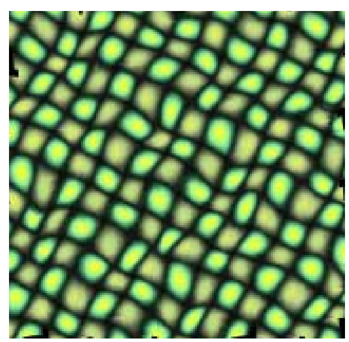




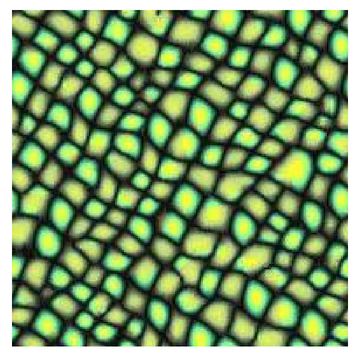
Efros & Freeman'01
G = 1.5, A = 0.14



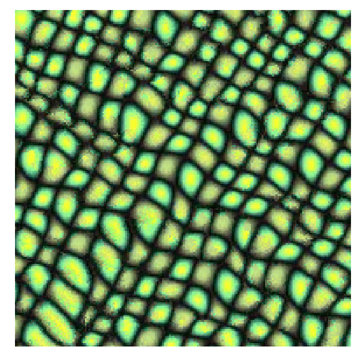
Kwatra et al'03
G = 1.7, A=0.15



Ours
G = 1.8, A = 0.11



Wei & Levoy'00
G = 3.8, A= 0.22



Efros & Leung;99
G = 3.9, A = 0.16

Firing Fields of Grid Cells in rats brains

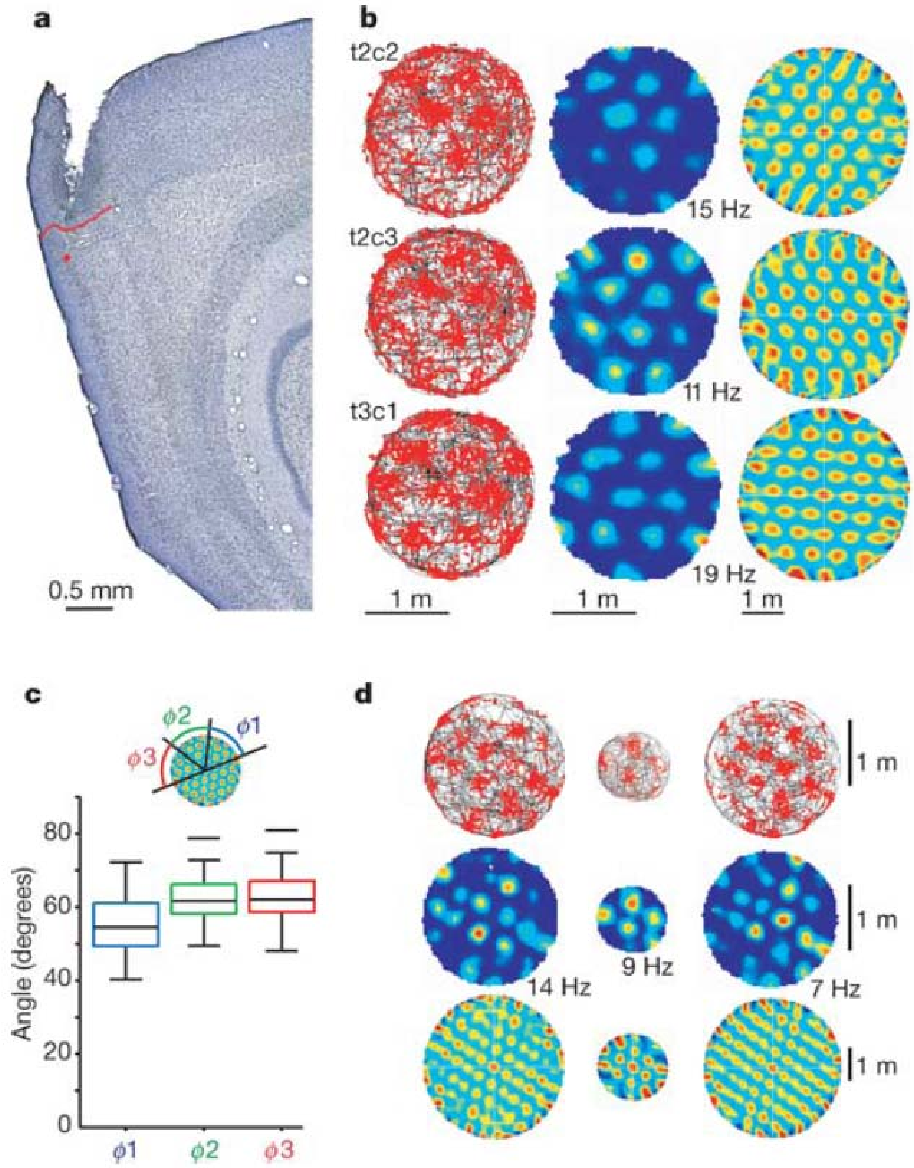


Figure 1 | Firing fields of grid cells have a repetitive triangular structure.

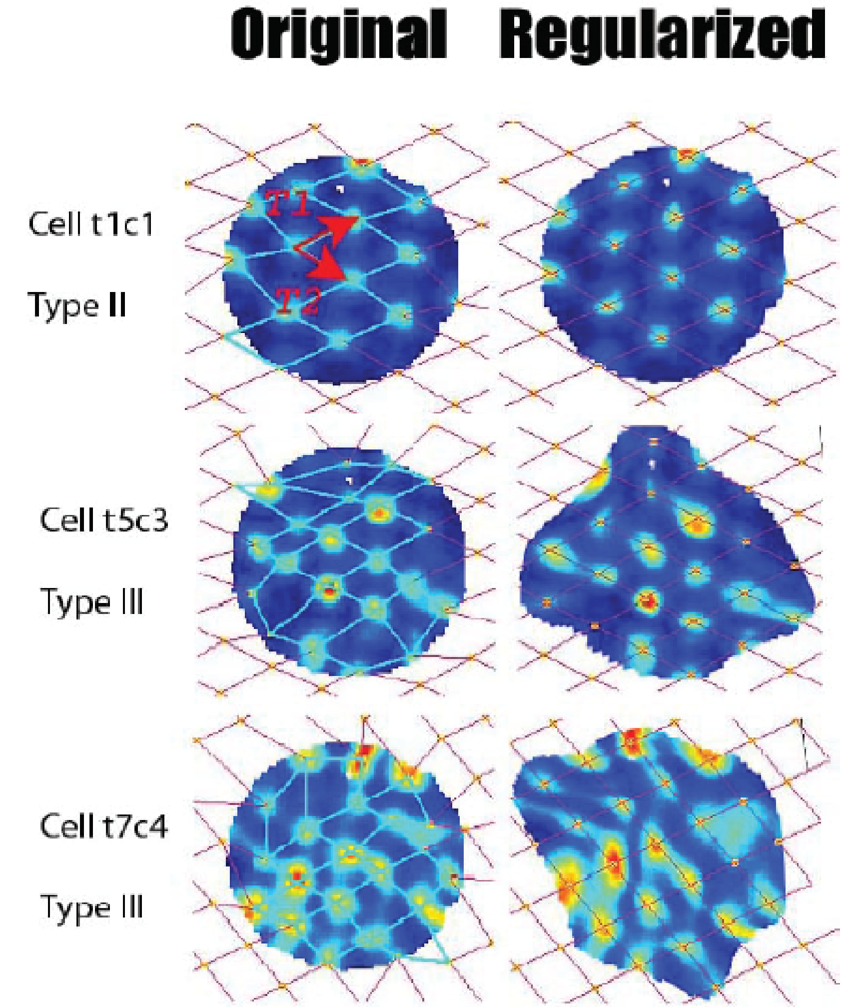
Nature **436**, 801-806 (11 August 2005) doi:10.1038/nature03721

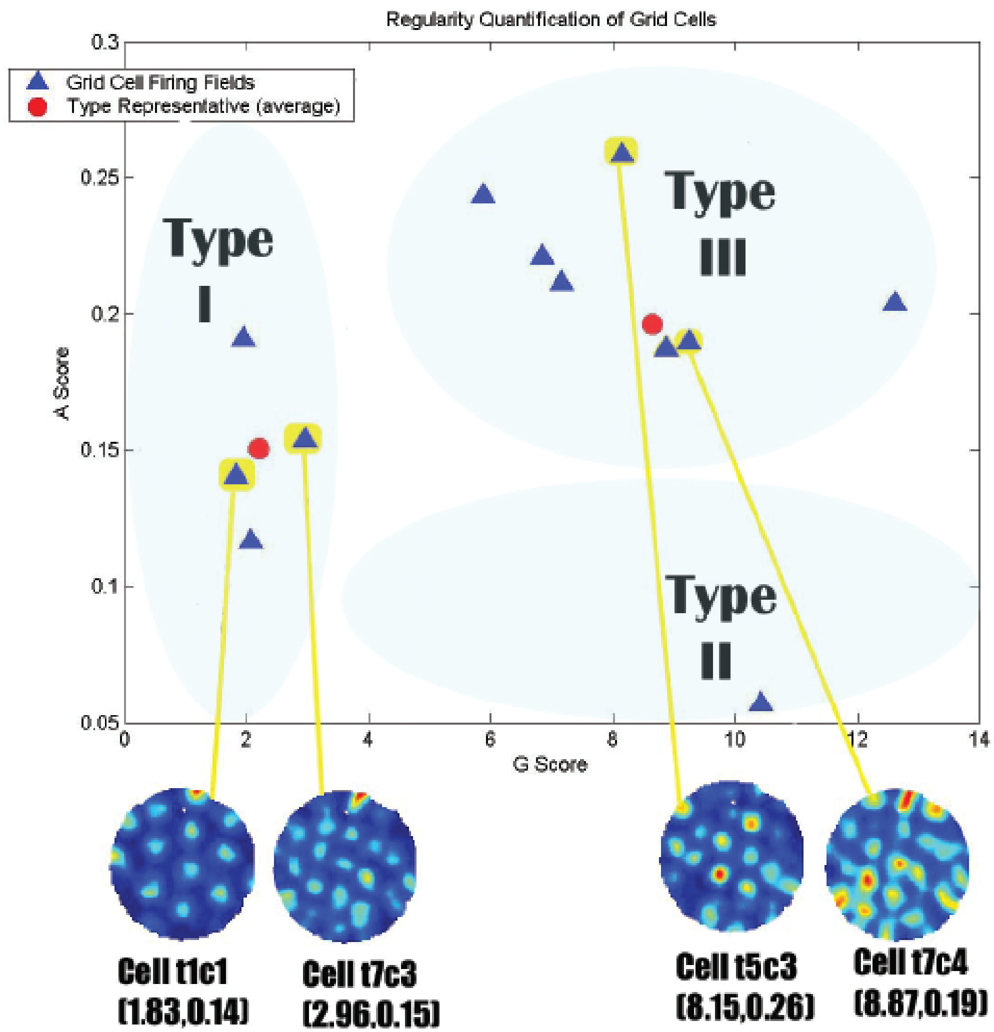
Microstructure of a spatial map in the entorhinal cortex

Torkel Hafting, Marianne Fyhn, Sturla Molden, May-Britt Moser and Edvard I. Moser

Quantified Symmetry for Entorhinal Spatial Maps

Chastain and Liu, Special Issue in Neurocomputing Journal, Vol. 70, No. 10 - 12, June, 2007, pp. 1723 - 1727

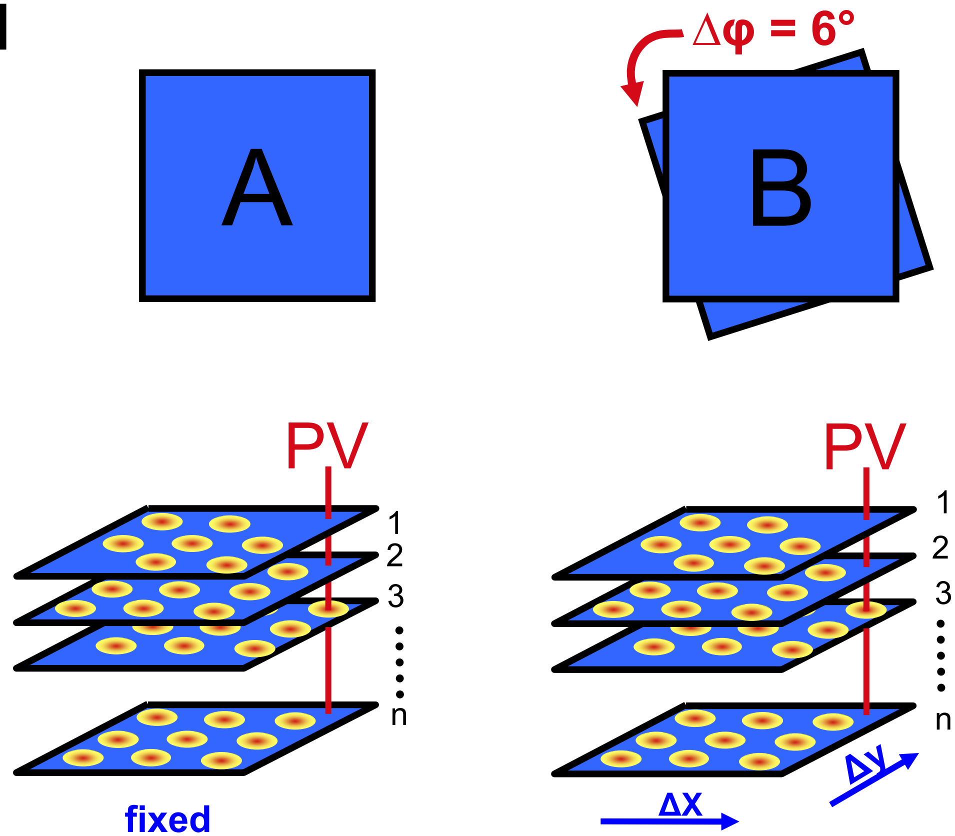




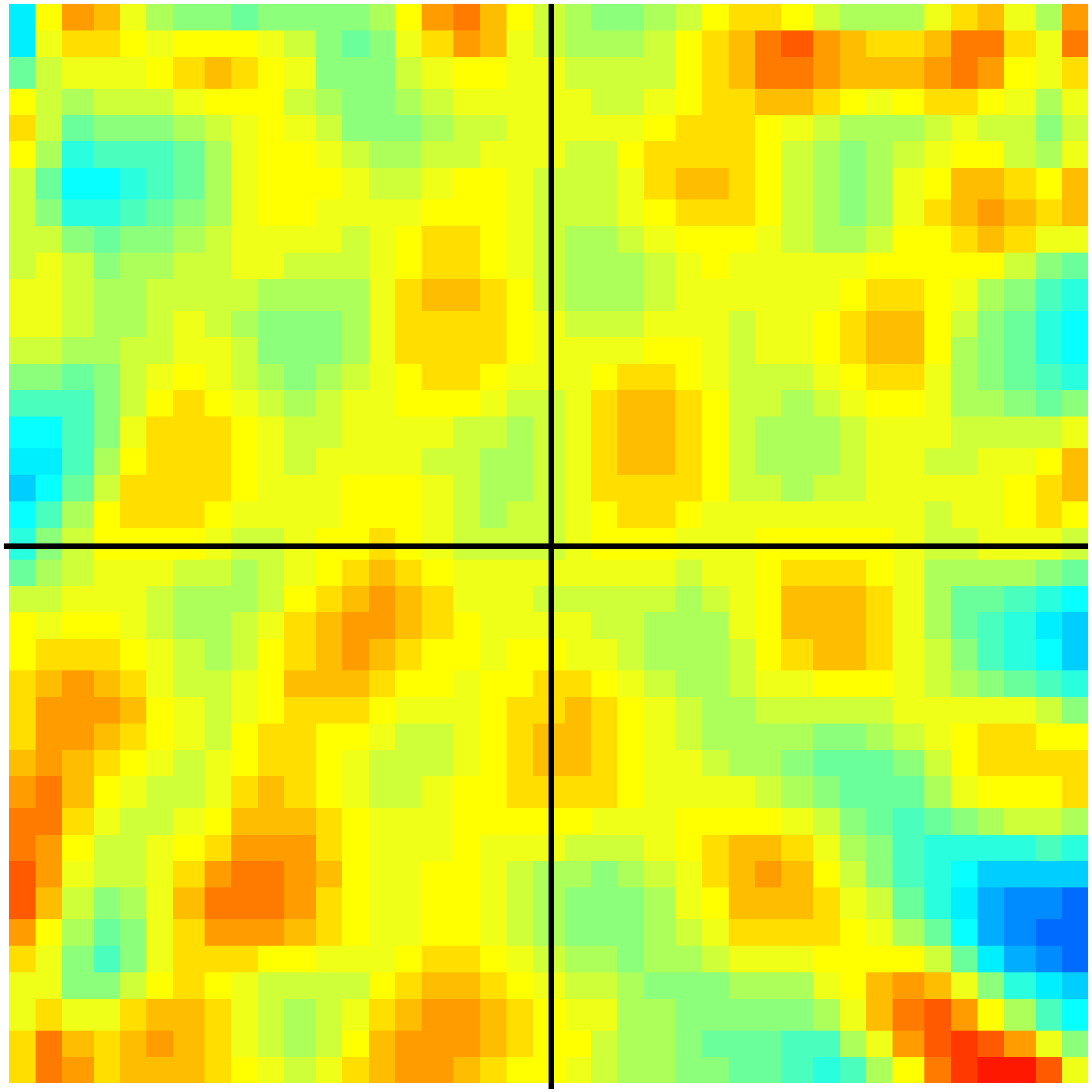
Comments from Reviewers of Neurocomputing Journal

- This is a mathematical quantification of the newly discovered entorhinal grid cells. (R1)
- This is a very interesting quantification method. It will be **useful in our understanding of grid cells**. (R2)
- **Very appropriate for Computational Neuroscience**. The work is sound and the conclusions interesting. (R3)

Coherence between grid maps in two rooms was observed only after relative rotation of the grid maps



PV(A) X PV(B)Δφ



Fyhn, Hafting et al., unpublished

Thus, the intrinsic structure of the map (spacing, orientation, spatial phase) is retained,

i.e. a single map may be applied rigidly in all environments

Without deviation from the norm progress is not possible. In order for one to ``deviate successfully'', one has to have at least a passing acquaintance with whatever norm one expects to deviate from.

--- Frank Zappa, "The Real Frank Zappa Book"

Back from deviations …

A detour to the land of symmetry…

Regularity → Symmetry → Symmetry Groups

Y. Liu ``Computational Symmetry'', *Symmetry-2000*, Wenner-Gren International Series, vol 80 Part I, pp. 231-245. Portland Press, London (ISBN I 85578 149 2). January 2002.

Definition

A **group** G is a set of elements with a binary operation * defined on the set that satisfy:

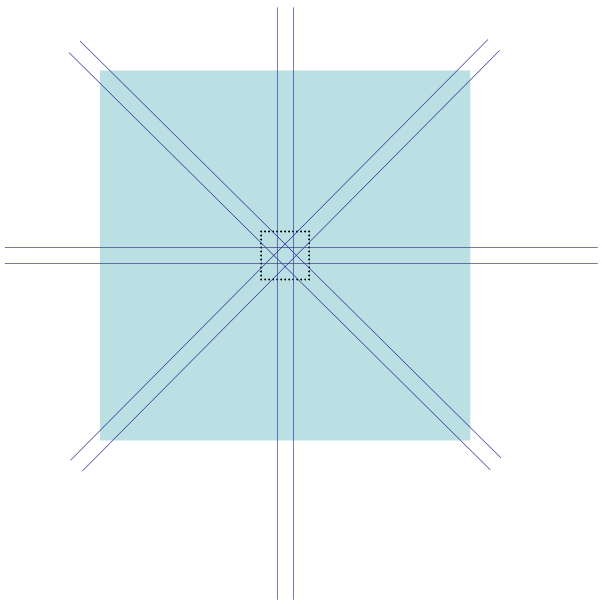
- * is associative a*(b*c) = (a*b)*c
- any element in G has an inverse
- there is an identity element id in G, id*a = a

An example:

G = symmetries of a square

* = transformation composition

a square plate in R(2)

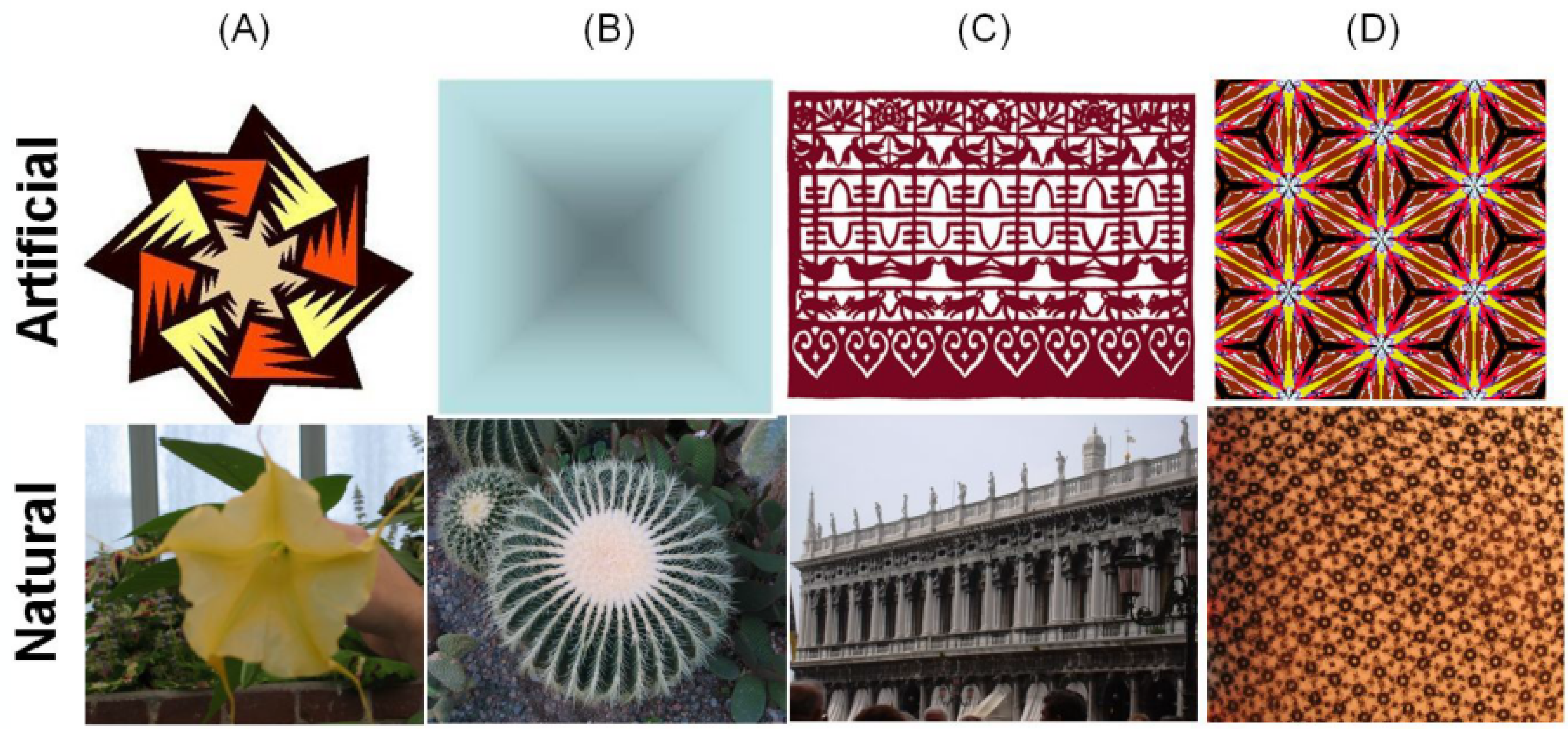


Symmetry Group

All symmetries of a subset S of Euclidean space R^n have a group structure G, and G is called the **symmetry group** of S.

TYPES of
SYMMETRY GROUPS G
in Euclidean Space

2D Euclidean Space



Cyclic Symmetry Group (rotation)

Dihedral Symmetry Group (rotation+ reflection)

Frieze symmetry Group (translation + reflection)

Wallpaper symmetry Group (translations + rotation + Reflection + glide-reflection)

E: Euclidean group

E^+: the proper Euclidean group

O: Orthogonal group

SO: Special Orthogonal group

T: Translation group

T_{dis} : Discrete Translation group

D: Dihedral group

C: Cyclic group

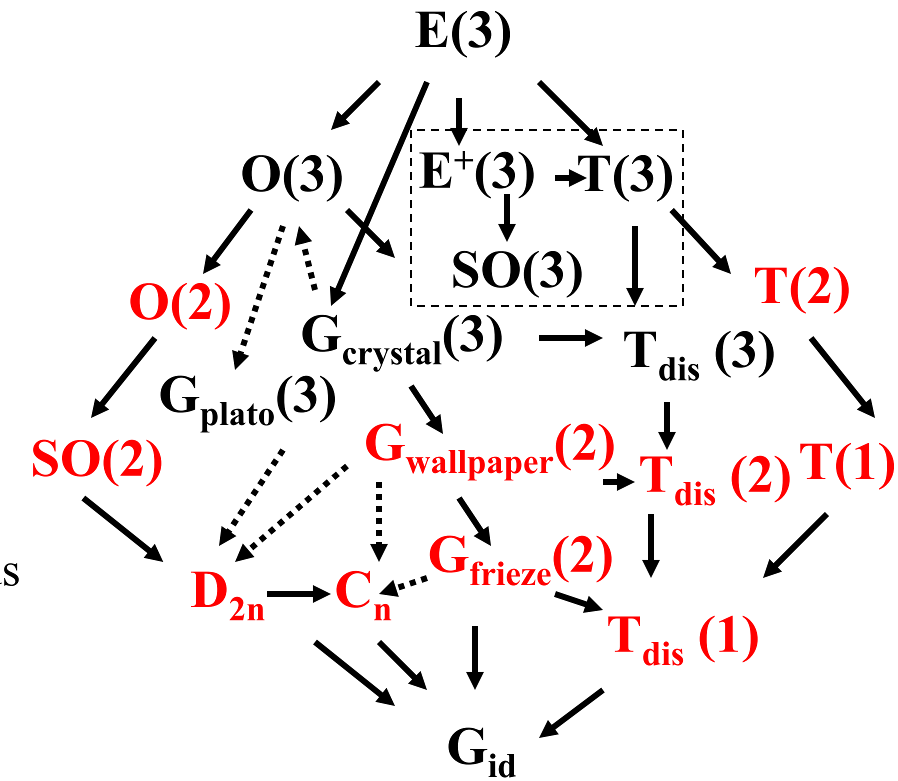
$G_{crystal}(3)$: Crystallographic groups in 3D

$G_{wallpaper}$ (2): wallpaper groups

$G_{frieze}(2)$: frieze groups

G_{plato} → Symmetry groups for Platonic solids

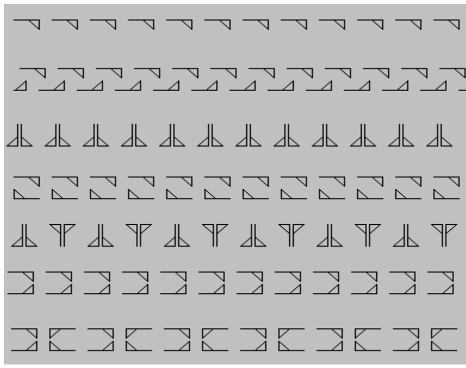
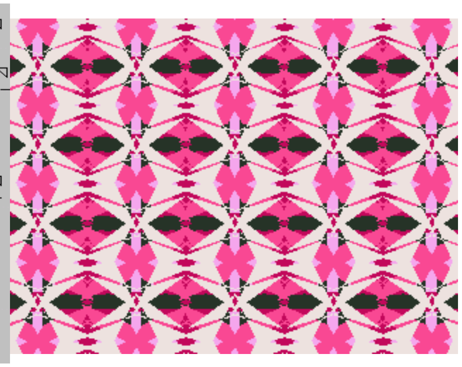
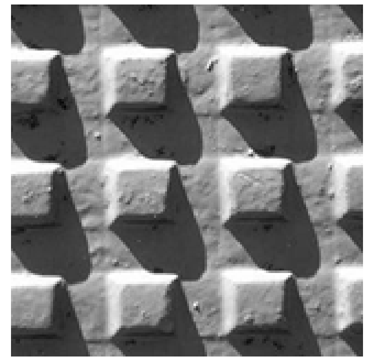
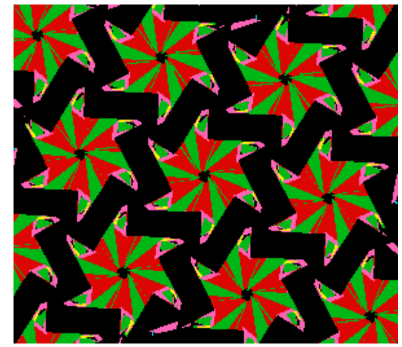
G_{id} → The identity group



A → B means: B is a subgroup of A

A - -▸ B means: B is a subgroup of A for some n

Hilbert's 18th Problem

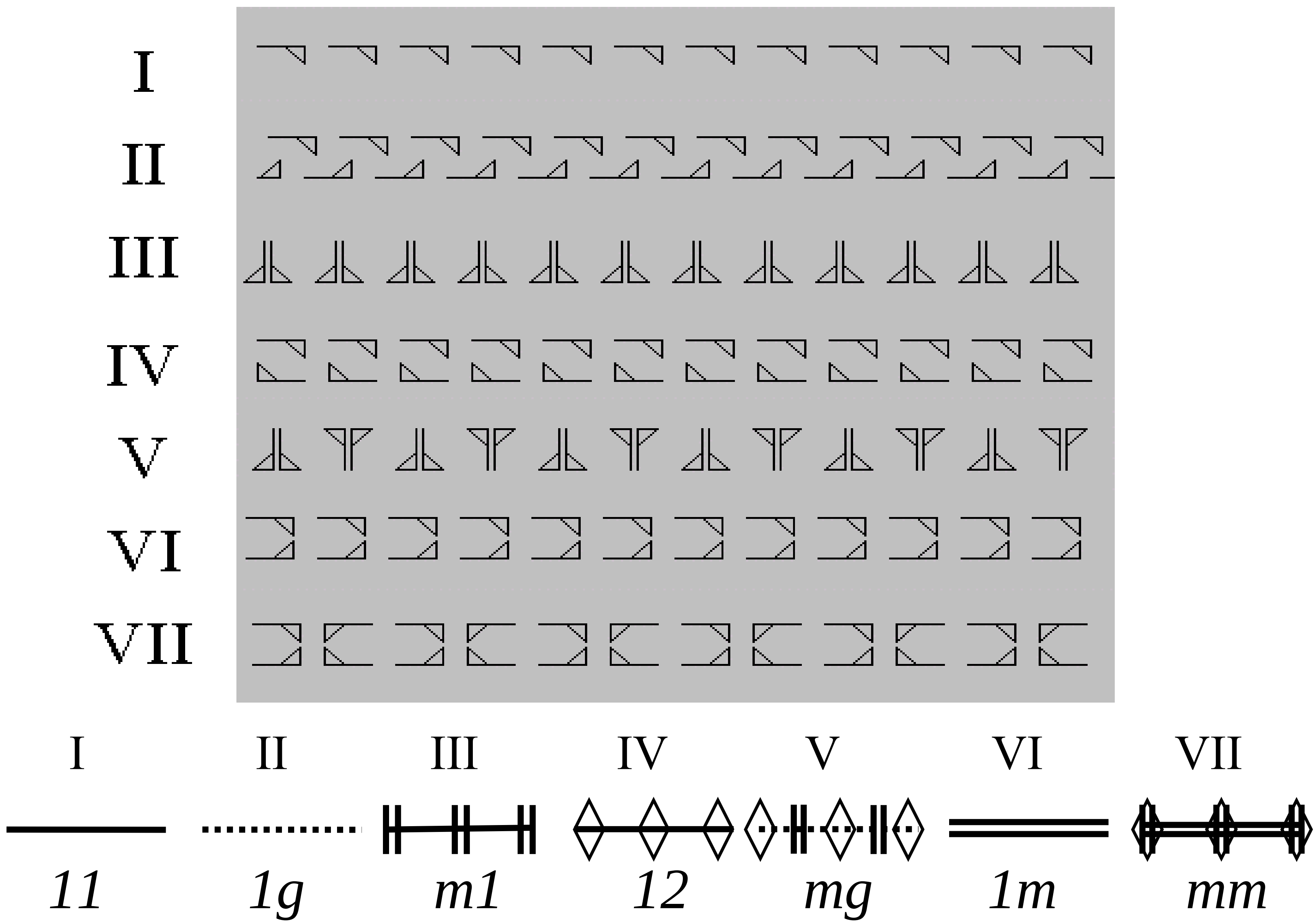
Question:
In n-dimensional euclidean space is there … only a finite number of **essentially different** kinds of ***(symmetry) groups*** of motions with a **fundamental region**?

Answer:
Yes! (Bieberbach and Frobenius 1910-1912)

Frieze Symmetry Groups

or **1D crystallographic groups**

Examples and Notations of the Seven Frieze Groups



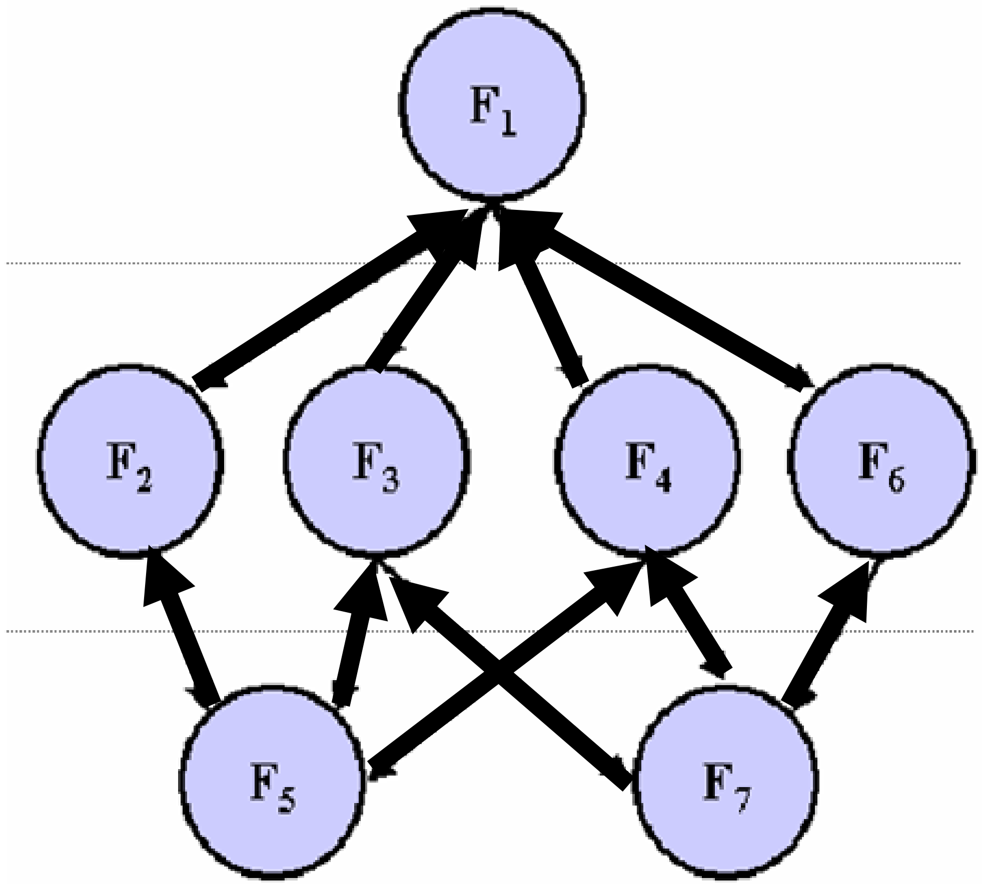
Inner Structure

Table 1. Symmetries of frieze pattern

Symmetry Group	translation	2-fold rotation	Horizontal reflection	Vertical reflection	Glide reflection
F1	yes	no	no	no	no
F2	yes	no	no	no	yes
F3	yes	no	no	yes	no
F4	yes	yes	no	no	no
F5	yes	yes	no	yes	yes
F6	yes	no	yes	no	no
F7	yes	yes	yes	yes	no

Inter Structure

Hierarchy of Frieze Groups



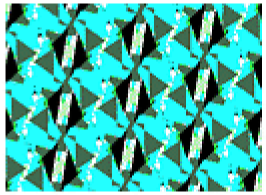
Wallpaper Symmetry Groups

or **2D crystallographic groups**

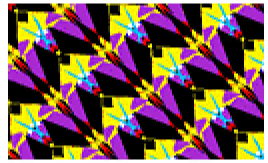
Examples of 17 Wallpaper Patterns

p1

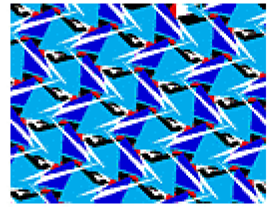
p2



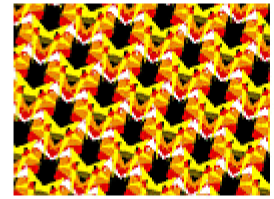
pm



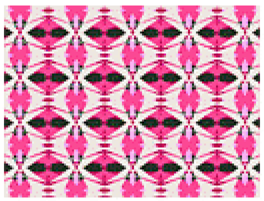
pg



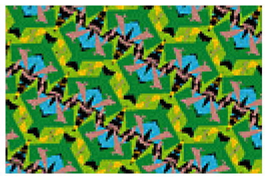
cm



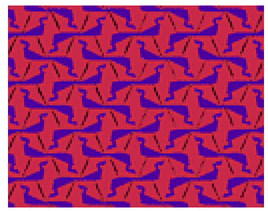
pmm



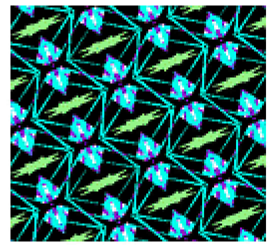
pmg



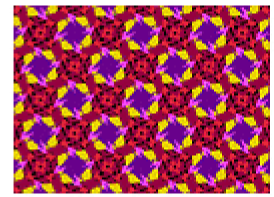
pgg



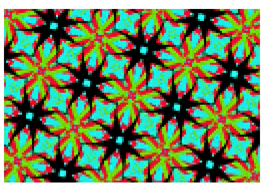
cmm



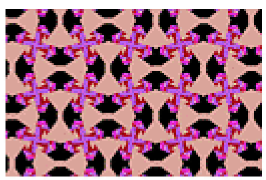
p4



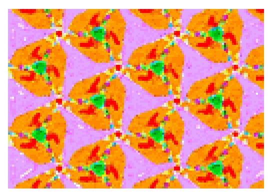
p4m



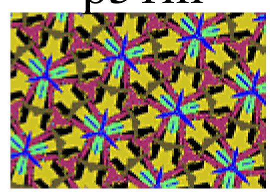
p4g



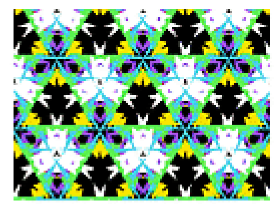
p3



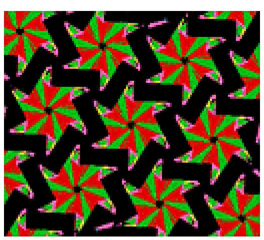
p31m



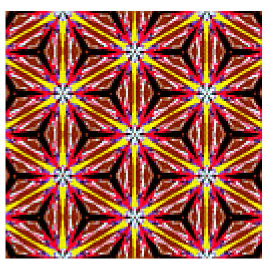
p3m1



p6



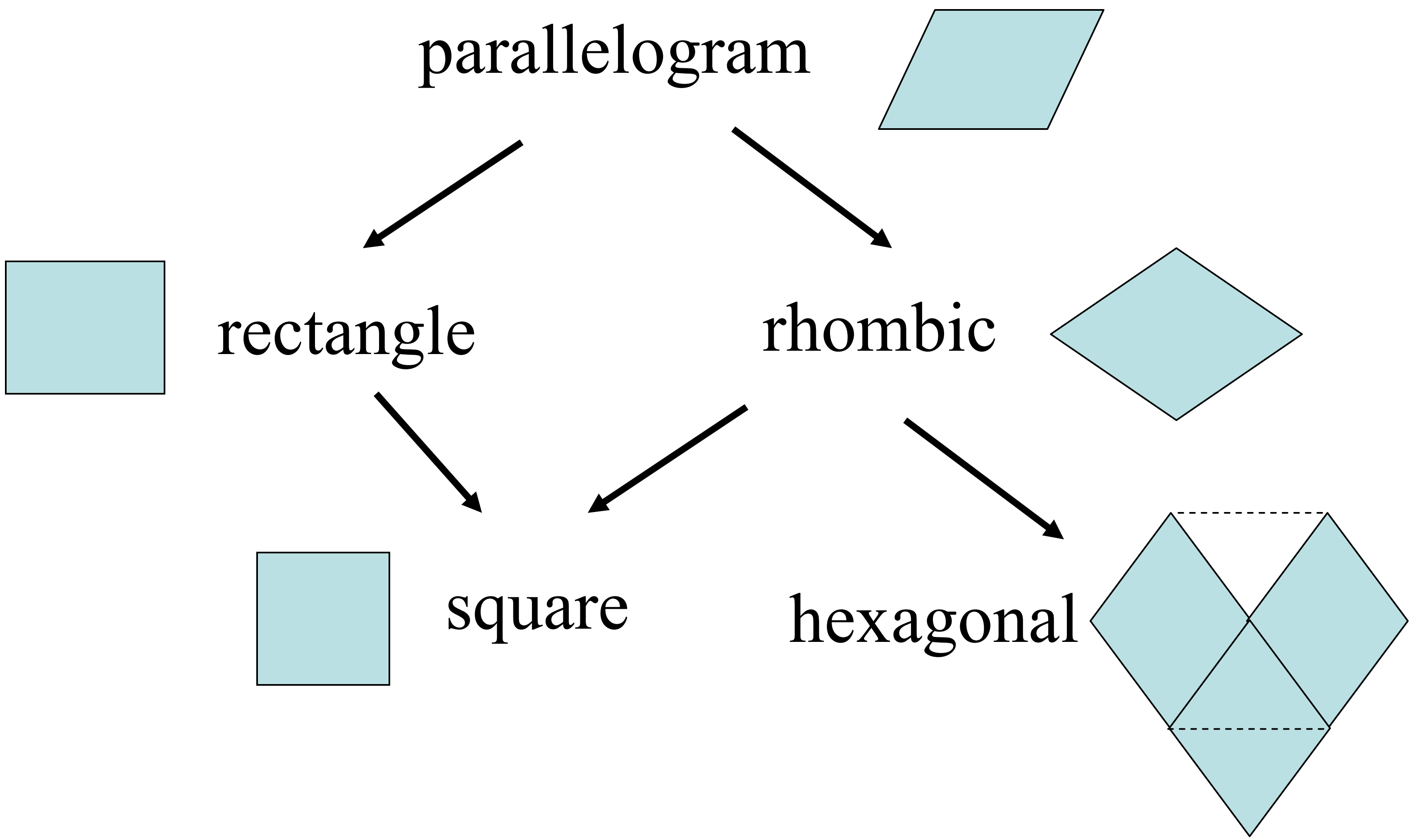
p6m



From a web page by:
David Joyce, Clark Univ.

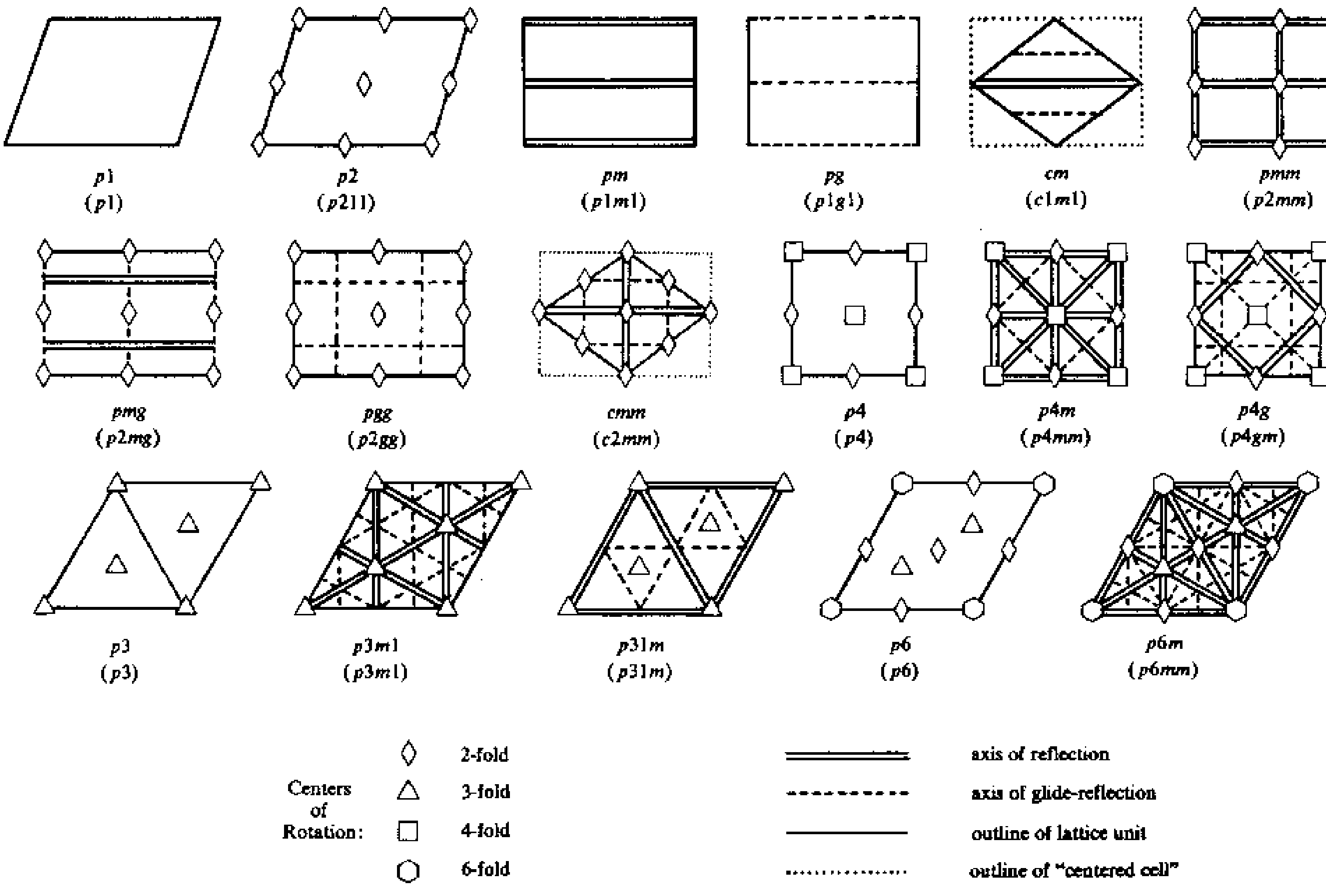
Types of Possible Lattices

formed by the two shortest vectors



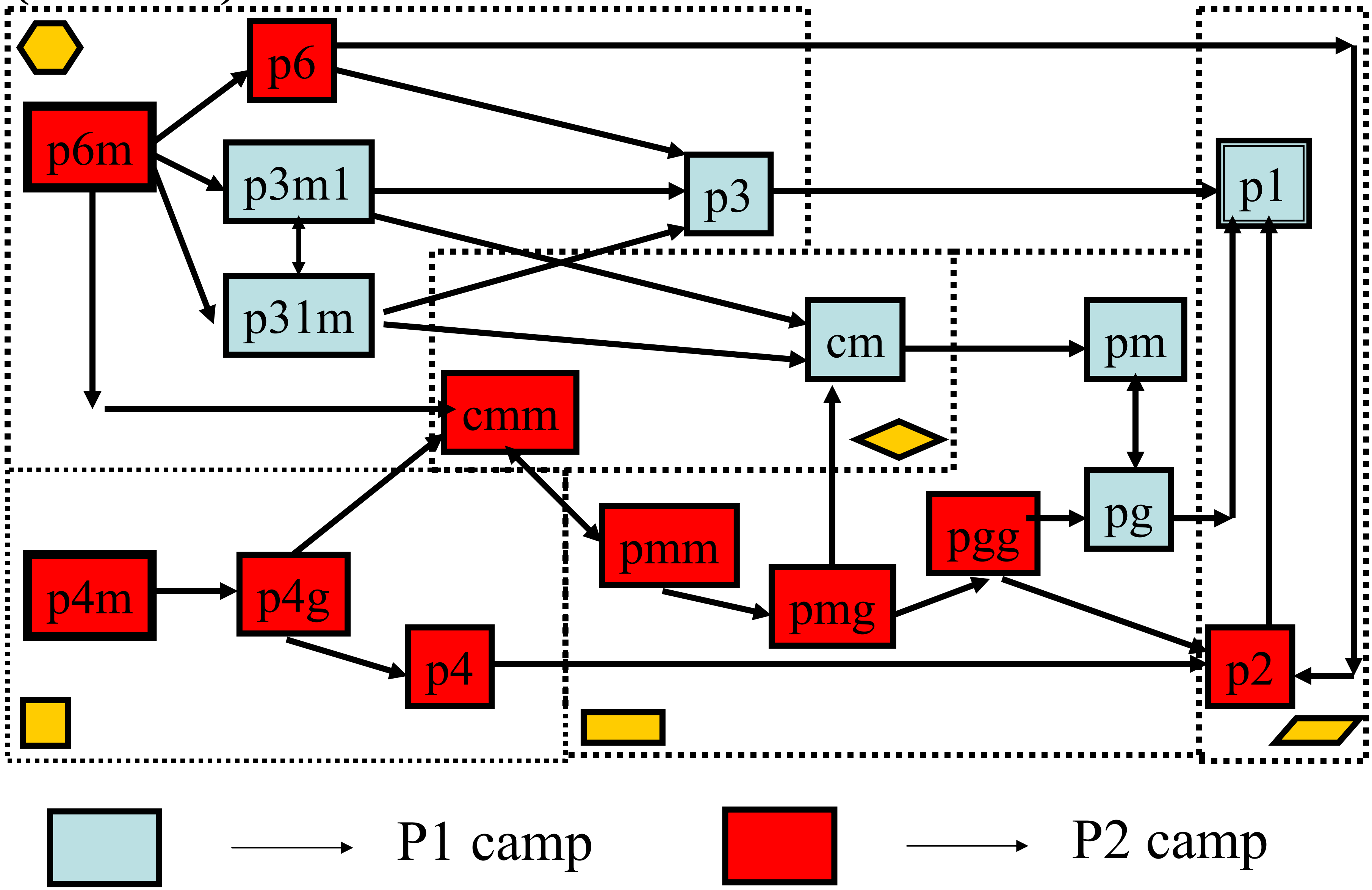
Inner Structure

Lattice Units Notation of the 17 Wallpaper Groups



Inter Structure

Subgroup Relationship Among the 17 Wallpaper Groups (Coxeter)

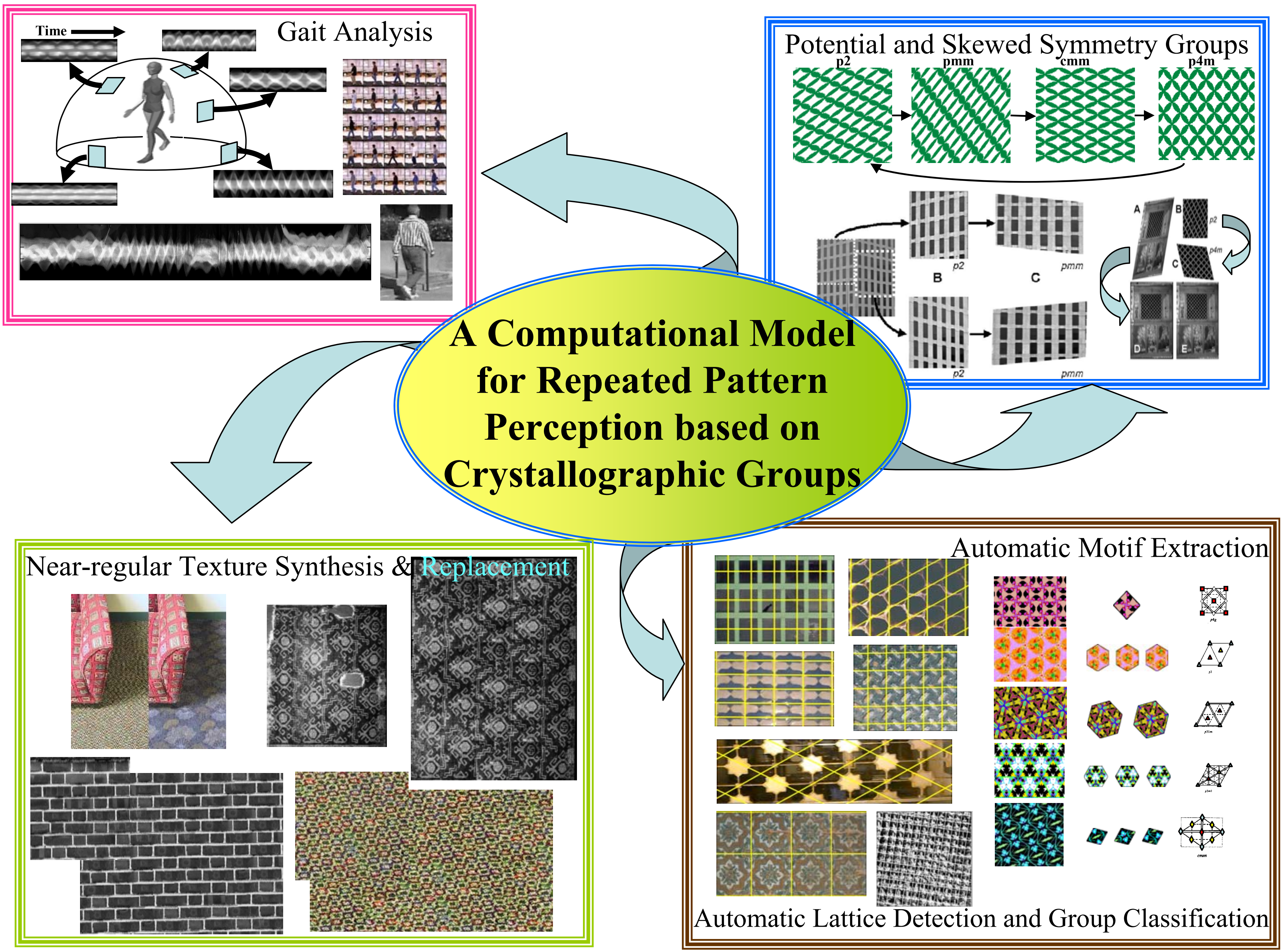


A Computational Model for Periodic Pattern Perception

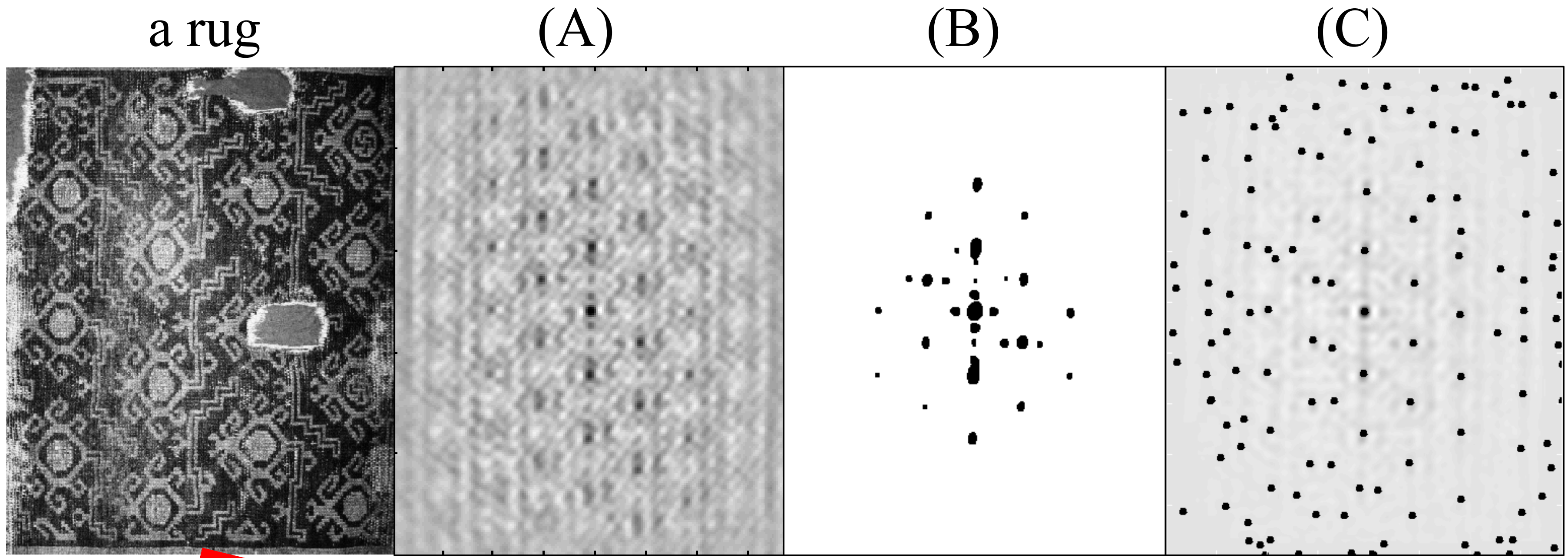
•Liu, Y., Collins,R.T. and Tsin, Y. ``A Computational Model for Periodic Pattern Perception Based on Frieze and Wallpaper Groups", IEEE TPAMI. Vol. 26, No. 3, March, 2004, pp. 354 – 371.
•Liu, Y. and Collins,R.T., ``A Computational Model for Periodic Pattern Perception based on Frieze and Wallpaper Groups", *Computer Vision and Pattern Recognition Conference* 2000 (CVPR'00), Hilton Head, 2000.

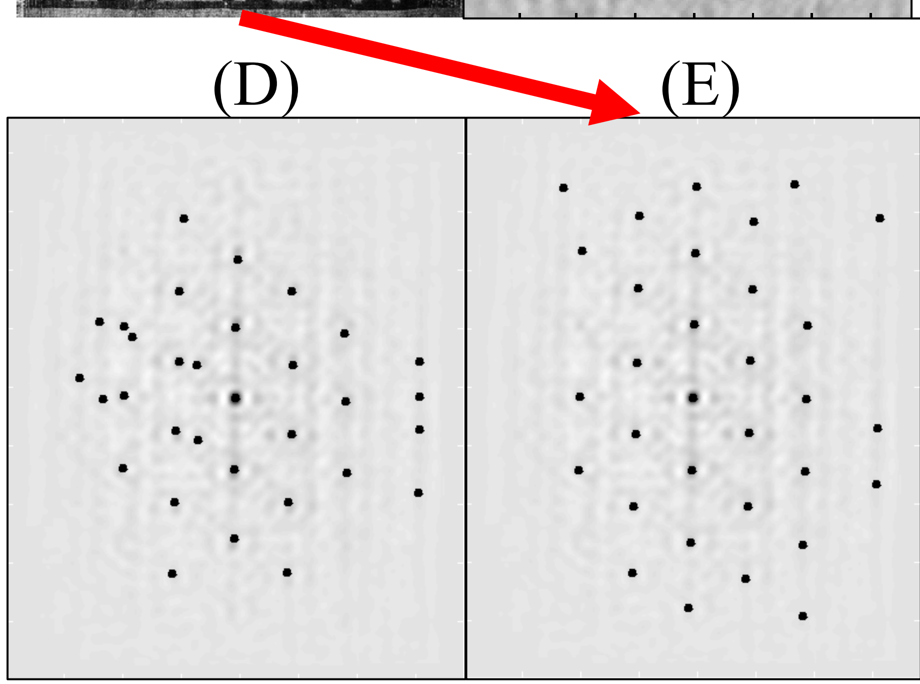
A Computational Model for Periodic Pattern Perception

- lattice detection
- symmetry group classification
- motif(s) extraction
- symmetry group migration under affine deformation
- applications
    - gaits analysis
    - image indexing
    - image compression
    - texture synthesis
    - texture replacement in real images


A Computational Model for Repeated Pattern Perception based on Crystallographic Groups
Gait Analysis
Time
Potential and Skewed Symmetry Groups
p2
pmm
cmm
p4m
Near-regular Texture Synthesis & Replacement
Automatic Motif Extraction
Automatic Lattice Detection and Group Classification

An Example of Lattice Extraction from Images





(A) its autocorrelation surface

(B) peaks found by global thresholding

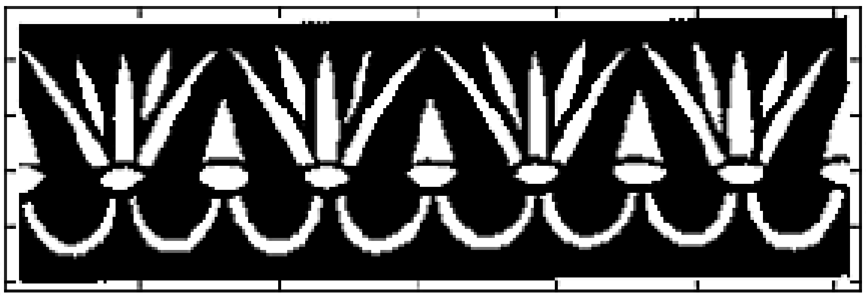
(C) peaks found by Lin et al's method

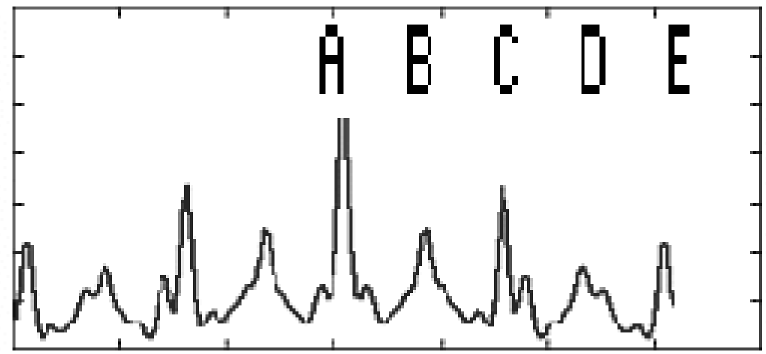
(D) the highest 32 peaks from (C)

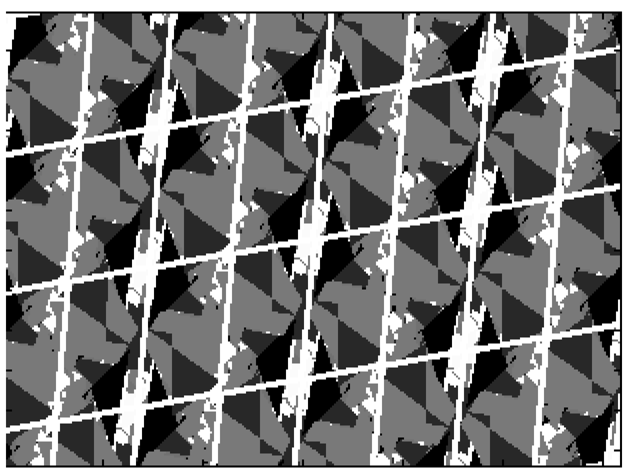
(E) the 32 most-dominant peaks by our method

Translational Lattice Extraction

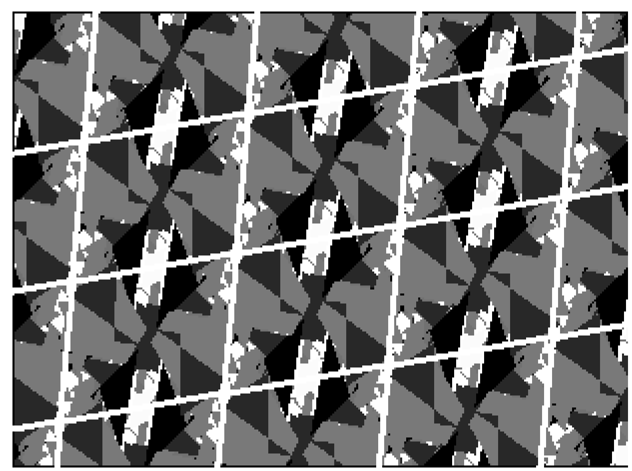
Region of dominance







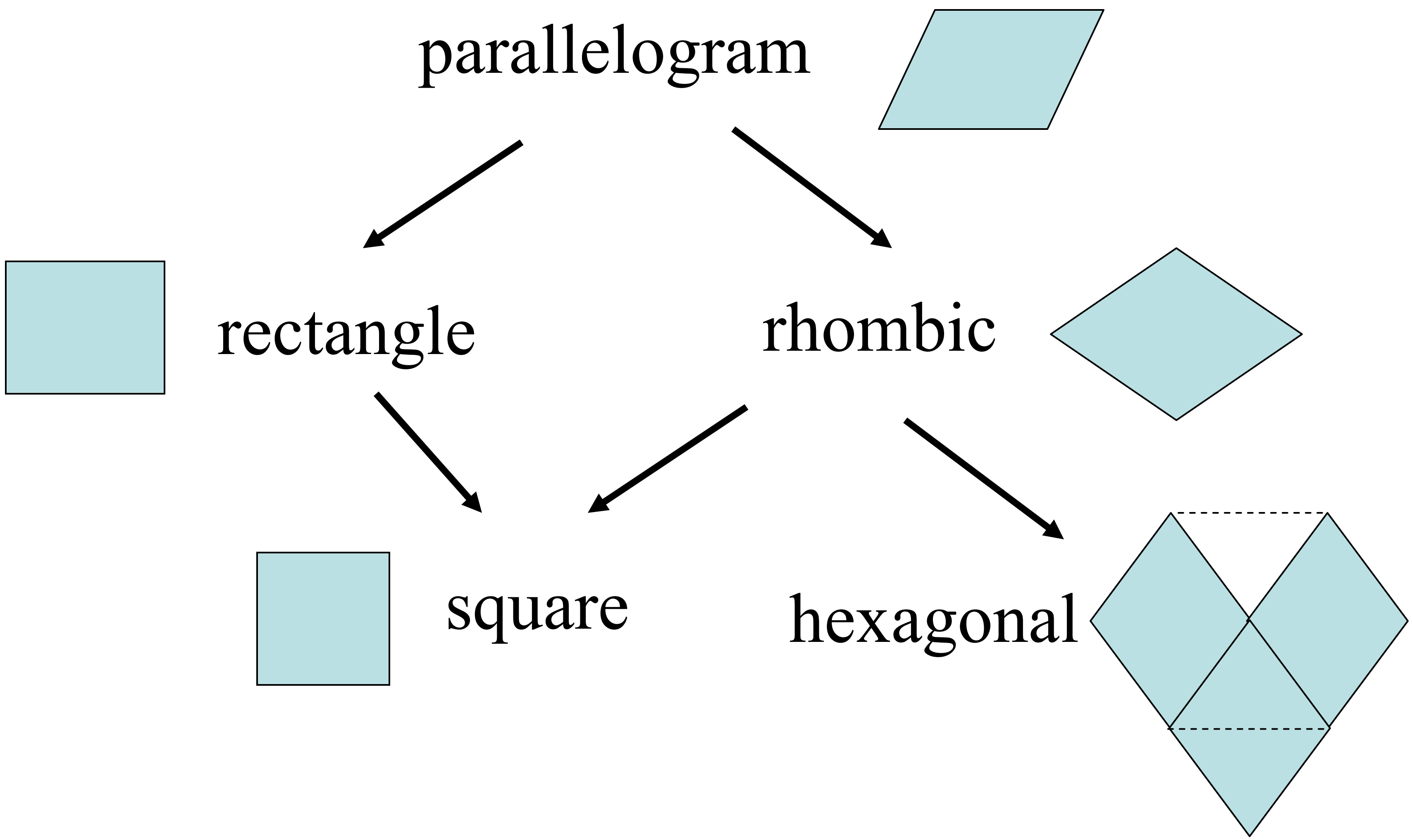
Lin et al's method



Our method

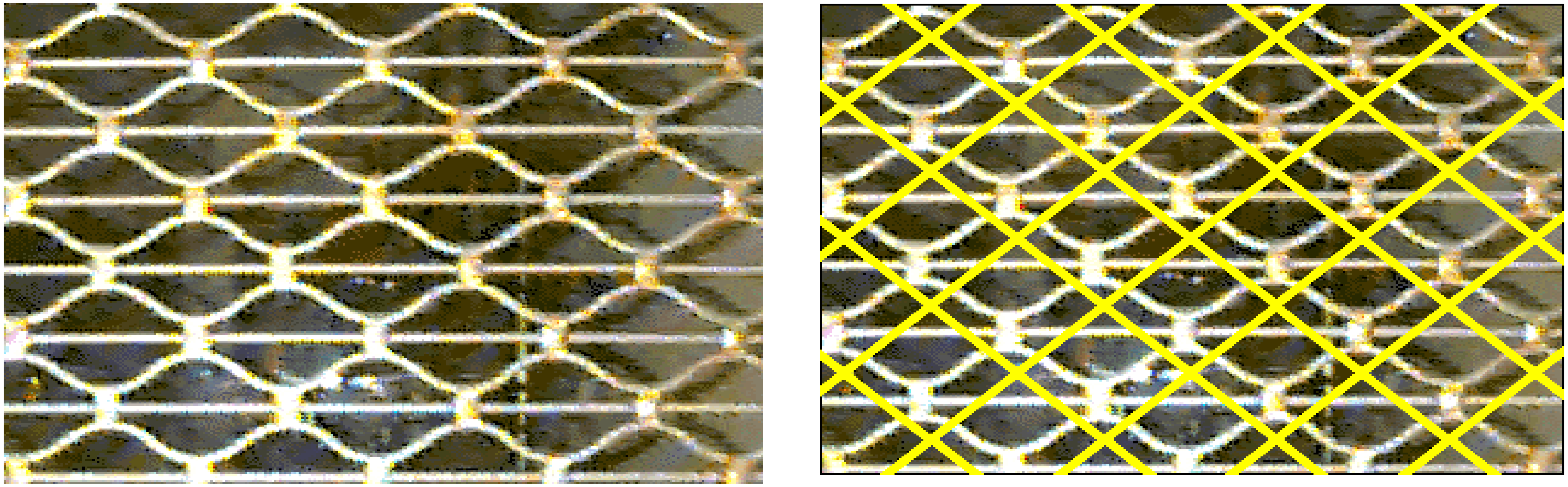
Types of Possible Lattices

formed by the two shortest vectors

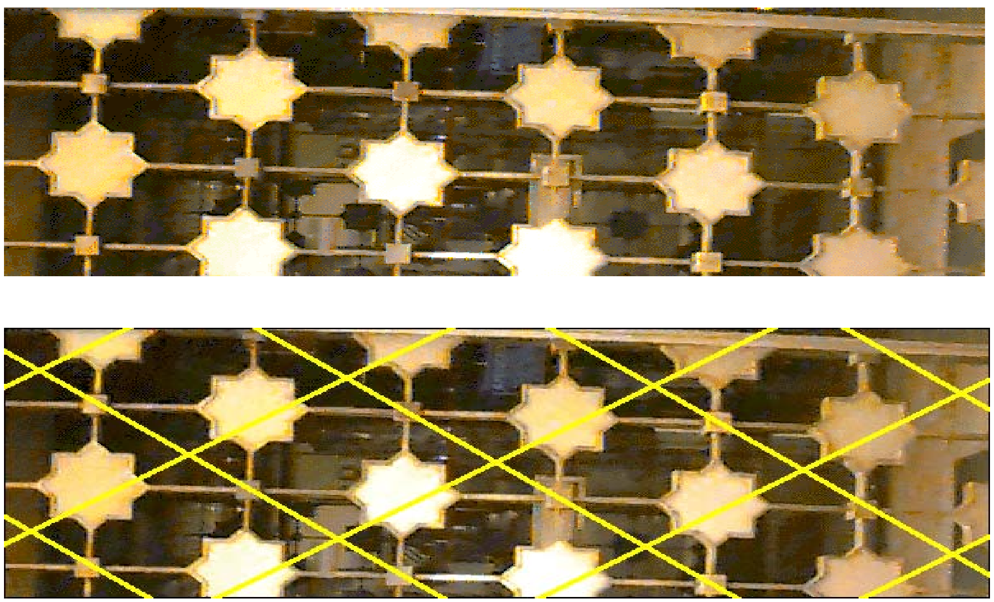


Wallpaper Group Classification Algorithm

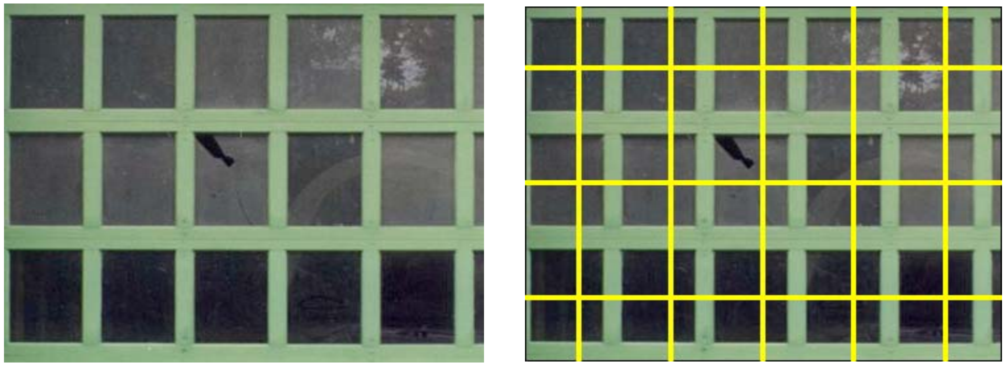
	p1	p2	pm	pg	cm	pmm	pmg	pgg	cmm	p4	p4m	p4g	p3	p3m1	p31m	p6	p6m
180		Y				Y	Y	Y	Y	Y	Y	Y				Y	Y
120													Y	Y	Y	Y	Y
90										Y	Y	Y					
60																Y	Y
T_1			Y	Y(g)		Y	Y(g)	Y(g)			Y	Y(g)			Y		Y
T_2						Y	Y	Y(g)			Y	Y(g)			Y		Y
D_1					Y				Y		Y	Y		Y	Y		Y
D_2									Y		Y	Y					Y



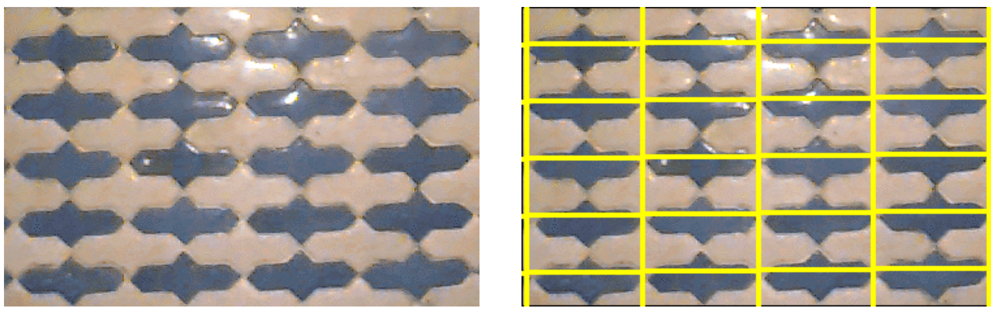
cmm



cmm

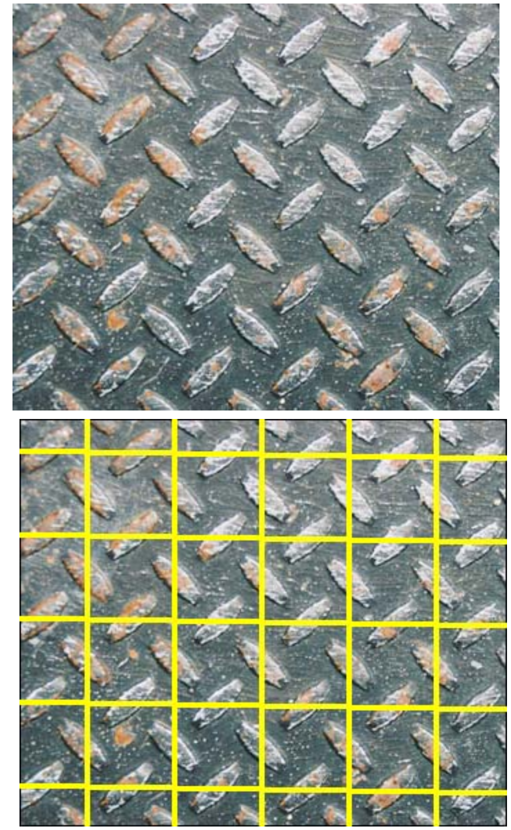


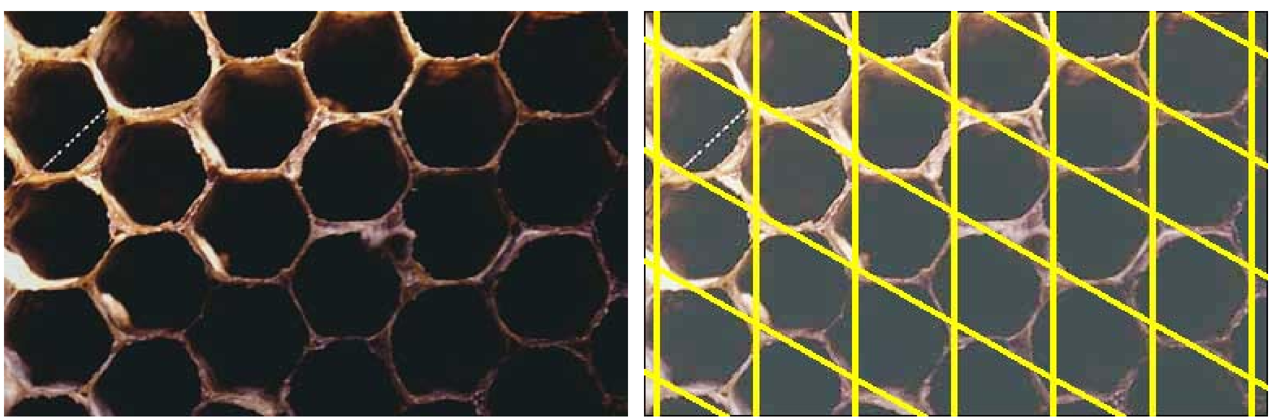
pmm



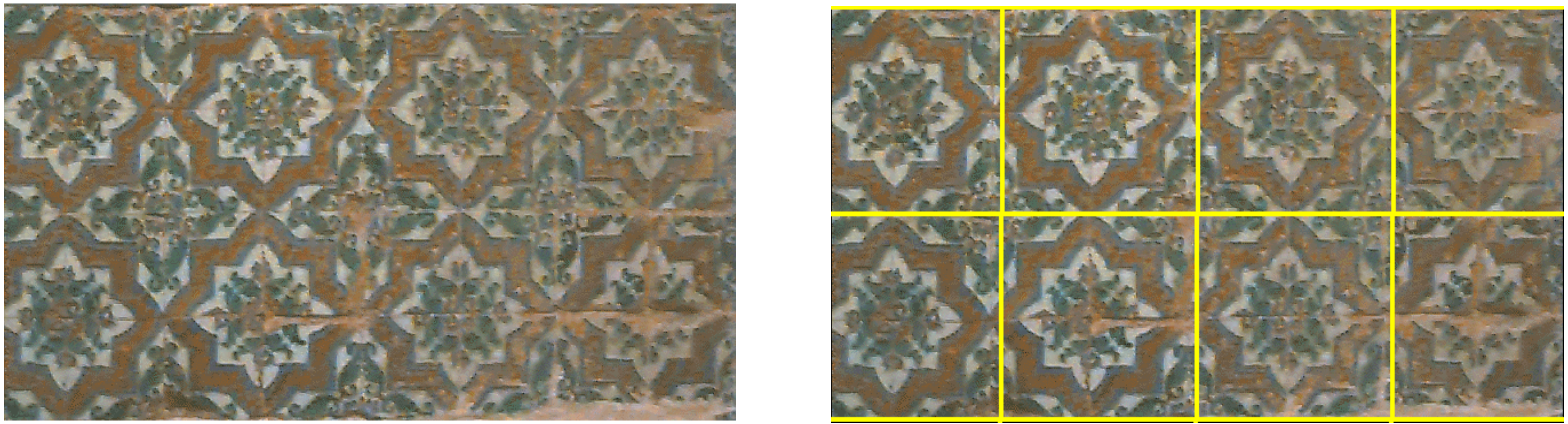
pmm

p4g

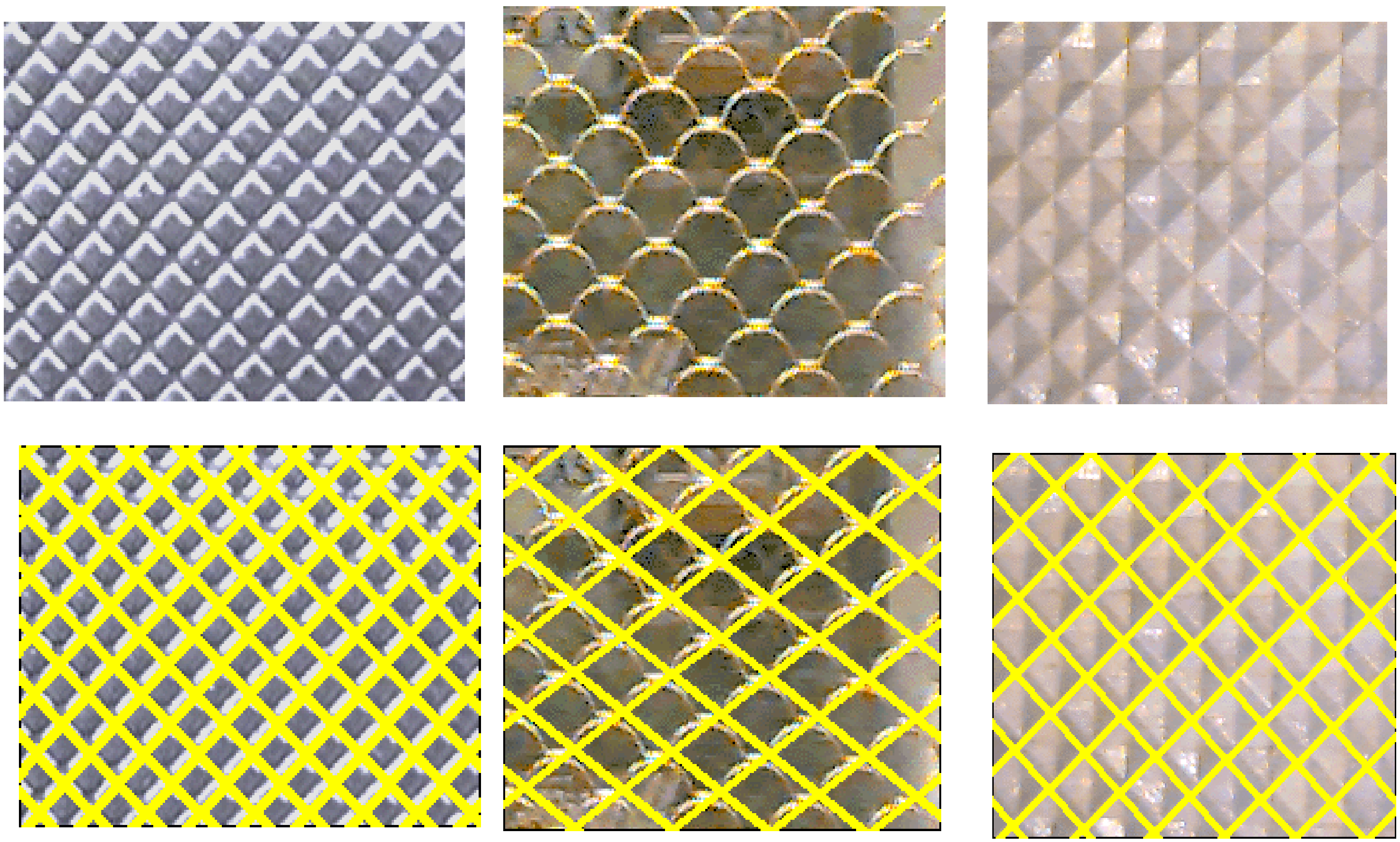




p6m

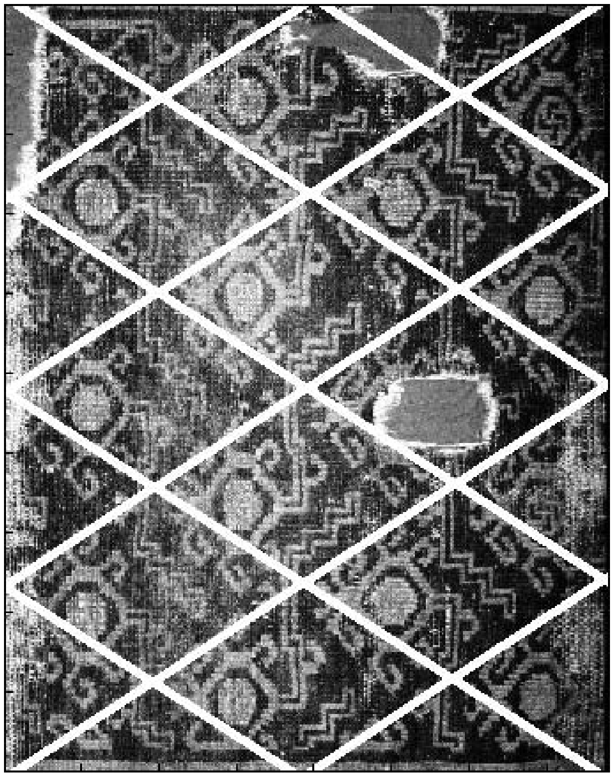


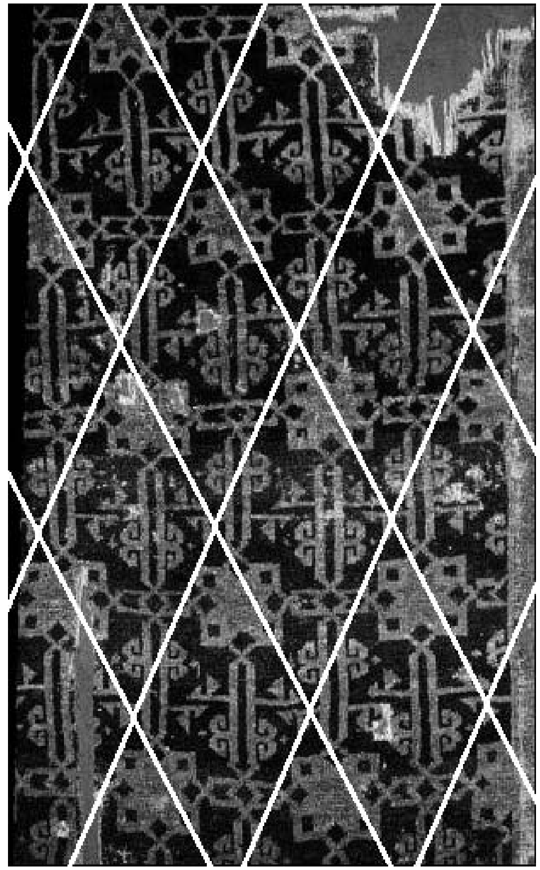
p4m

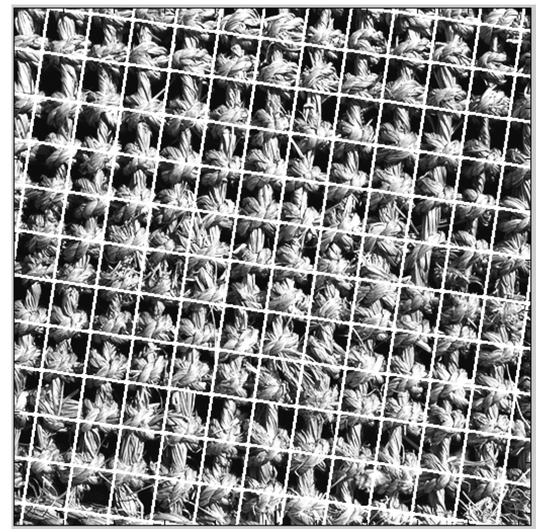


cm

Patches/windows differ in shape, size, and orientations:







Automatically Detected Tiles (patches) from Near-Regular Texture Patterns

(Liu/Collins CVPR 2000, Liu, Collins and Tsin PAMI 2003)

Machine Perception of Periodic Patterns

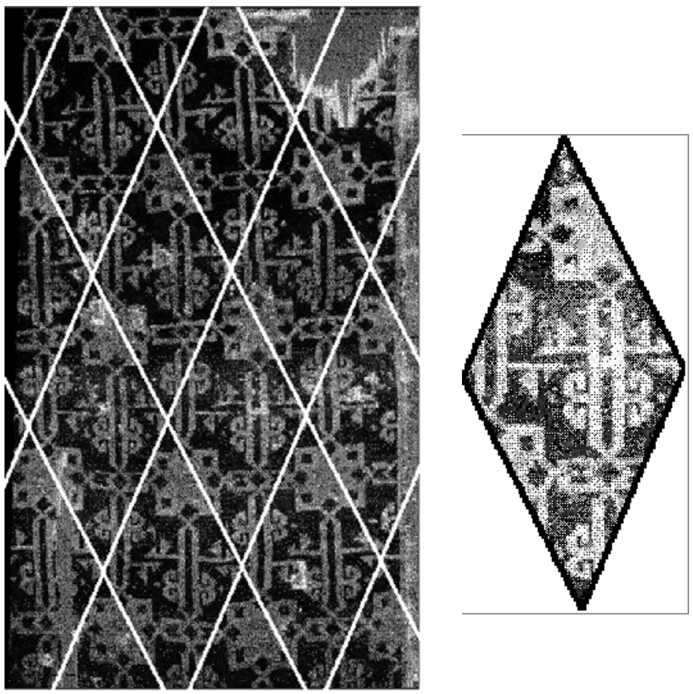
Motif

--- the reoccurring theme

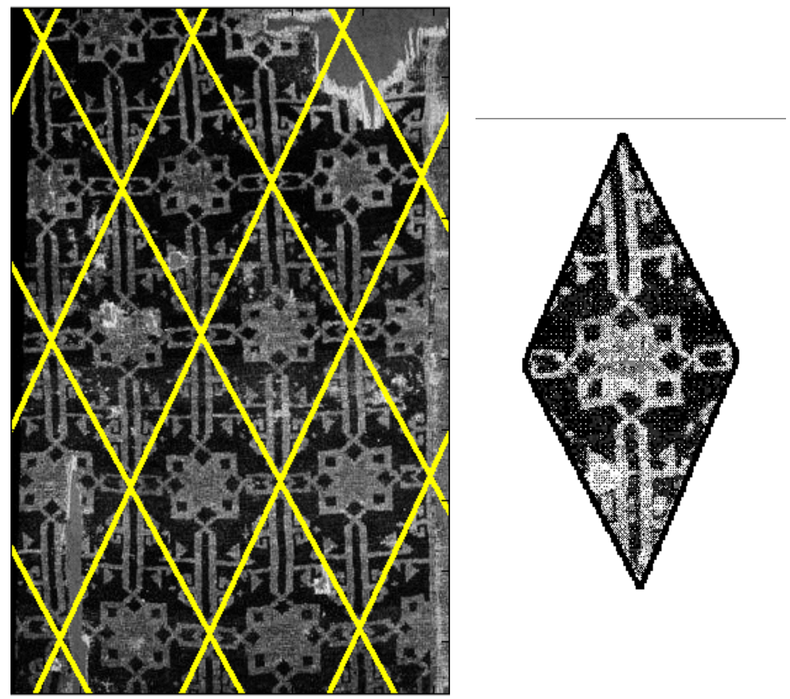
Tiles and Better Tiles

The rug lattice and its motifs

CMM

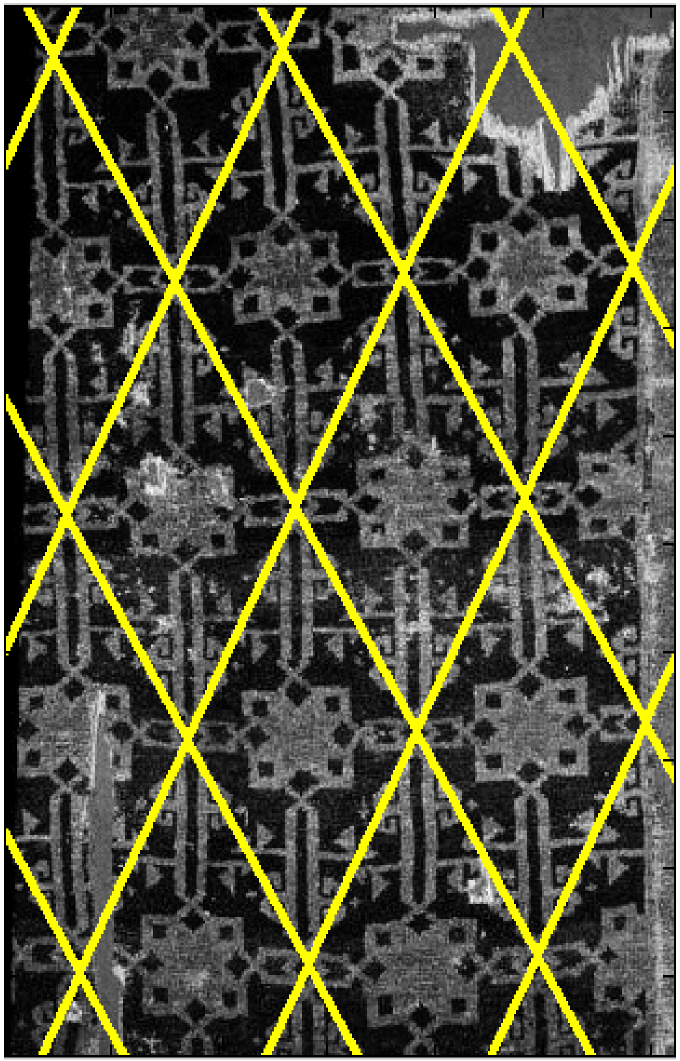


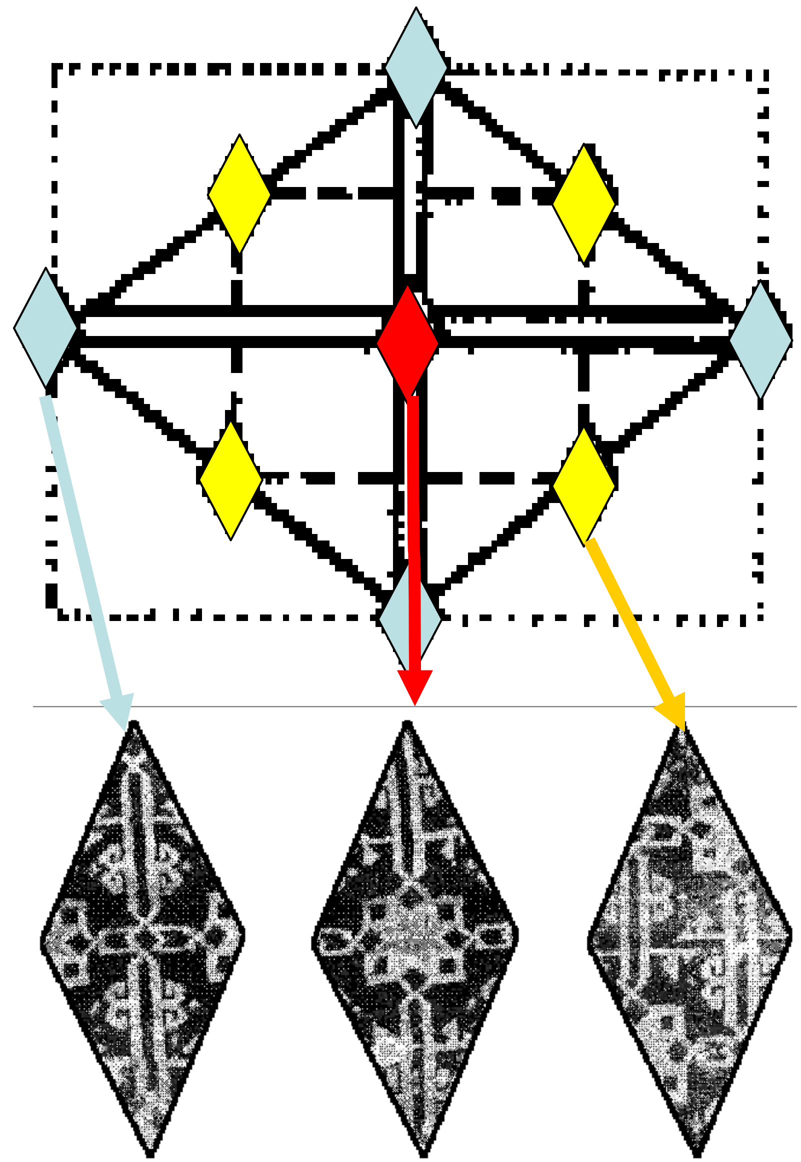
CMM



Orbits of 2-fold rotation centers

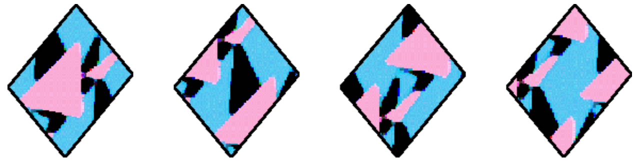
CMM

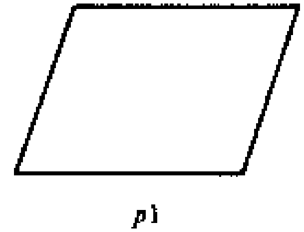


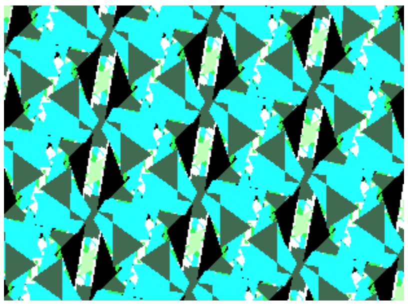


Motifs

p1







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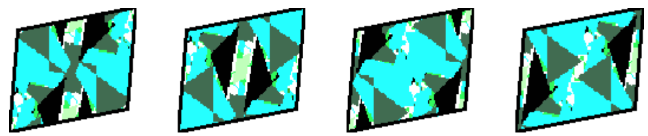
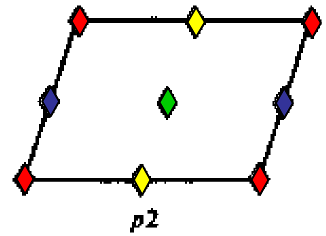
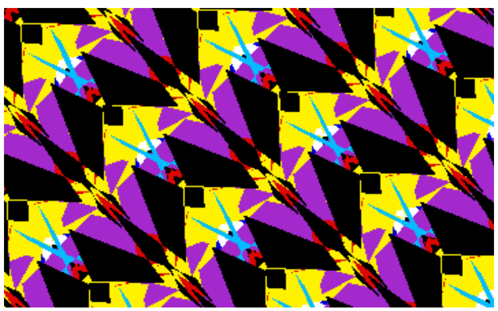
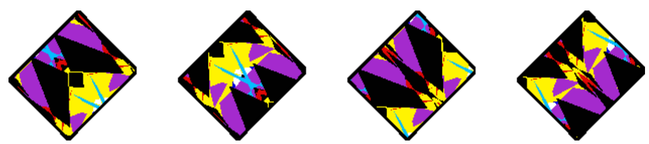
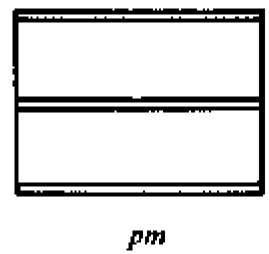


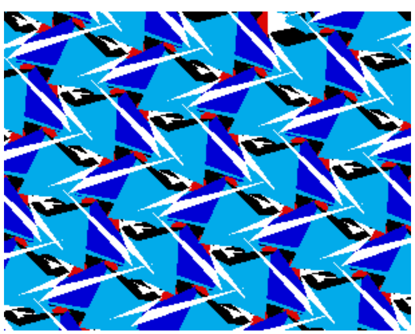




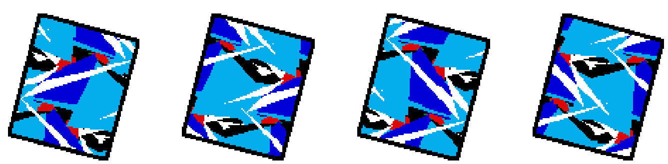
pm

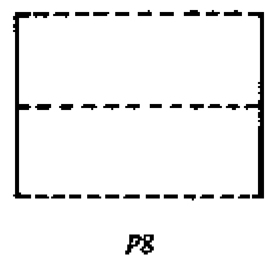






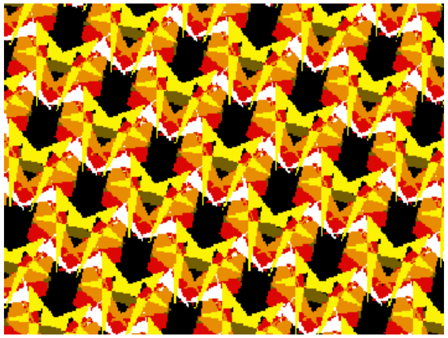
pg

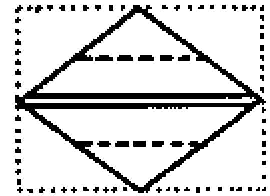




motifs

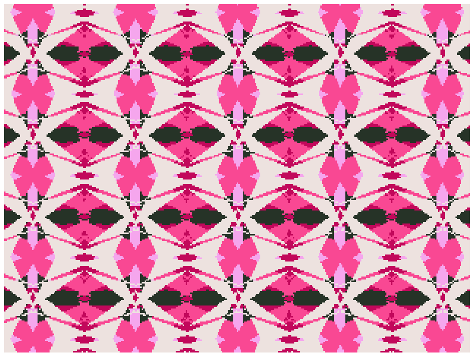
cm

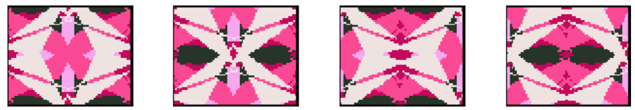


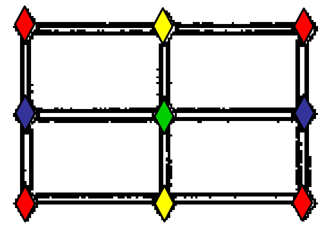


cm

pmm

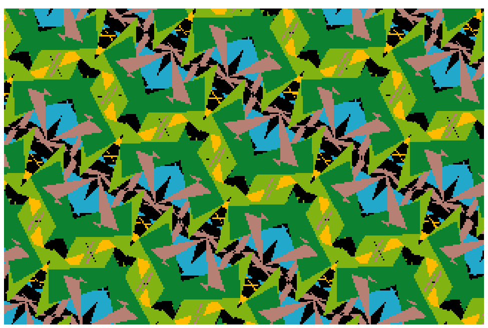


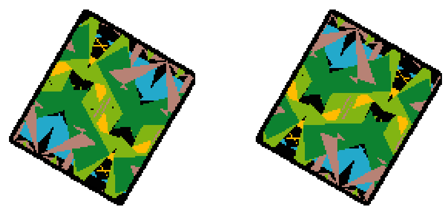


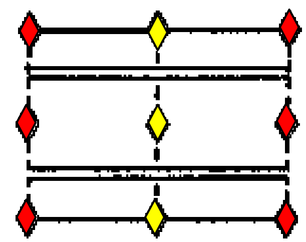


pmm

pmg

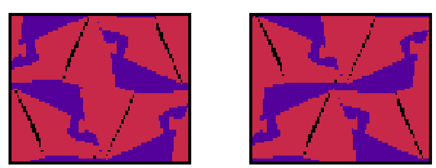


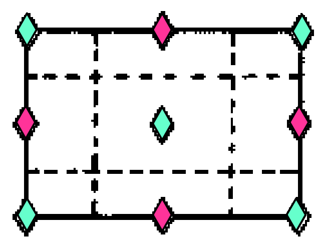




pmg

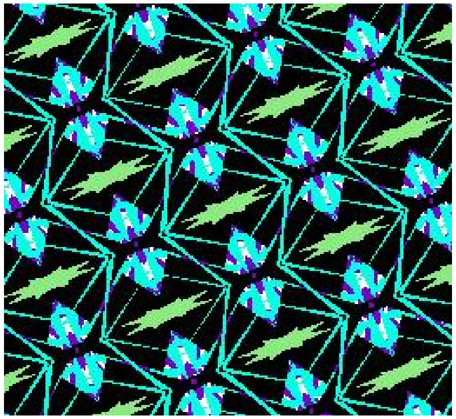
pgg





pgg

motifs



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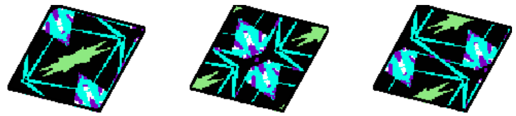

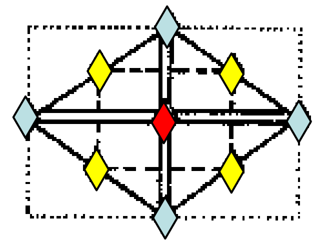

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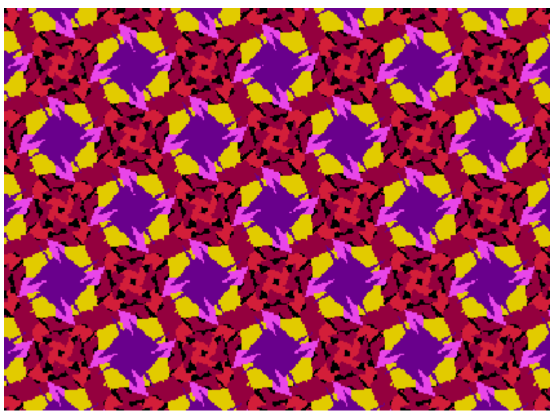

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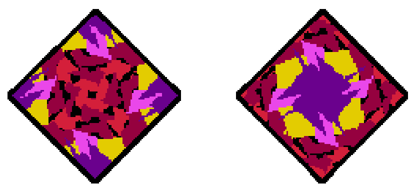

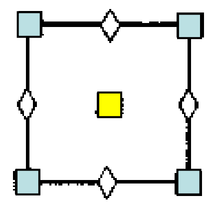

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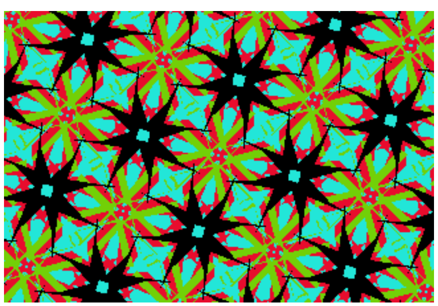

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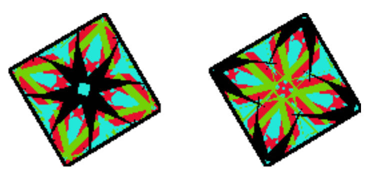

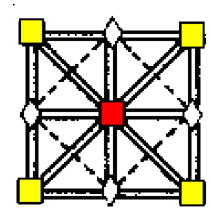

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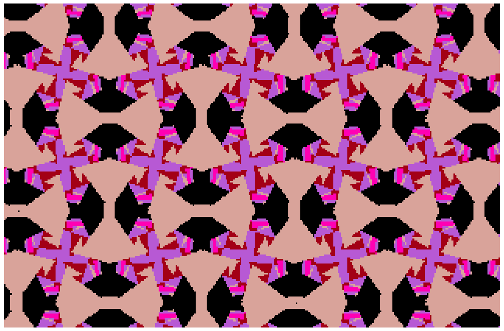
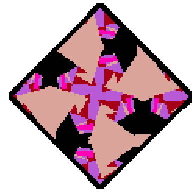
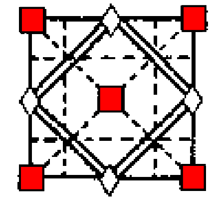


p4g





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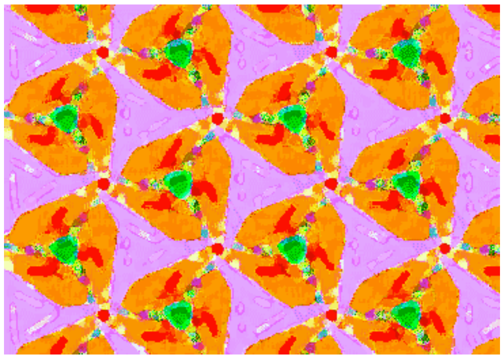

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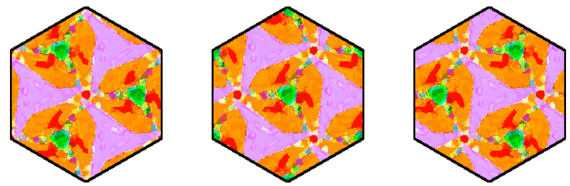

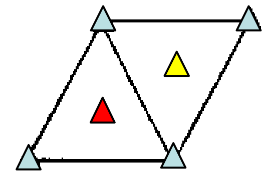

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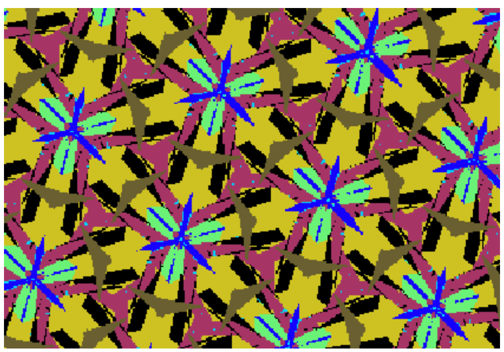

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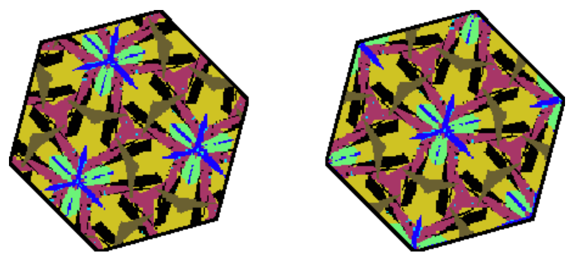

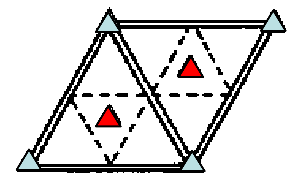

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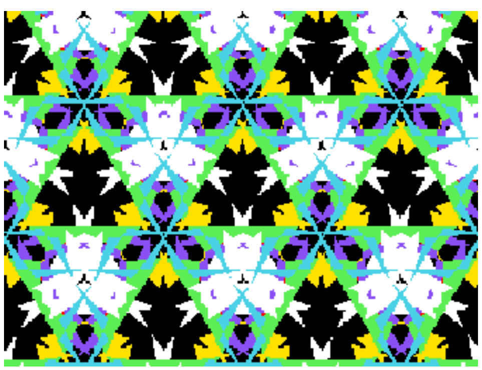
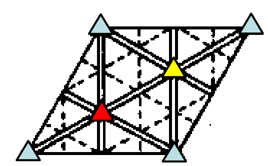
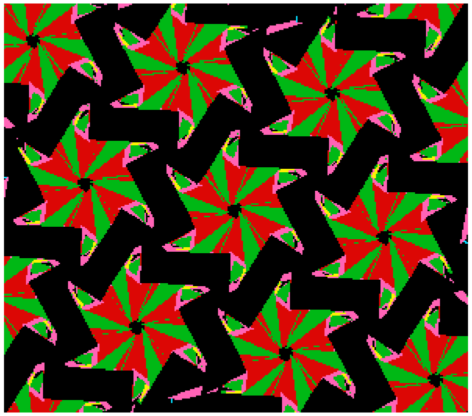


p3m1



p3m1

motifs



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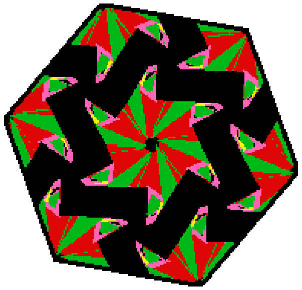

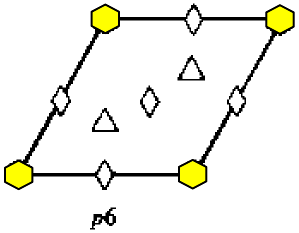

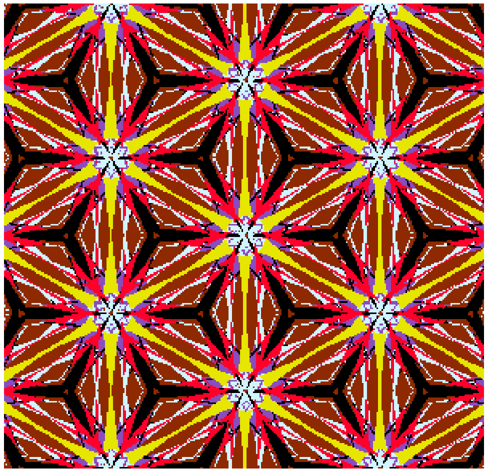

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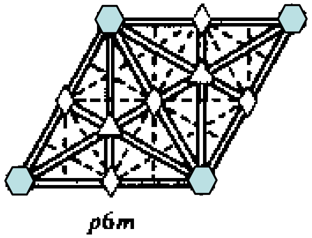
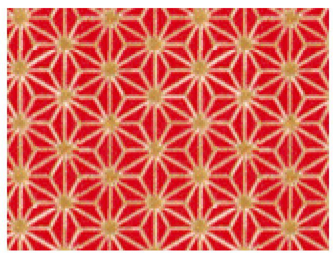
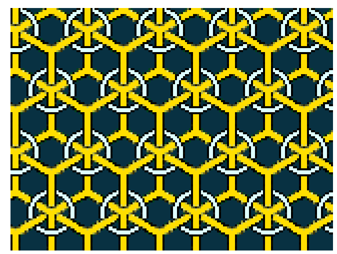
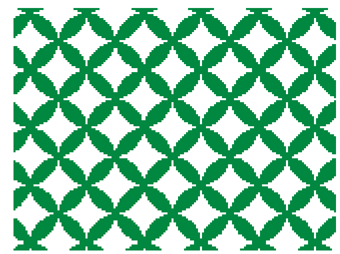


Skewed Symmetry Group

--- a global phenomena

Skewed Symmetry Groups

Liu, Y. and Collins,R.T., ``Skewed Symmetry Groups'', Computer Vision and Pattern Recognition Conference (CVPR'01), Kauai, Hawaii. December, 2001.

Skewed Symmetry Groups:
Euclidean symmetry groups conjugated by affine transformations

$$G' = AGA^{-1},\ S' = A\,(S)$$

where A --- an affine transformation

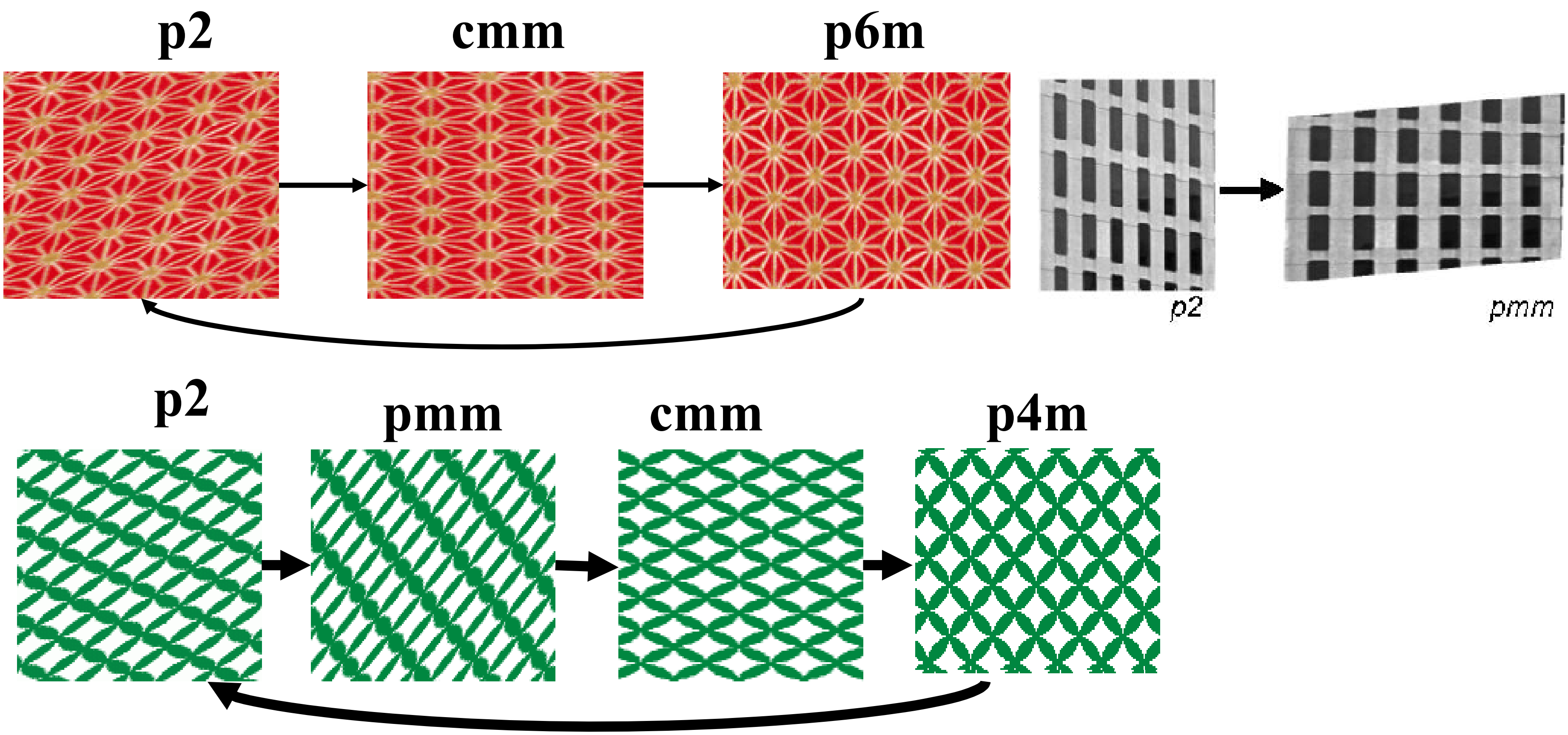
G --- the original Euclidean symmetry group of S

Question: what is the symmetry group of S'?

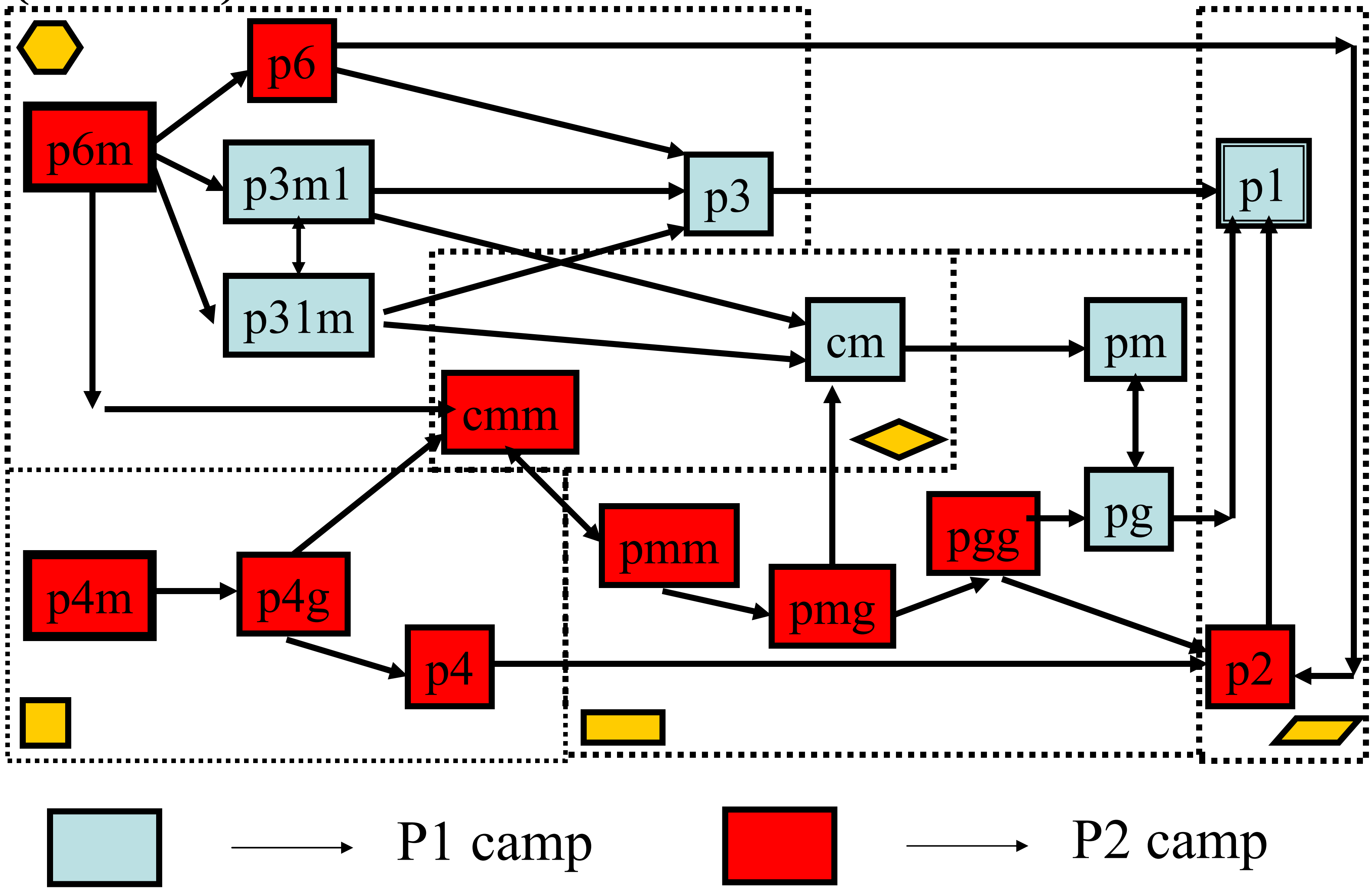
Potential Symmetry

= largest Euclidean subgroup in AGA-1

An inherent, affine-invariant, <u>non-face-value</u> property of a pattern



Subgroup Relationship Among the 17 Wallpaper Groups (Coxeter)



	p1	p2	pm	pg	cm	pmm	pmg	pgg	cmm	p4	p4m	p4g	p3	p3m1	p31m	p6	p6m
p1	A	1	P	P	P	1	1	1	1	1	1	1	P	P	P	1	1
p2	1	A	1	1	1	P	P	P	P	P	P	P	1	1	1	P	P
pm	A	1	N	2	3	1	1	1	1	1	1	1	4	3	3	1	1
pg	A	1	2	N	3	1	1	1	1	1	1	1	4	3	3	1	1
cm	A	1	3	3	N	1	1	1	1	1	1	1	4	P	P	1	1
pmm	1	A	1	1	1	N	2	2	P	3	P	3	1	1	1	4	3
pmg	1	A	1	1	1	2	N	2	3	3	3	3	1	1	1	4	3
pgg	1	A	1	1	1	2	2	N	P	3	3	P	1	1	1	4	3
cmm	1	A	1	1	1	P	3	P	N	3	P	P	1	1	1	3	P
p4	1	A	1	1	1	3	3	3	3	S	2	2	1	1	1	4	3
p4m	1	A	1	1	1	N	3	3	N	2	S	2	1	1	1	3	3
p4g	1	A	1	1	1	3	3	N	N	2	2	S	1	1	1	3	4
p3	A	1	4	4	4	1	1	1	1	1	1	1	S	2	2	1	1
p3ml	A	1	3	3	N	1	1	1	1	1	1	1	2	S	2	1	1
p3lm	A	1	3	3	N	1	1	1	1	1	1	1	2	2	S	1	1
p6	1	A	1	1	1	4	4	4	3	4	3	3	1	1	1	S	2
p6m	1	A	1	1	1	3	3	3	N	3	3	4	1	1	1	2	S

Skewed Symmetry Groups

Can transform under:

- **A** General affine
- **N** Nonuniform scaling
- **S** Similarity transform
- **P** Pattern dependent

Can not transform because:

1) G1 has a 2-fold rotation G2 does not
2) they have the same lattice type
3) surviving symmetry exists;
4) they do not have a subgroup relationship.

Applications

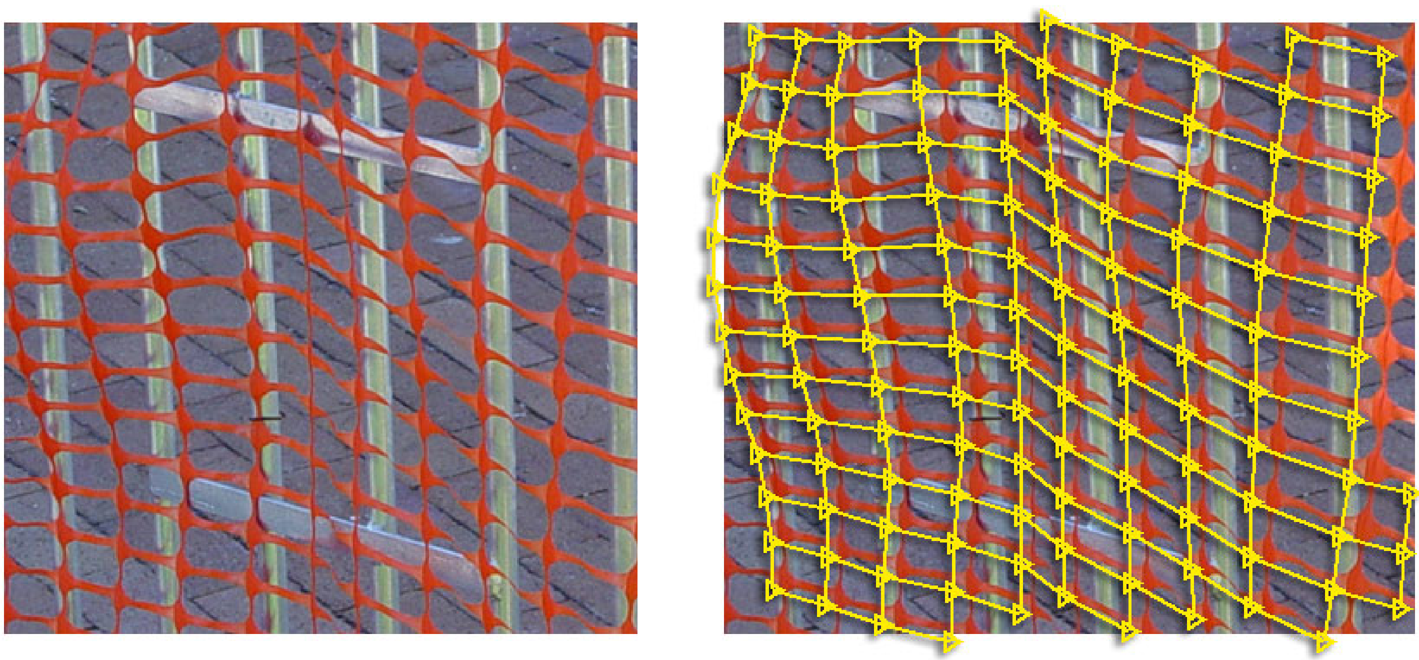
Research Direction #1: beyond global affine transformation

Automatic Lattice Detection

- **Discovering Texture Regularity as a Higher-Order Correspondence Problem**
J.H. Hays, M. Leordeanu, A.A. Efros, and Y. Liu
***9th European Conference on Computer Vision*, May, 2006**.
- **Deformed Lattice Detection via Mean-Shift Belief Propagation**
Minwoo Park, Robert T. Collins, and Yanxi Liu
European Conference on Computer Vision (ECCV), Marseille, France, October 2008.
- **Automatic Lattice Detection in Near-Regular Histology Array Images**
B.A. Canada, G.K. Thomas, K.C. Cheng, J.Z. Wang, and Y. Liu
Proceedings of the IEEE International Conference on Image Processing, October 2008.
- **Deformed Lattice Detection from Real Images via Mean-Shift Belief Propagation**
Minwoo Park, Robert T. Collins, and Yanxi Liu
PAMI to appear 2009.

ECCV 2006 Hays et al

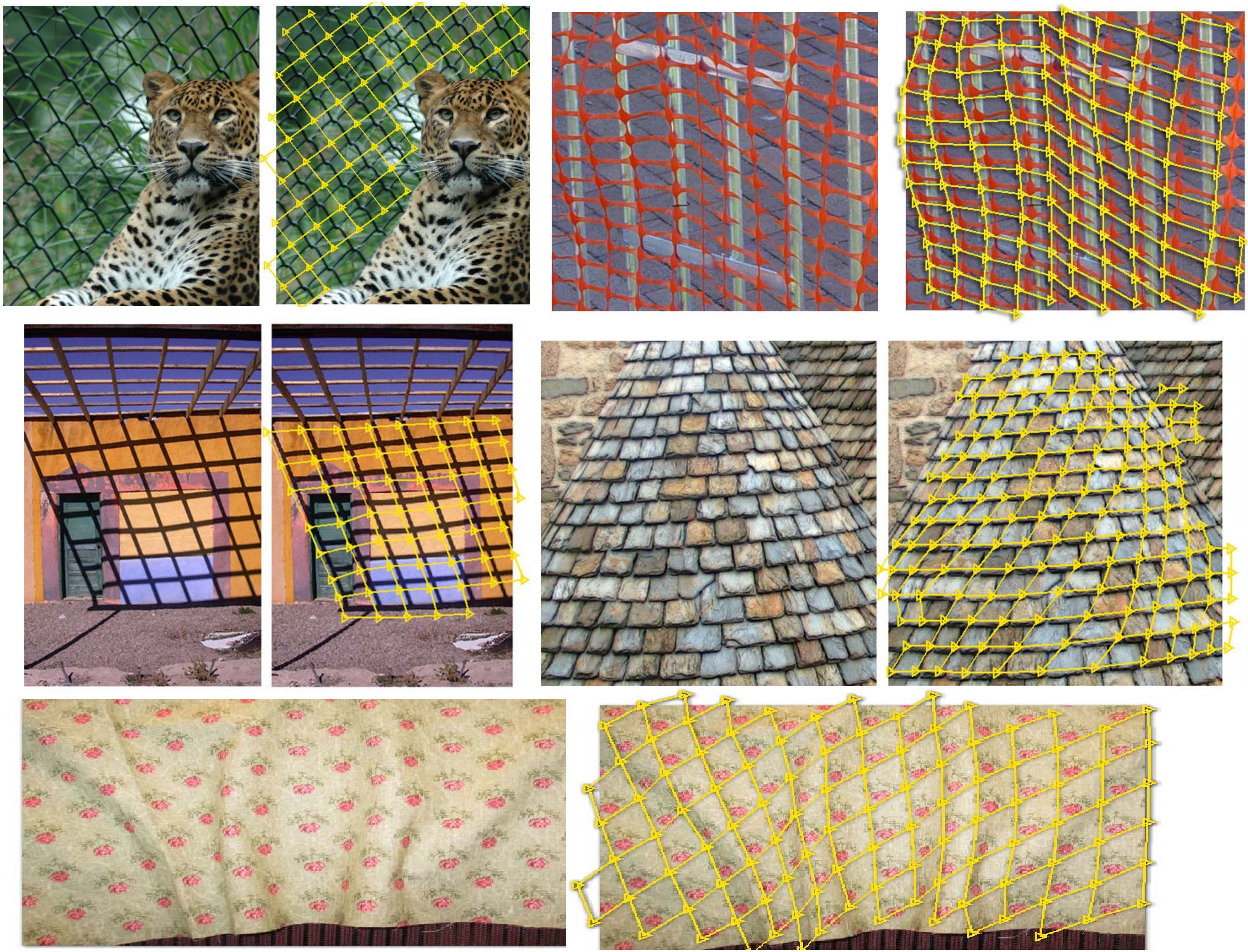
A texture's lattice unambiguously identifies all texels as well as the spatial and topological relationship among the texels.

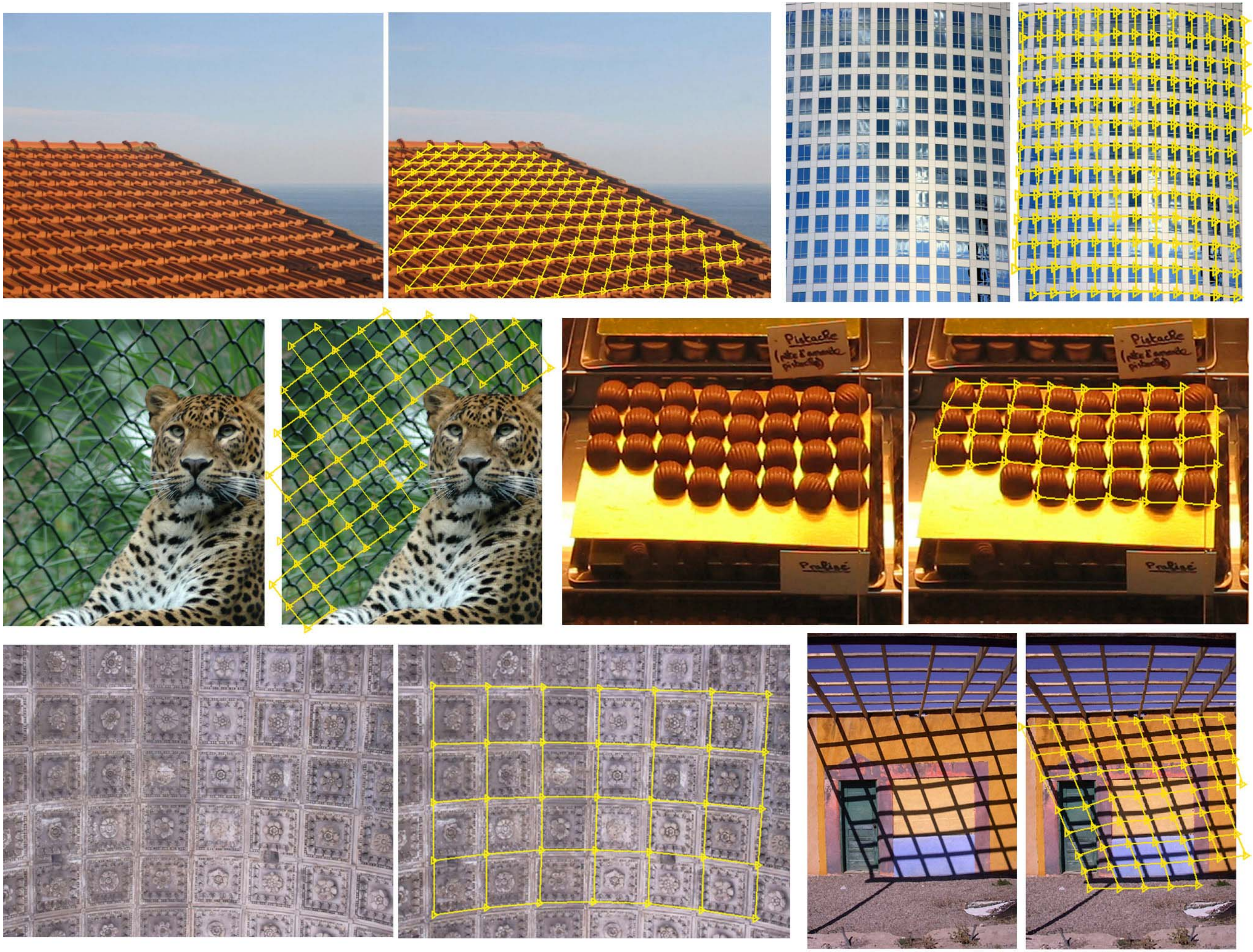


Discovering Texture Regularity as a Higher-Order Correspondence Problem

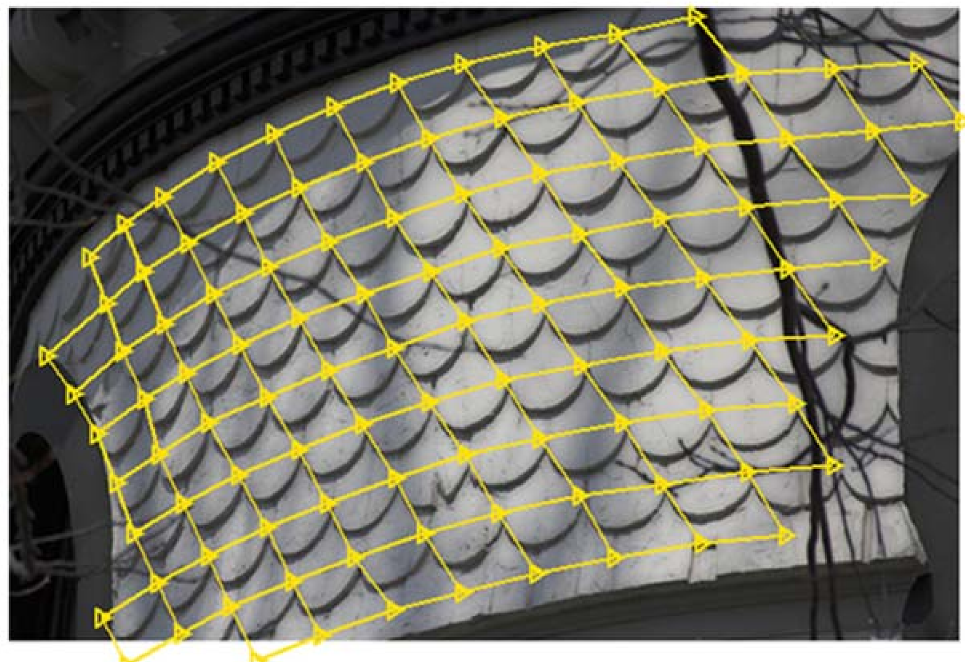
James Hays, Marius Leordeanu,
Alexei A. Efros, and Yanxi Liu

Movie ECCV 2006




Pistache
Praliné

Lattice on curved surface …



New Advance 2008/2009

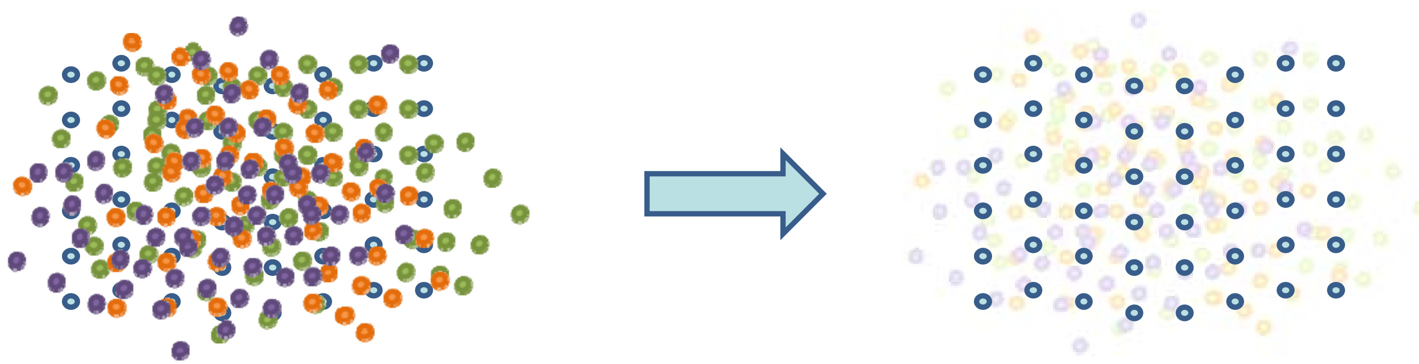
- **Deformed Lattice Detection via Mean-Shift Belief Propagation**
  M. Park, R. Collins, and Y. Liu
  European Conference on Computer Vision (ECCV), October, 2008.
- **Deformed Lattice Detection in Real-World Images using Mean-Shift Belief Propagation**

  Park, Brocklehurst, Collins and Liu

  To appear ***PAMI*** 2009

Phase I

- We use
  - Block-wise KLT corner detector to expose the 2DDLP.
  - A normalized image patch centered at each KLT point as a descriptor.
  - Mean-shift clustering with a bandwidth 7 to cluster repeating points.

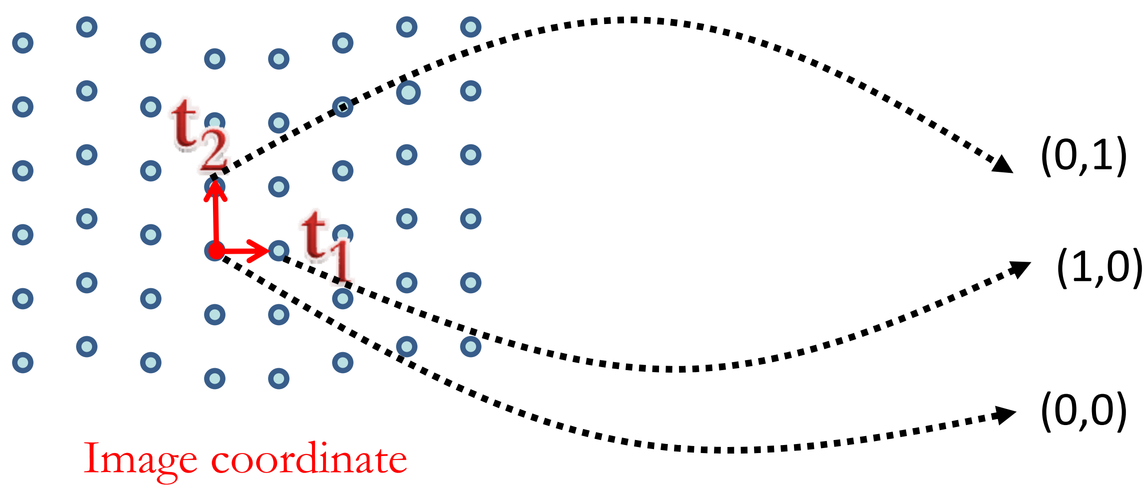


Block-wise KLT detection

Clustered points

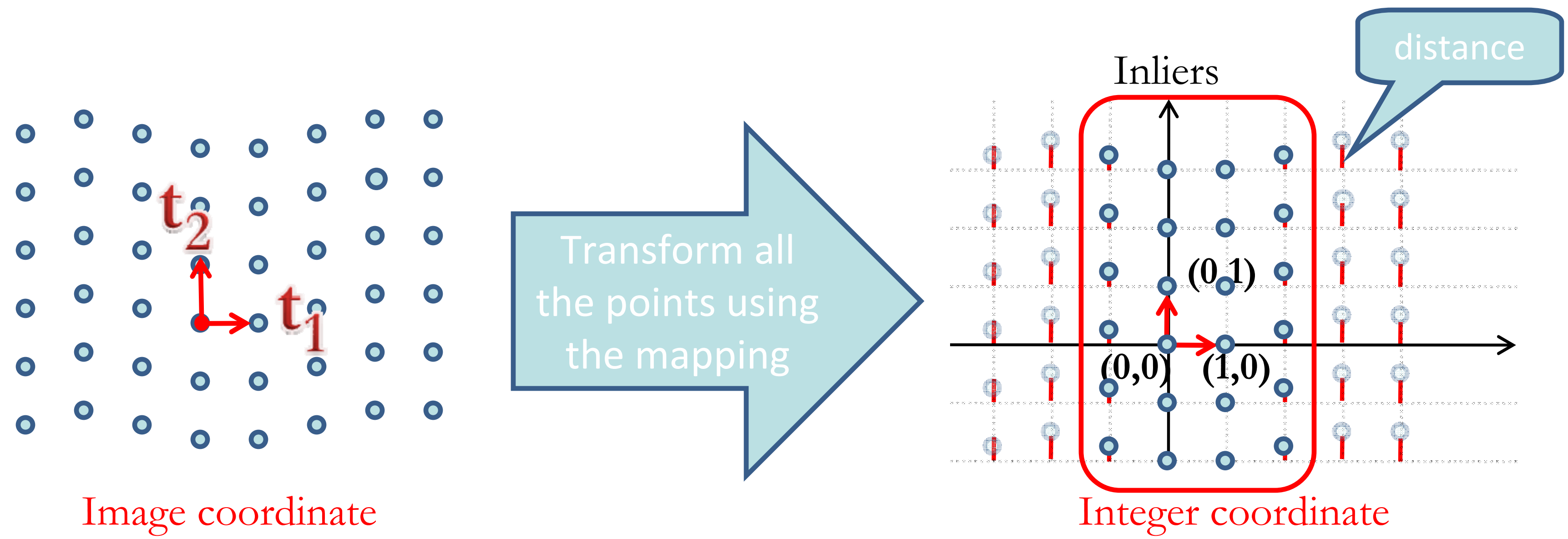
Phase I (continued)

- IF the clustered points form a lattice, there should be a mapping that transforms every repeating point to the integer coordinate at least locally.
- From the clustered points, we choose 3 points randomly to propose a mapping to integer coordinates (0,0) (0,1) (1,0).

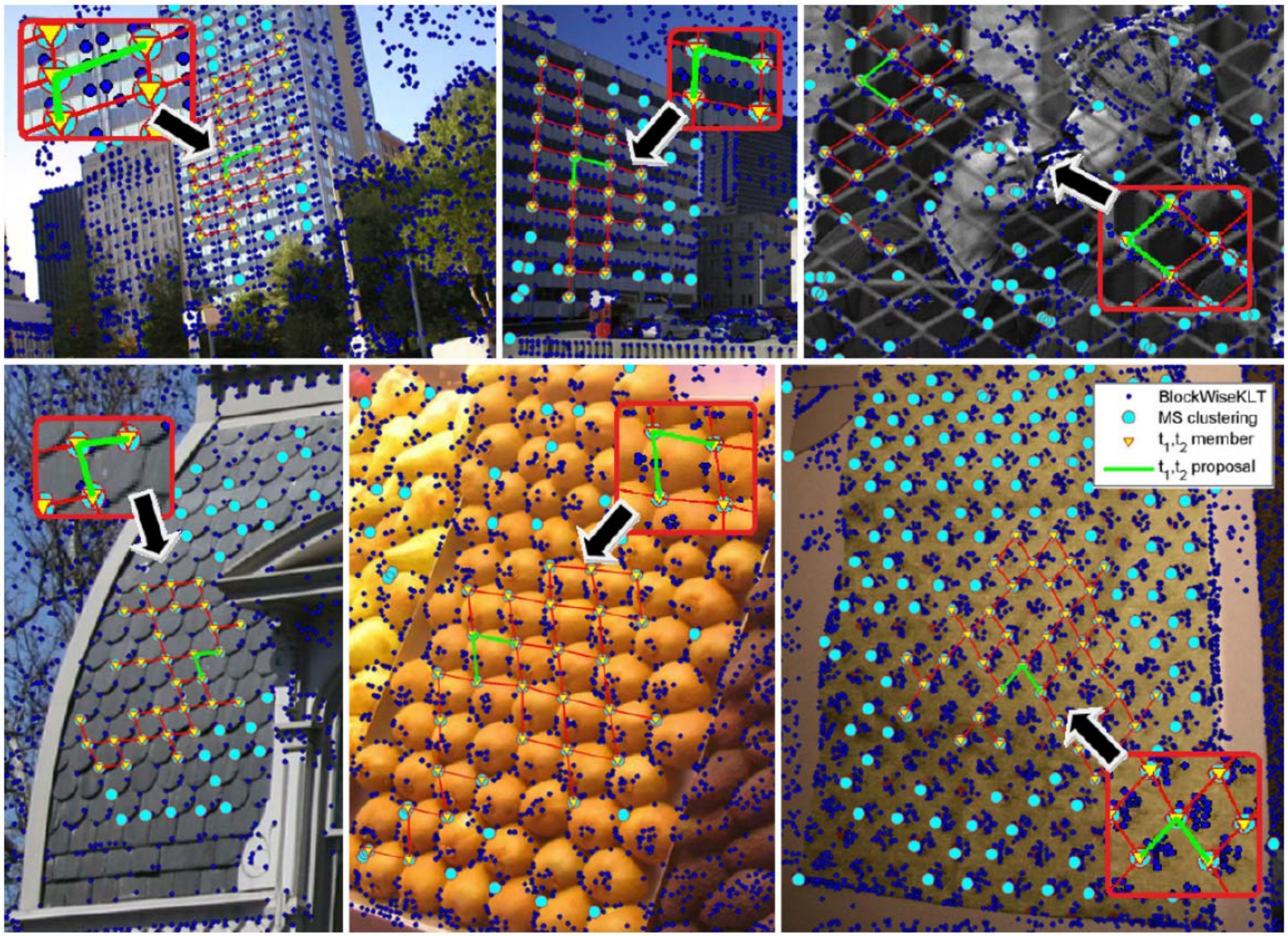


Phase I (continued)

- We transform all the clustered points using the mapping and measure the distance between the integer coordinates and transformed points.
- If the distance is less than 0.1, we increase inliers count.
- We choose a mapping with the maximum inliers and propose the 3 points as (t_1, t_2) proposal.



Sample Result – Phase I

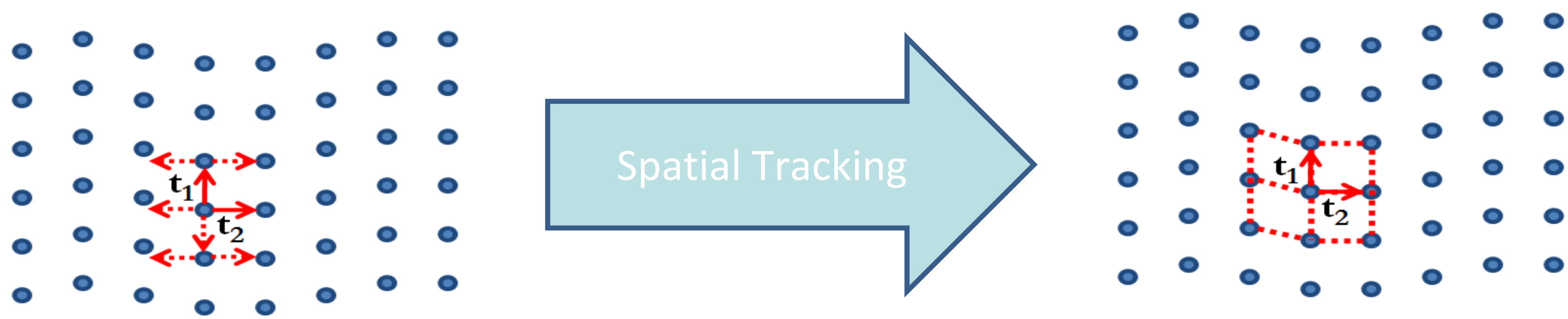


The green L-shape inside the red enlarged rectangular window is the proposed (t_1, t_2) vectors pair, and the red L-shapes are its supporting members (inlier votes).

Phase II – Spatial Tracking using Markov Random Field (MRF)

- Tracking lattice points using the proposed (t_1,t_2) as a prediction and the texel for image likelihood measurement under the MRF formulation.

- Mean-shift Belief Propagation (MSBP) on the MRF for inference. (CVPR 2008)



Prediction by (t_1, t_2) Inference using MSBP

MRF formulation

Joint compatibility
- image likelihood

$$\phi(\mathbf{x}_{[i,j]}, z_{[i,j]}) = \exp(-\alpha(1 - z_{[i,j]})),$$
$$z_{[i,j]} = NCC(T_0, I(\mathbf{x}_{[i,j]})) \times NCC(e_m(T_0), e_m(I(\mathbf{x}_{[i,j]})))$$

T_0 → texel, **$\mathbf{x}_{[i,j]}$** → hidden variable at node (i,j), **I(x)** → image patch at location x, **e_m(T)** → edge image of T, **NCC(A,B)** → normalized cross correlation between image patch A and B.

Pair-wise compatibility

$\psi(\mathbf{x}_{[i,j]}, \mathbf{x}_{[i,j\pm1]})$ For left-right and right to left message passing

$$= \exp(-\beta \times E(\overrightarrow{\mathbf{x}^{(it)}_{[i,j]}\mathbf{x}^{(it)}_{[i,j\pm1]}}, \overrightarrow{\mathbf{x}^{(it)}_{[i,j]}\mathbf{x}^{(0)}_{[i+1,j]}}, \overrightarrow{\mathbf{x}^{(0)}_{[i,j]}\mathbf{x}^{(0)}_{[i,j\pm1]}}, \overrightarrow{\mathbf{x}^{(0)}_{[i,j]}\mathbf{x}^{(0)}_{[i+1,j]}})^2)$$

$\psi(\mathbf{x}_{[i,j]}, \mathbf{x}_{[i\pm1,j]})$ For up-down and down-up message passing

$$= \exp(-\beta \times E(\overrightarrow{\mathbf{x}^{(it)}_{[i,j]}\mathbf{x}^{(it)}_{[i\pm1,j]}}, \overrightarrow{\mathbf{x}^{(it)}_{[i,j]}\mathbf{x}^{(0)}_{[i,j+1]}}, \overrightarrow{\mathbf{x}^{(0)}_{[i,j]}\mathbf{x}^{(0)}_{[i\pm1,j]}}, \overrightarrow{\mathbf{x}^{(0)}_{[i,j]}\mathbf{x}^{(0)}_{[i,j+1]}})^2)$$

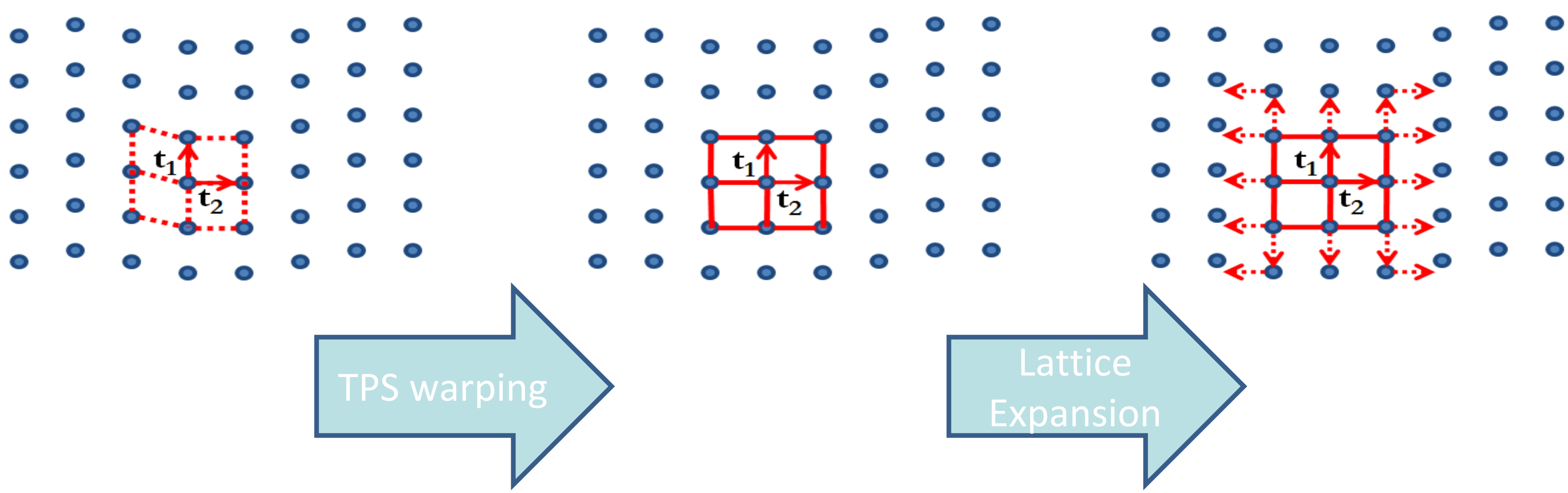
$$E(t_1^i, t_2^i, t_1^j, t_2^j) = max(\left\|t_1^i - t_1^j\right\|_2 / \left\|t_1^i\right\|_2, \left\|t_2^i - t_2^j\right\|_2 / \left\|t_2^i\right\|_2)$$

α and **β** are sensitivity parameter for normalized variables **z** and **E. $\mathbf{x}^{(0)}$** is a location of the initial model lattice built by (t_1,t_2) proposal. Since **E** basically measures the difference between two sets of (t_1,t_2) vectors pair, the pair-wise compatibility measures the compatibility between the current predicted lattice and the initial model lattice.

Phase III -

Thin plate spline (TPS) warping and lattice expansion

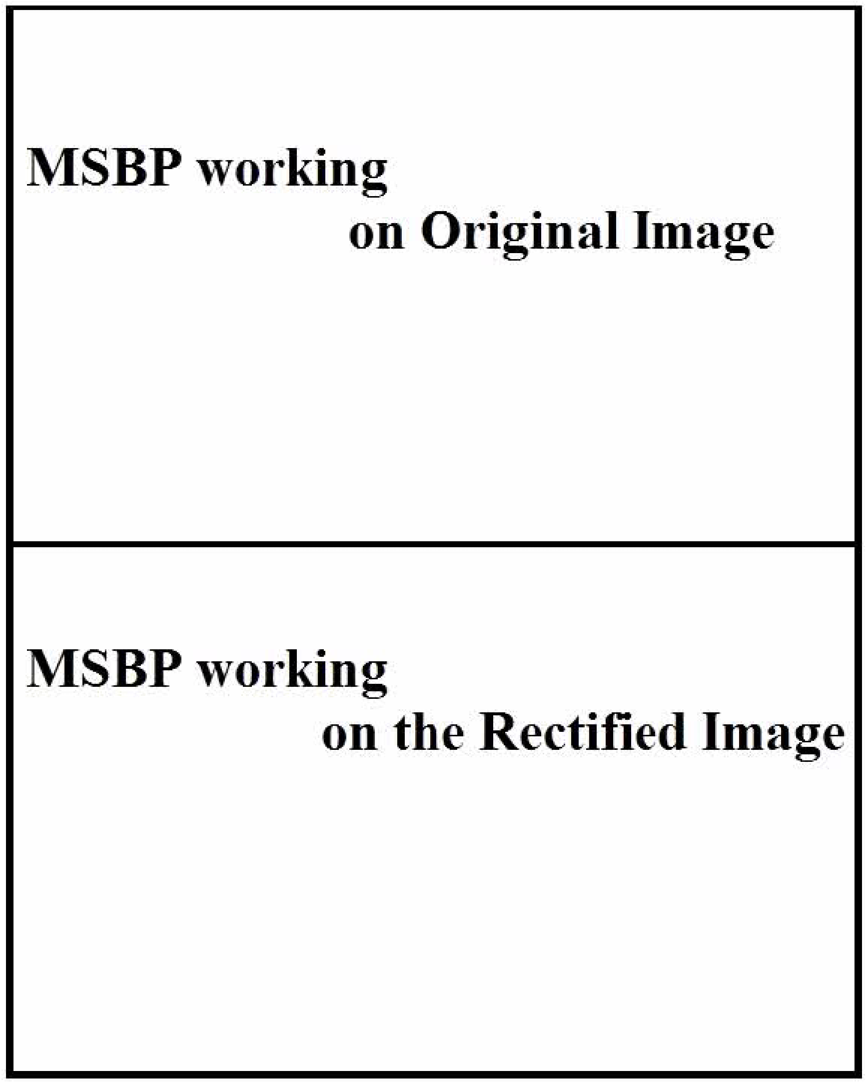
- Regularized TPS warping is computed using the correspondences between the current tracked lattice and the initial model lattice.
- Then warping is performed the regularized TPS warping.
- Then the lattice is expanded using the (t_1,t_2) proposal.



Sample result

Original Coordinate

Rectified Coordinate



Test Data Set

Data set 1
(Opaque)

Data set 1 (D1) contains 67 images where the texels are opaque, and appearance variations of the repeating elements come from different viewpoint and lighting conditions.

Data set 2
(See-through)

Data set 2 (D2) contains 73 images with see-through or wiry structures, thus high variation in texel appearance often occurs due to the changing background.

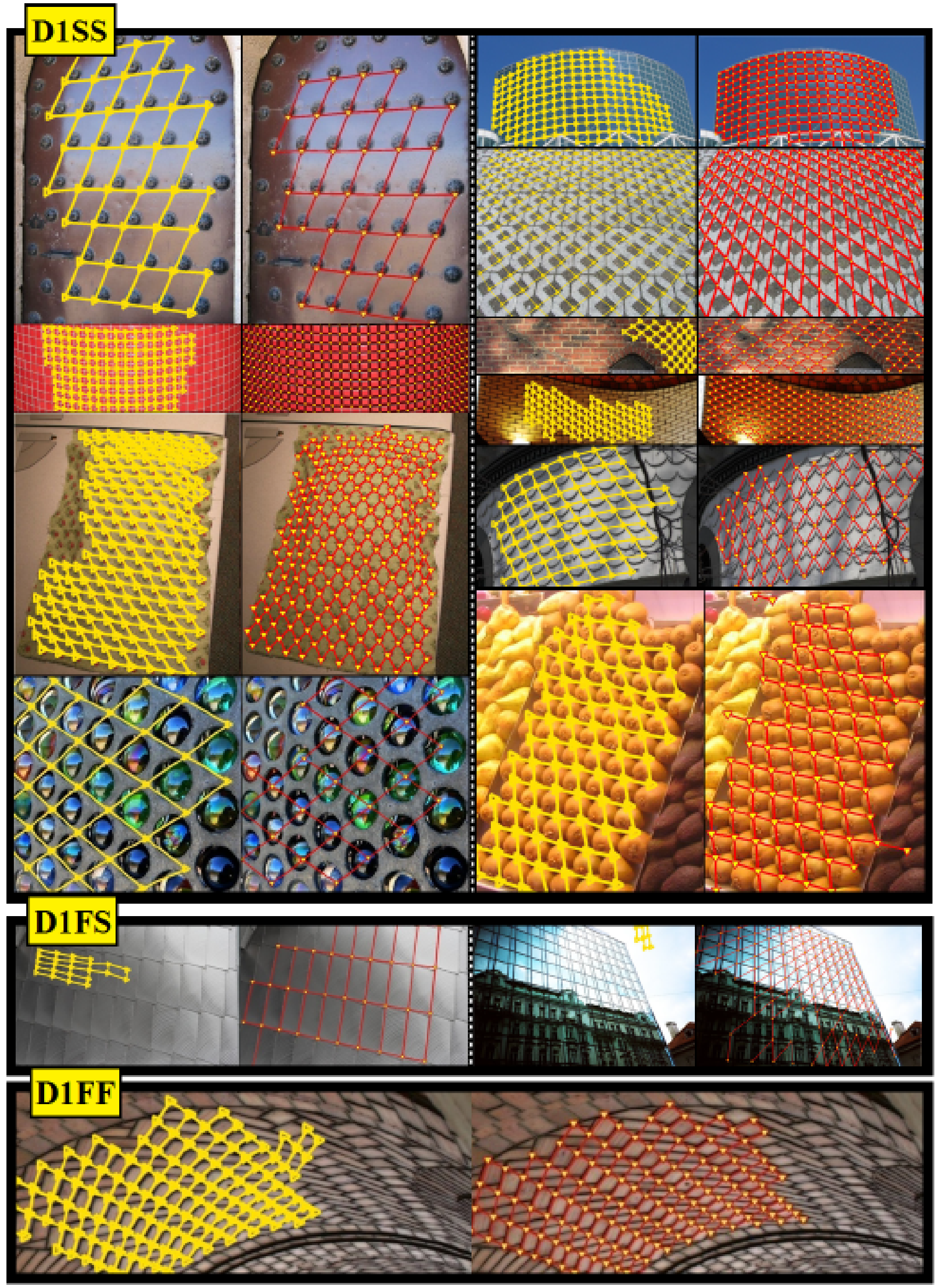
Data set 3
(Urban view)

Data set 3 (D3) contains 121 images with urban views of city buildings where there are multiple repeating patterns with perspective distortion

Comparison Result

Left: Hays et al

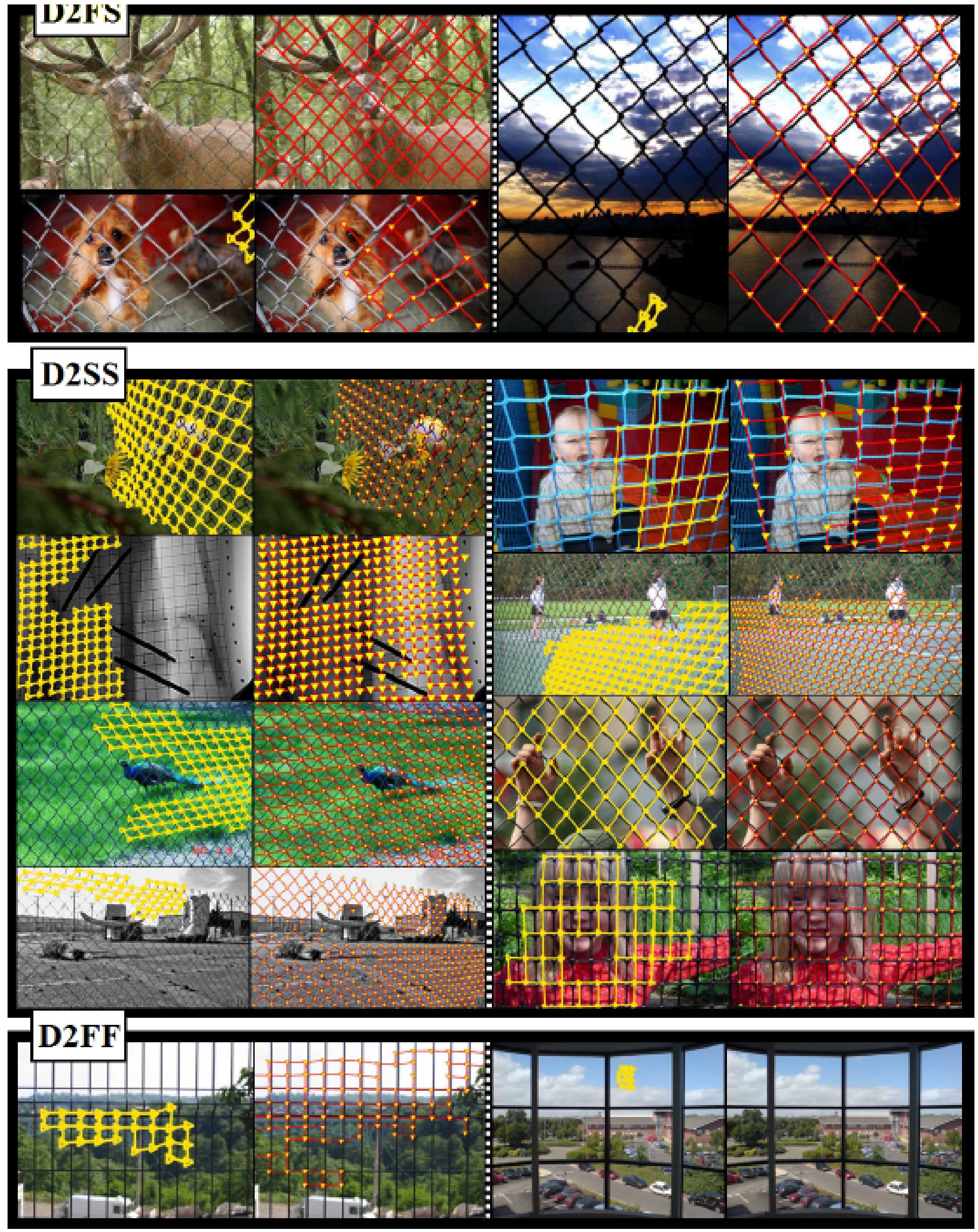
Right: our method



Comparison Result

Left: Hays et al

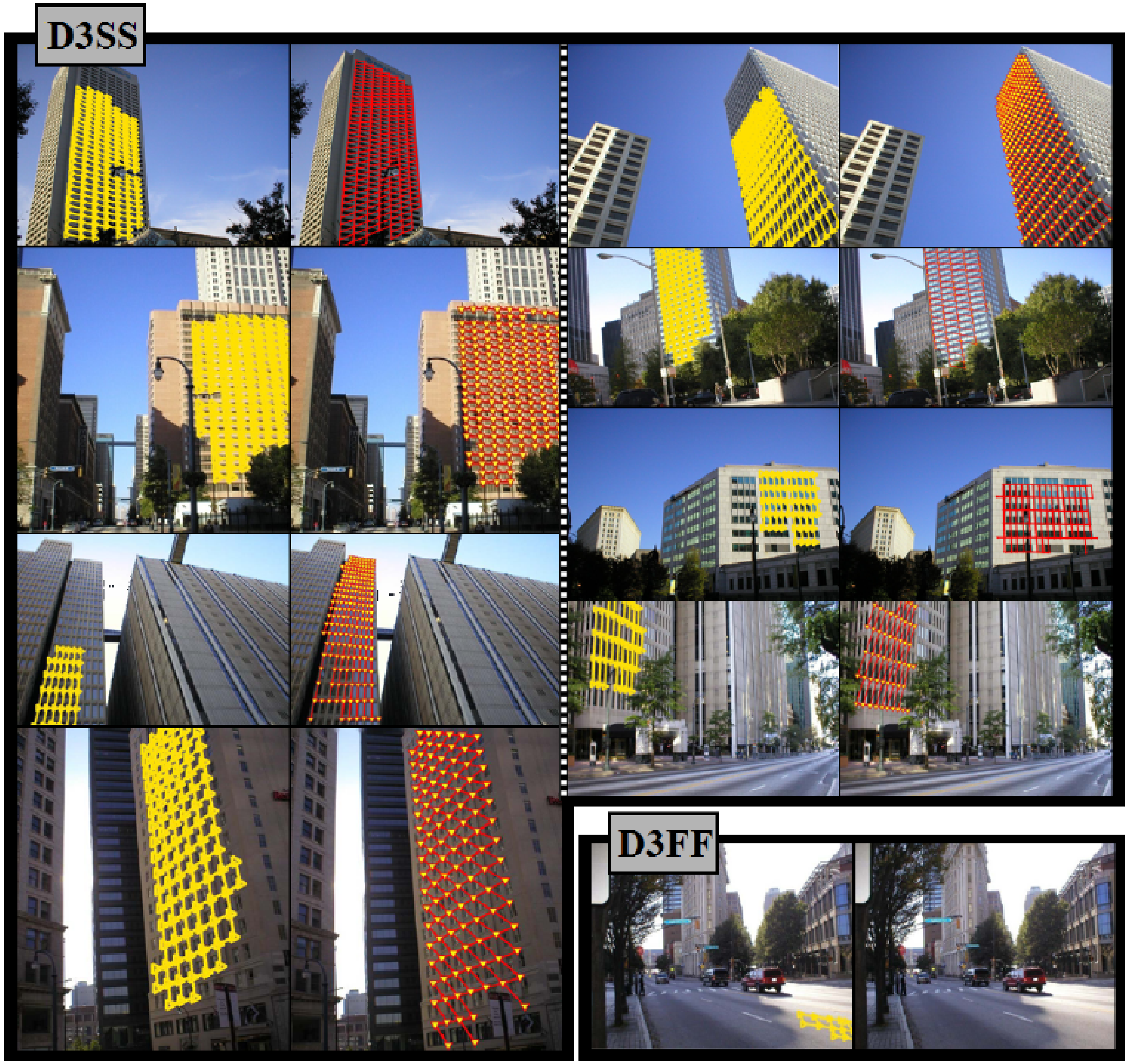
Right: our method



Comparison Result

Left: Hays et al

Right: our method


D3SS
D3FF

Detection Rate (%)	ECCV2008 [25] set (32 images)	subset of $D1$ (58 images)	subset of $D2$ (51 images)	subset of $D3$ (34 images)	subset of D (143 images)
Lin and Liu [13]	18.3 ± 20.0	N/A	N/A	N/A	N/A
Hays et al [11]	33.0 ± 35.2	65.76 ± 37.05	21.49 ± 31.10	14.93 ± 26.35	35.72 ± 39.28
Ours	69.9 ± 21.5	75.69 ± 23.26	50.19 ± 31.17	75.78 ± 20.54	65.78 ± 28.79

(a) Detection Rate

Significant quantitative improvements

Park, Brocklehurst, Collins, Liu PAMI 2009

Average Run Time Ratio	Data set 1 (67 images)	Data set 2 (73 images)	Data set 3 (121 images)	D (261 images)
$\frac{RunTime\ of\ [11]}{RunTime\ of\ Ours}$	12.61 ± 8.58	8.76 ± 6.28	9.33 ± 6.93	9.98 ± 7.34

(c) Average running time ratio

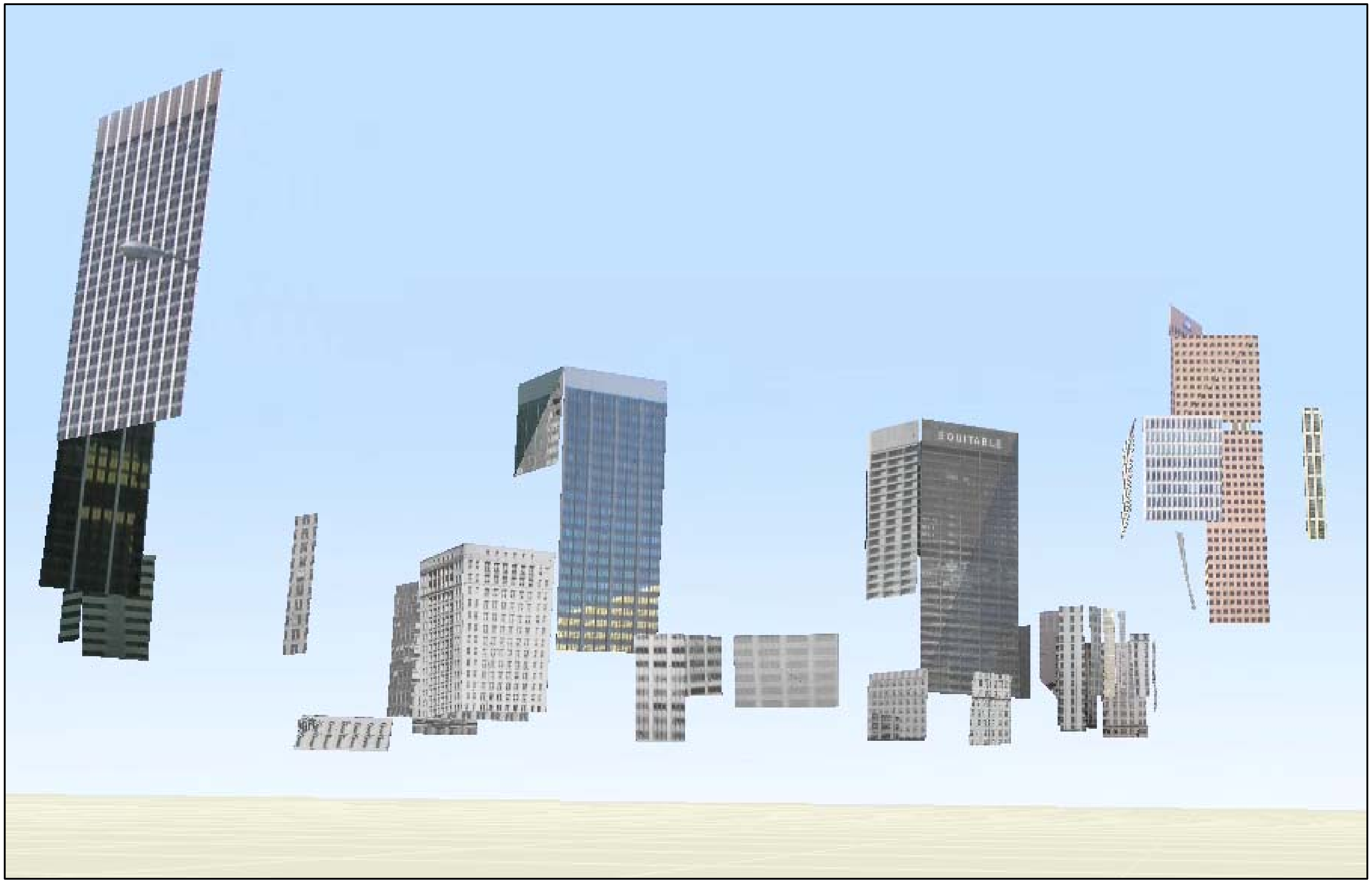
Direction #2: Skewed Wallpaper Groups-based Image Matching

Detecting and Matching Repeated Patterns for Automatic Geo-tagging in Urban Environments

Grant Schindler, Panchapagesan Krishnamurthy, Roberto Lublinerman, Yanxi Liu and Frank Dellaert

Computer Vision and Pattern Recognition Conference (CVPR '08)

Automatic Geo-tagging (CVPR 2008)



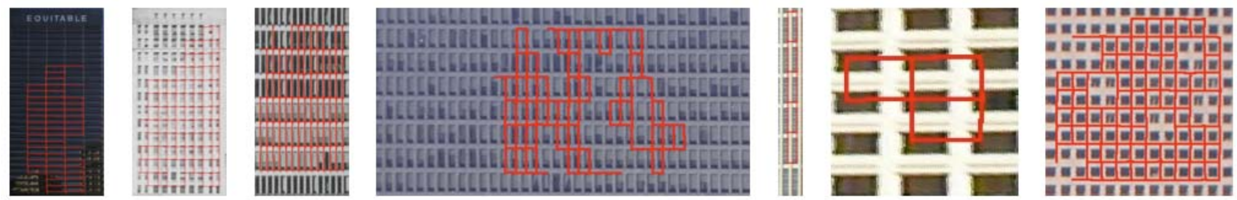
Begin with a Database of Textured 3D Facades

Urban Environments are Highly Repetitive

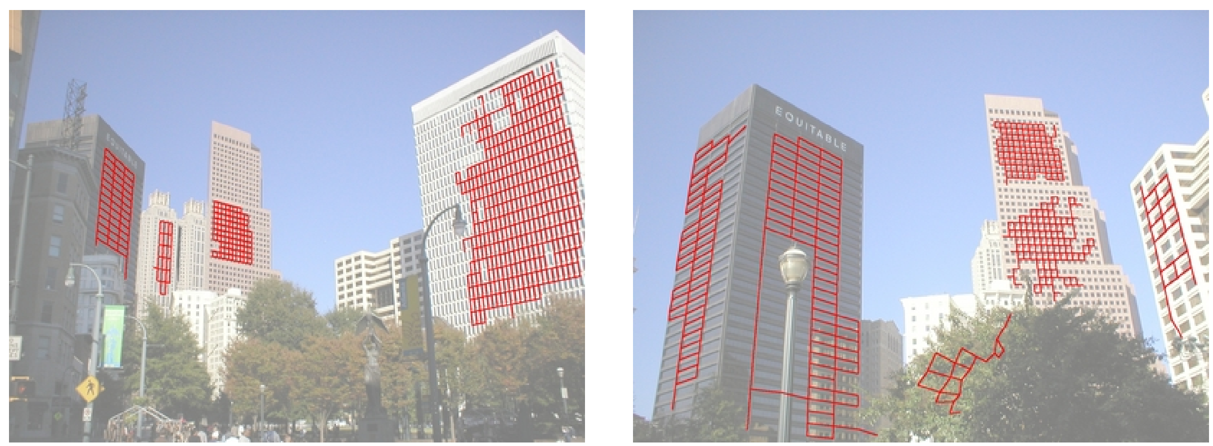
Wide-baseline matching is difficult in highly repetitive environments -- can we exploit this repetition instead?

Lattice Detection

Database Facades

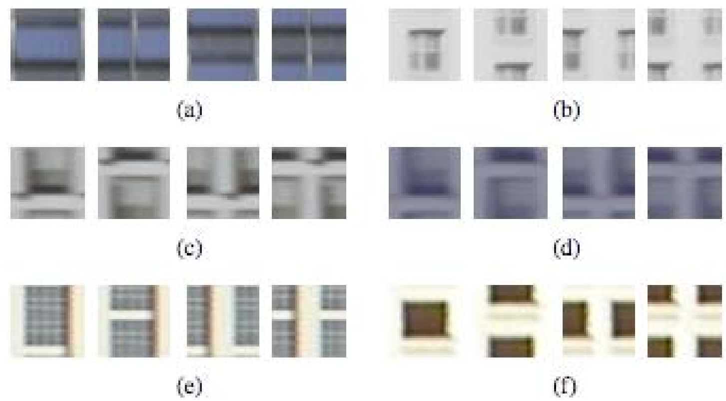


Query Images

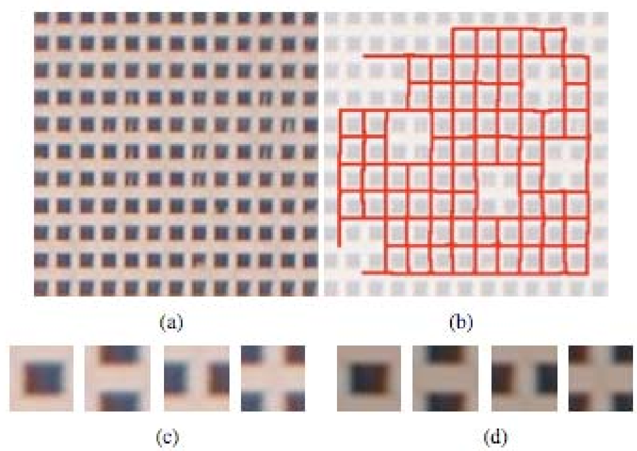


Lattice Detection: Homography-based RANSAC on clusters of SIFT features

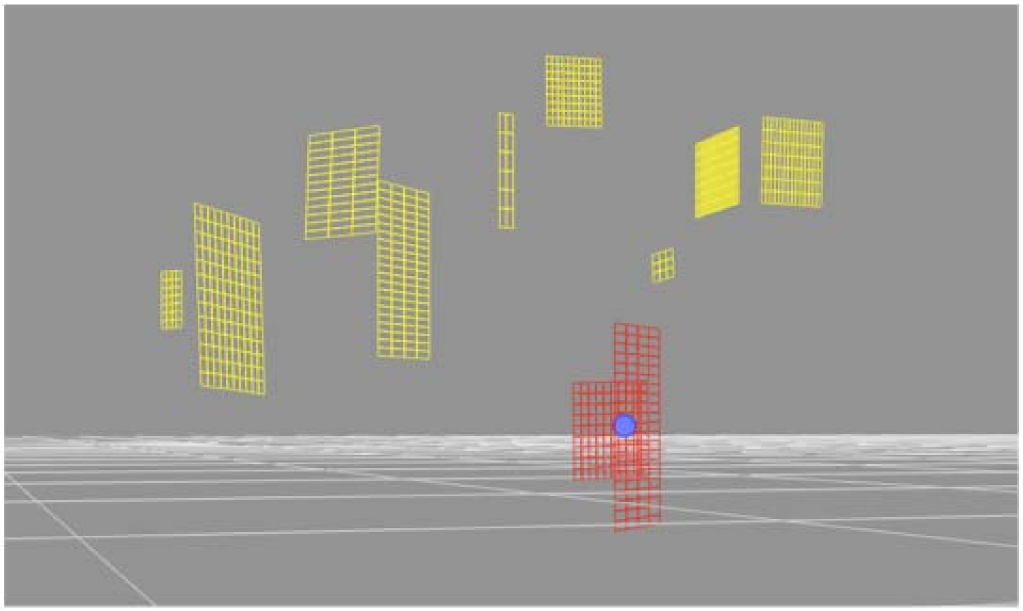
Motifs from Lattice Correspondence (2D-to-3D) Using **Skewed Symmetry Groups** and **unique motifs** mathcing

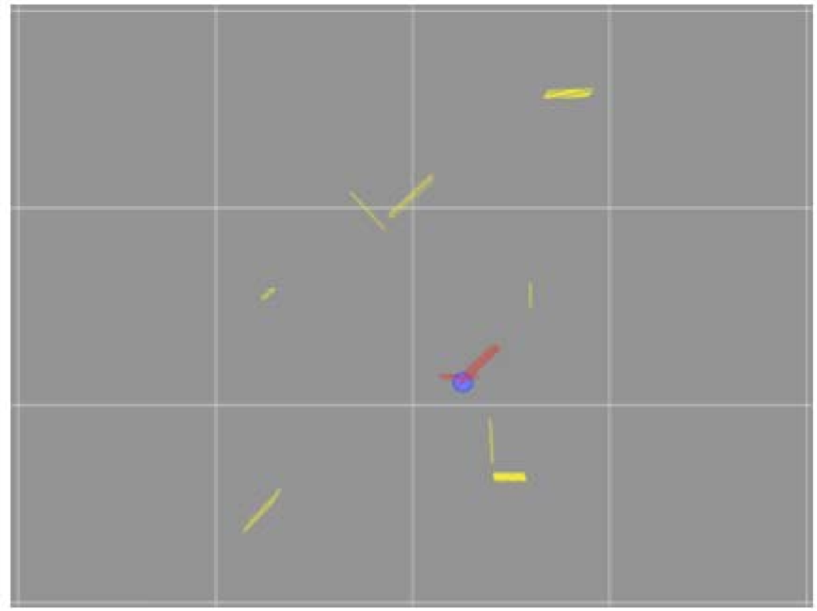


Motif = canonical representation of appearance of repeating element in a lattice



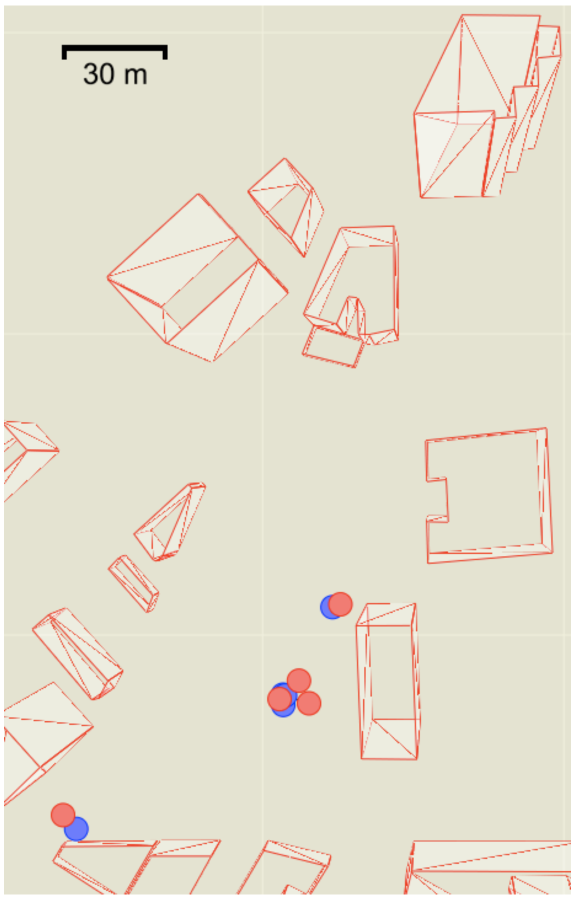
Intersecting Families of Camera Centers





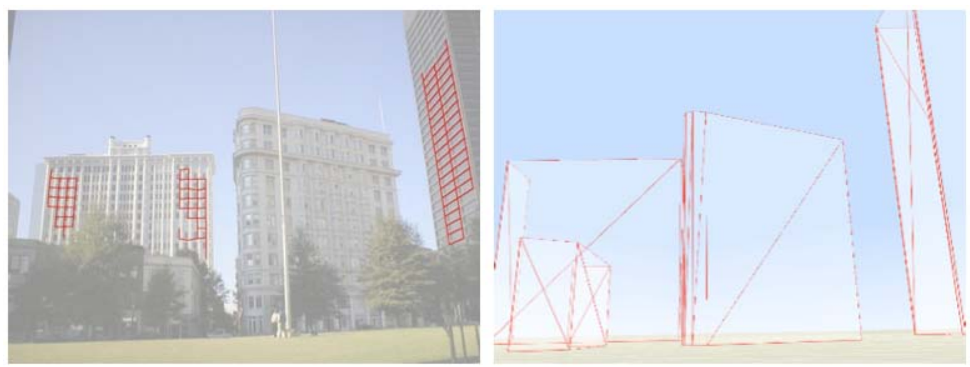
Each 2D-to-3D lattice correspondence produces a family of camera centers due to row/column shift ambiguity. Solution lies at intersection of families.

Geo-tagging Results



Blue - Ground Truth
Red - Estimated Position

Mean Errors
  Distance: 6.04 m
  Yaw: 1.51°
  Pitch: 0.75°



Y. Tsin, Liu, Y., and V. Ramesh. ``Texture Replacement in Real Images", Computer Vision and Pattern Recognition Conference 2001 (CVPR'01), Kauai, December, 2001.

Texture Replacement in Real (un-segmented) Images

Problem definition:

To replace near-regular texture in a scene while preserve the same occlusions, shadowing, and homography using regularity.

New Research Direction #3

- **Image De-fencing**
  Yanxi Liu, Tamara Belkina, James H. Hays, and Roberto Lublinerman
  Computer Vision and Pattern Recognition Conference (CVPR '08)

- Website: http://vision.cse.psu.edu/defencing.htm

Image De-fencing

CVPR 2008

(a) Leopard in a Zoo

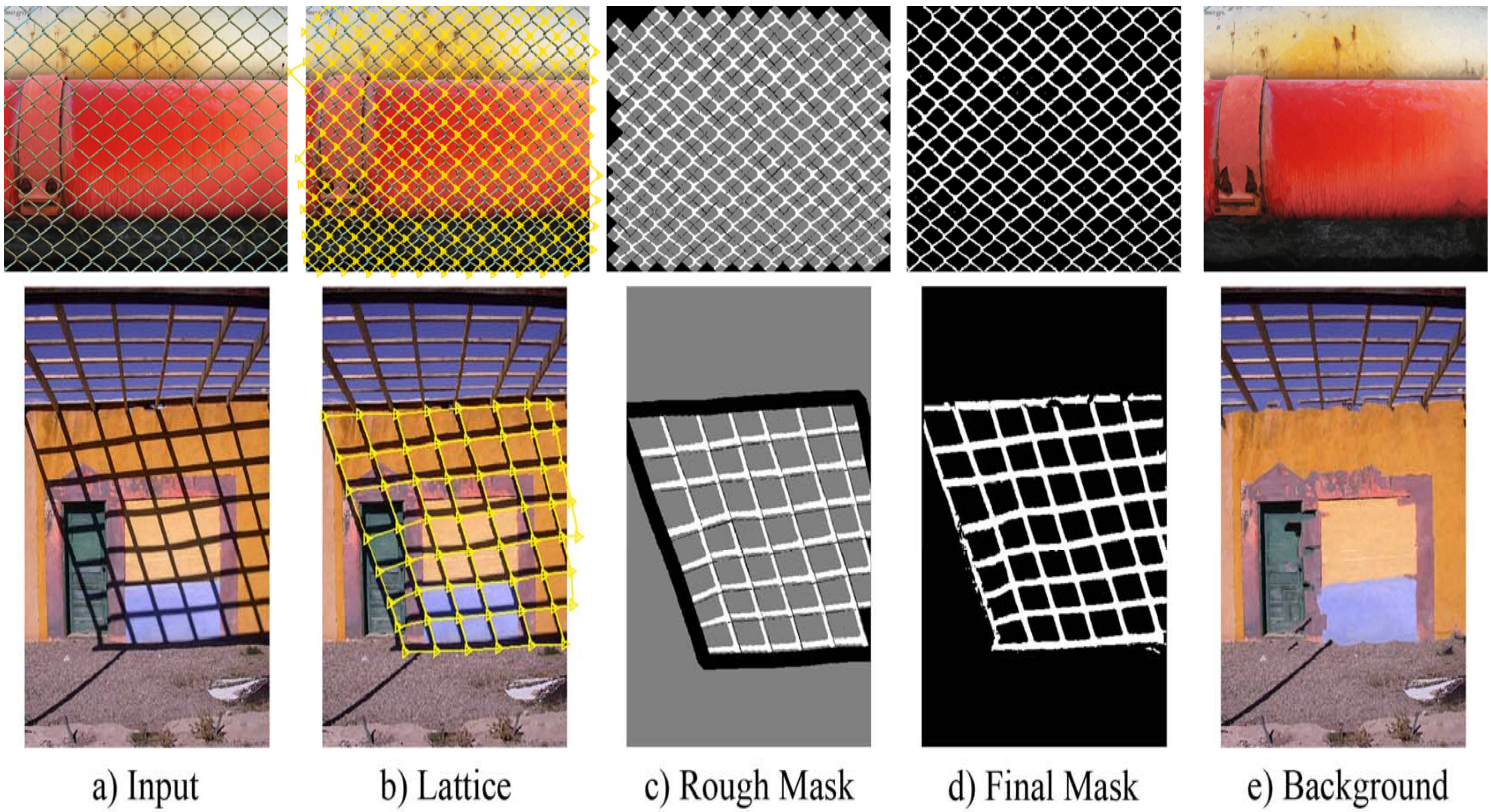
(b) Leopard in the wild

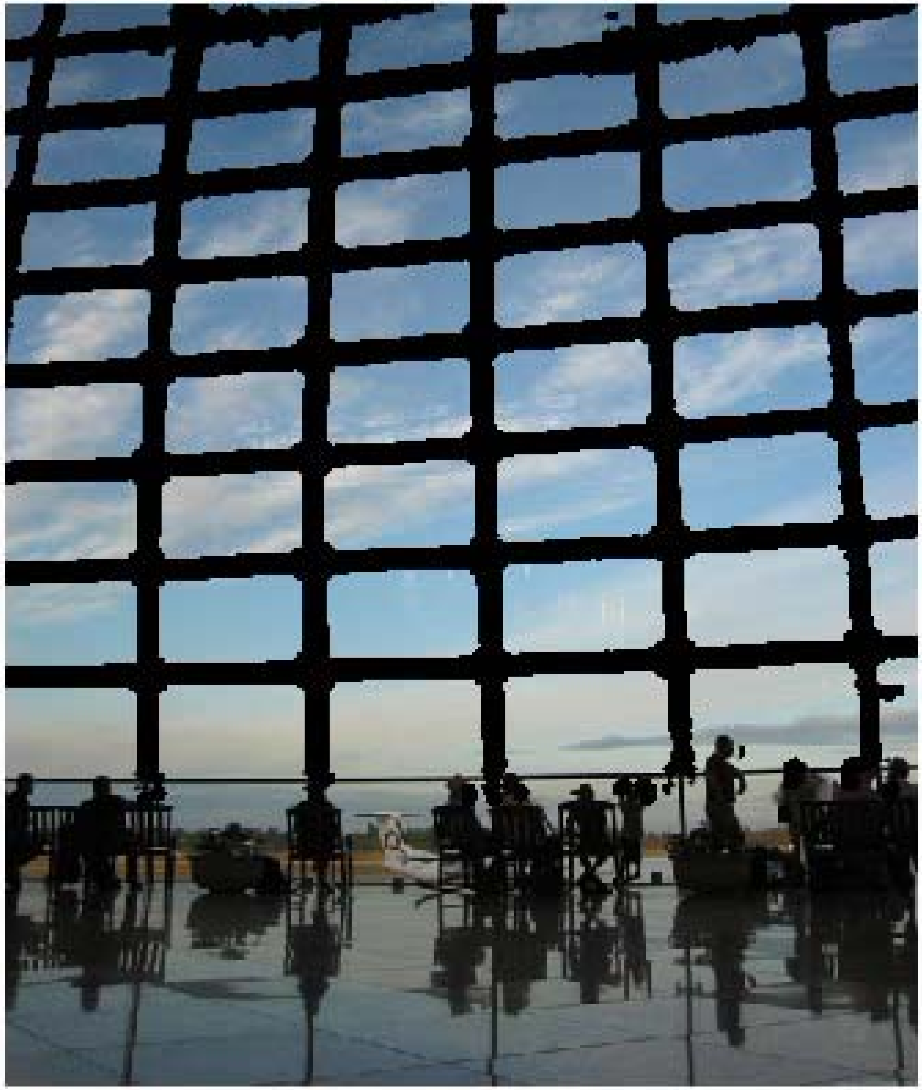
(c) People in an airport

(d) People on a deck

Image De-fencing (cont.)

Basic Idea: (1) lattice detection → (2) build a foreground/background classifier (a binary mask) → (3)in-painting








New Research Direction #4
Dynamic NRT Tracking

• **Tracking Dynamic Near-regular Textures under Occlusion and Rapid Movements**
W. Lin* and *Y. Liu
***9th European Conference on Computer Vision*, May, 2006.**

• **A Lattice-based MRF Model for Dynamic Near-regular Texture Tracking**
W. Lin* and *Y. Liu
***IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 29, No. 5, May, 2007, pp. 777 - 792.**

Various Forms of Dynamic NRTs

Cloth motion

Underwater pattern

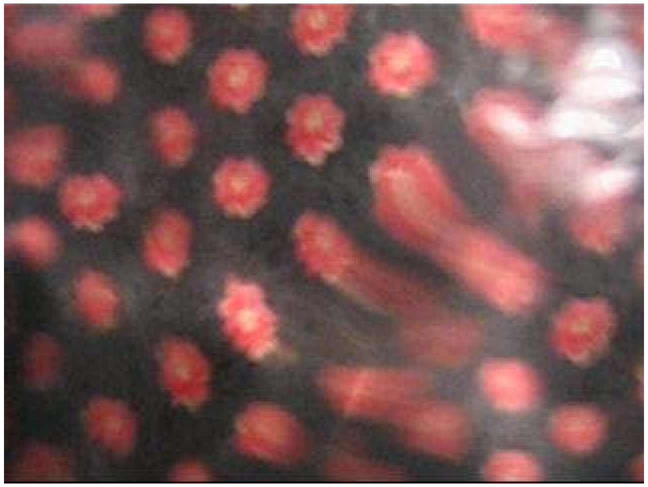
Towel folding

Crowd Marching

Challenges of Dynamic NRT Tracking

Ambiguity

Motion blur



Occlusions

Static Near-Regular Texture (NRT)

Tiles = T(x,y)

A set of statistically varied samples (in shape, color, intensity) of a canonical tile $T_0(t_1, t_2)$

Dynamic NRT (dNRT): NRT with Motion

Tiles = T(x,y,time)

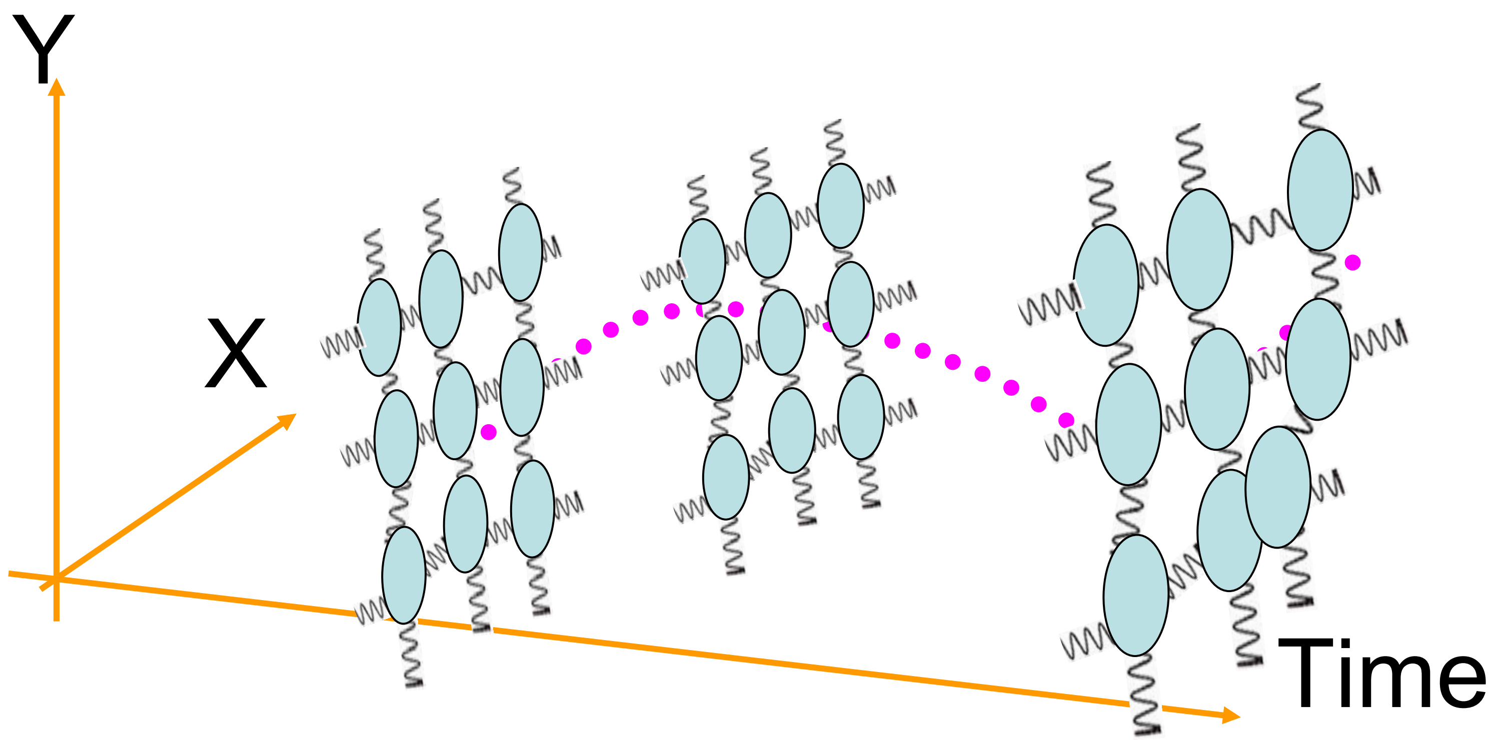
Appearance varies spatiotemporally

Basic Idea for Dynamic NRT Tracking

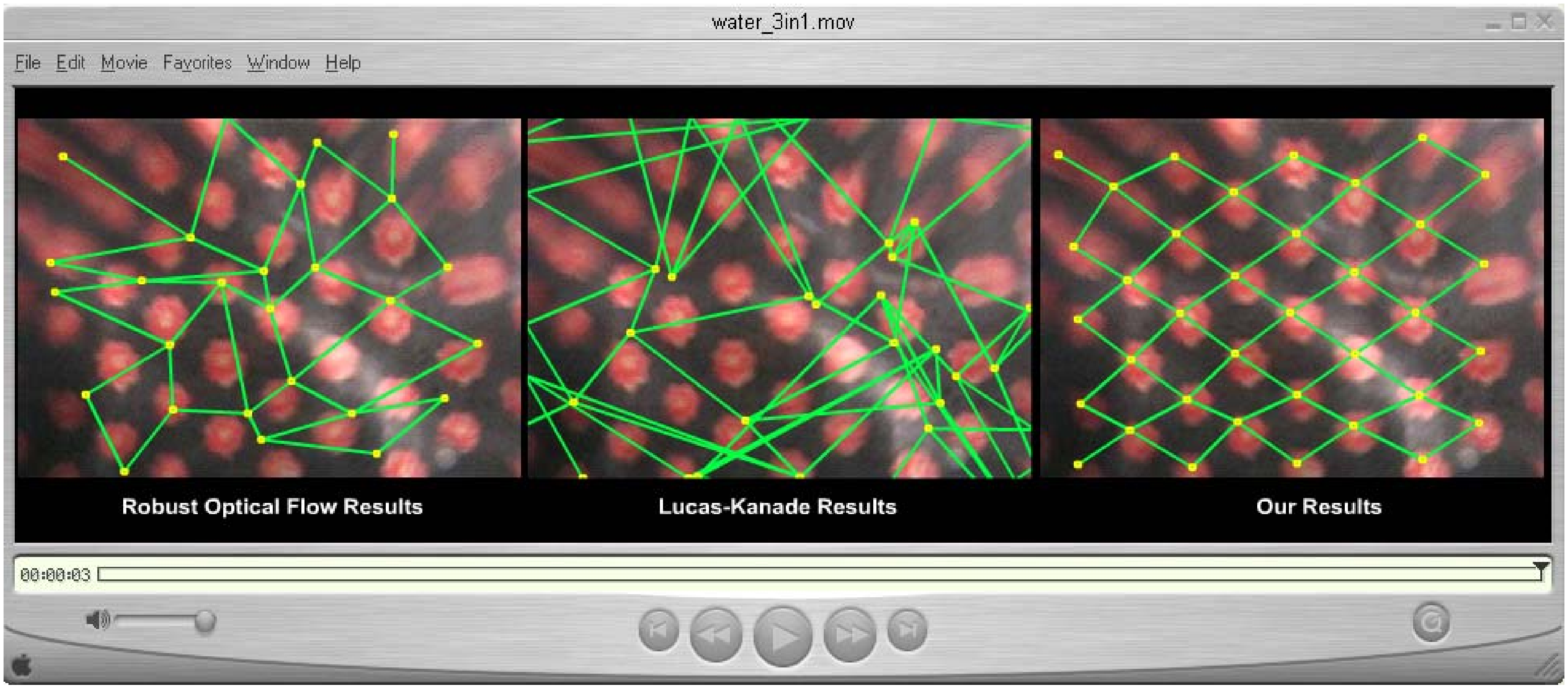
Topological Invariance: The topological structure of a dynamic NRT remains invariant regardless of the geometry and photometry variations

Lattice-based Markov Random Field Model

View spatiotemporal constraints as a network of springs and model them as an MRF



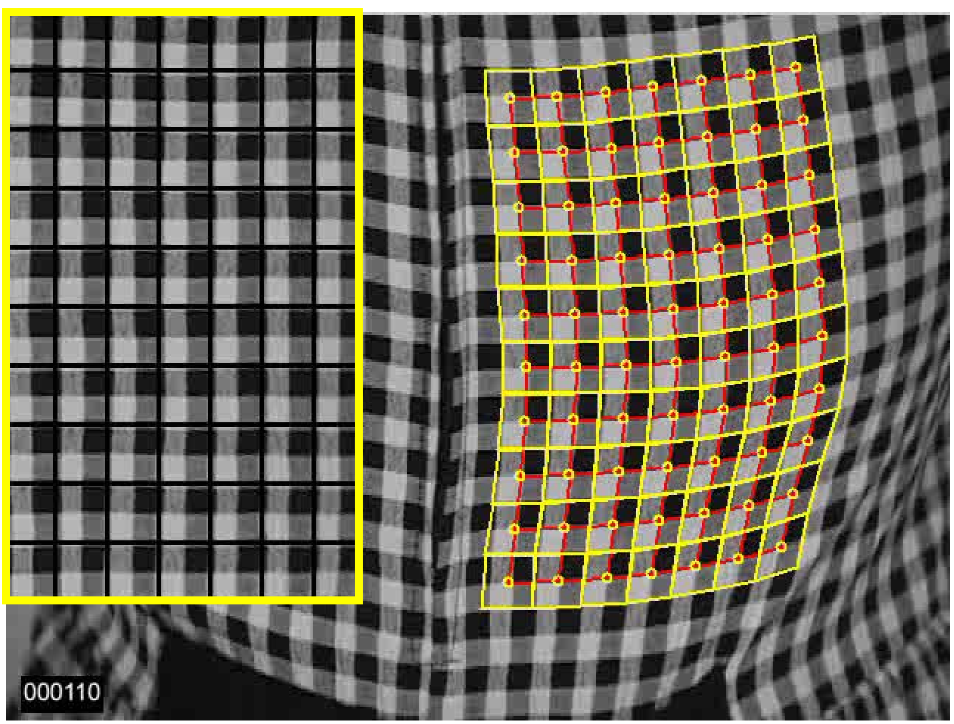
Tracking Underwater Texture



(play outside of ppt)

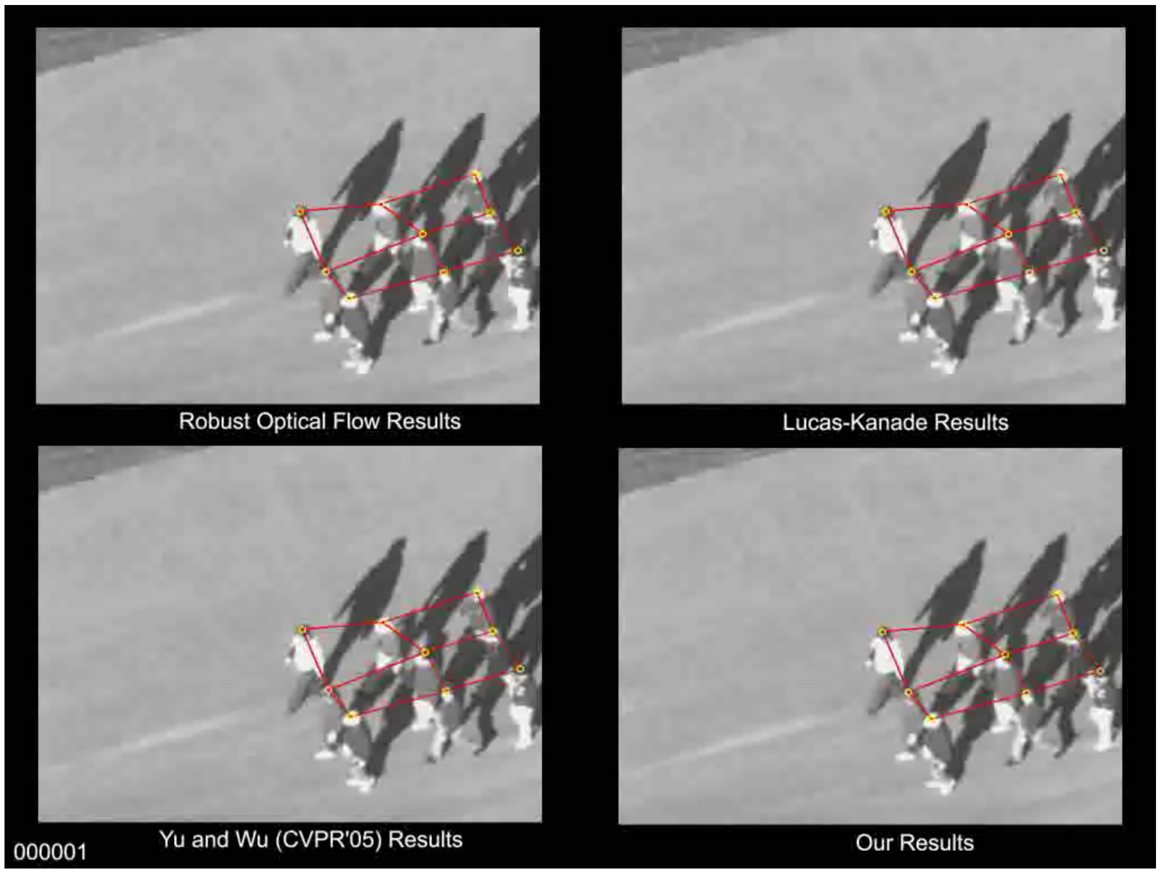
Tracking Fabric Texture under Occlusion

visibility map

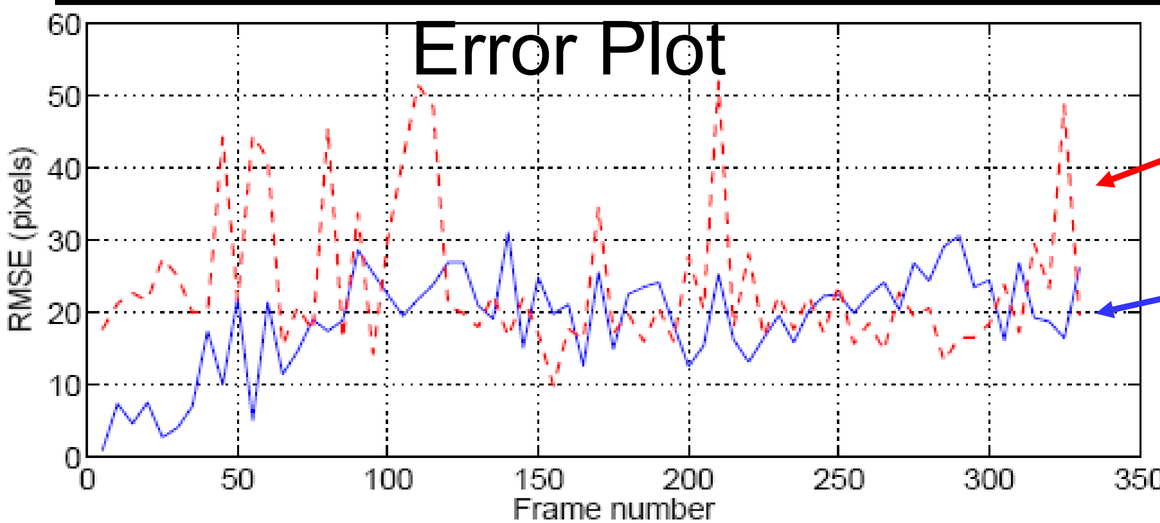


play movies

Tracking Marching Crowd



Tracking Marching Crowd



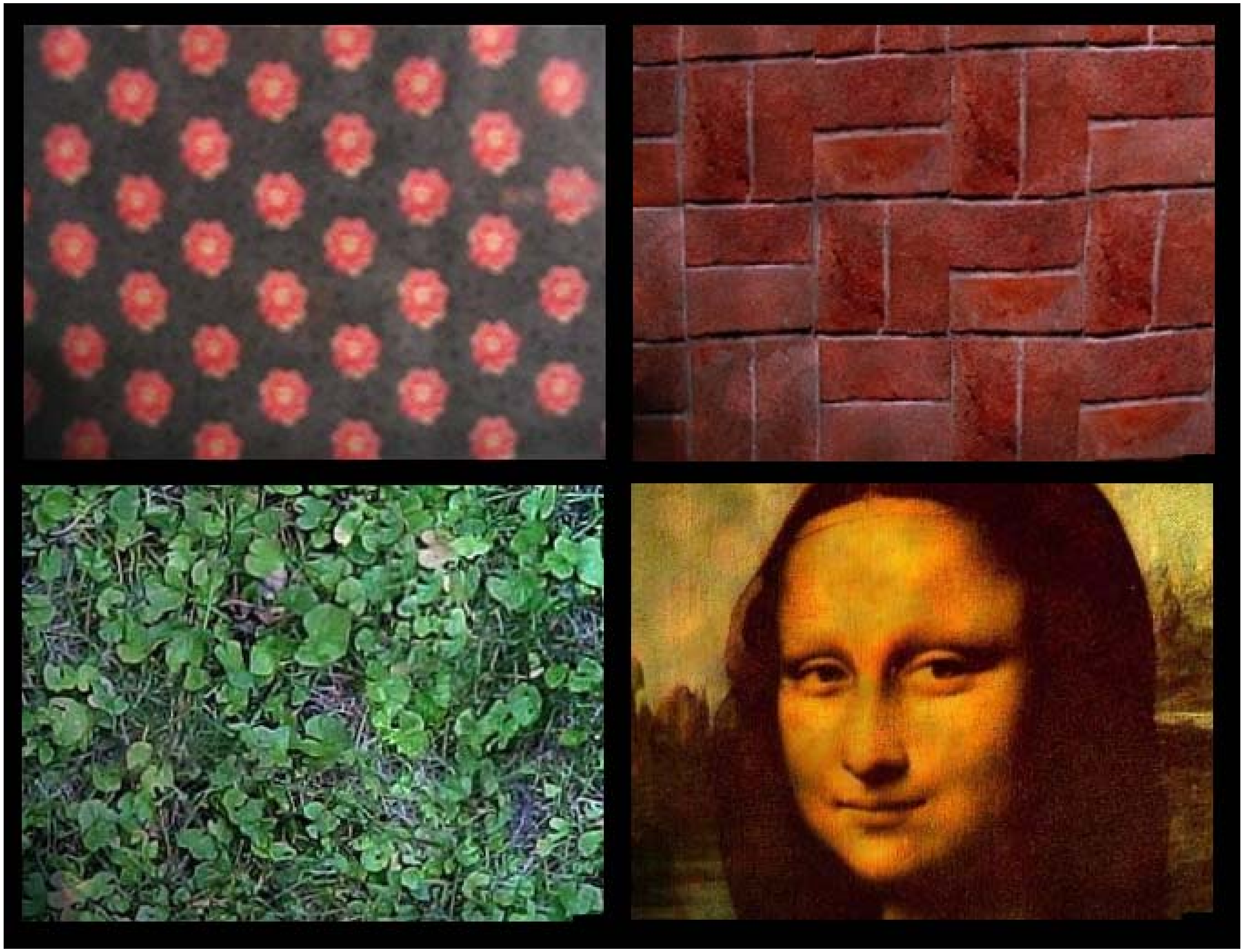
Yu & Wu

Total RMSE = 25.9

Ours

Total RMSE = 20.2

Dynamic NRT Tracking and Replacement



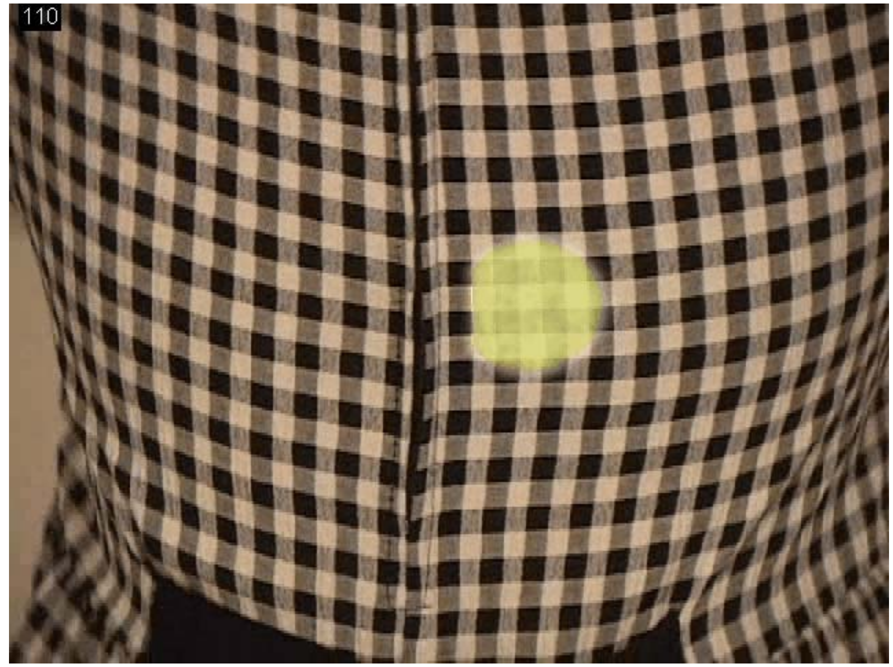
ECCV 2006 Lin and Liu

Dynamic Texture Replacement Results

Word “USA” Overlaid video

PAMI 2007/ECCV 2006 Lin and Liu

Word "USA" Overlaid video



PAMI 2007/ECCV 2006 Lin and Liu

New Advance

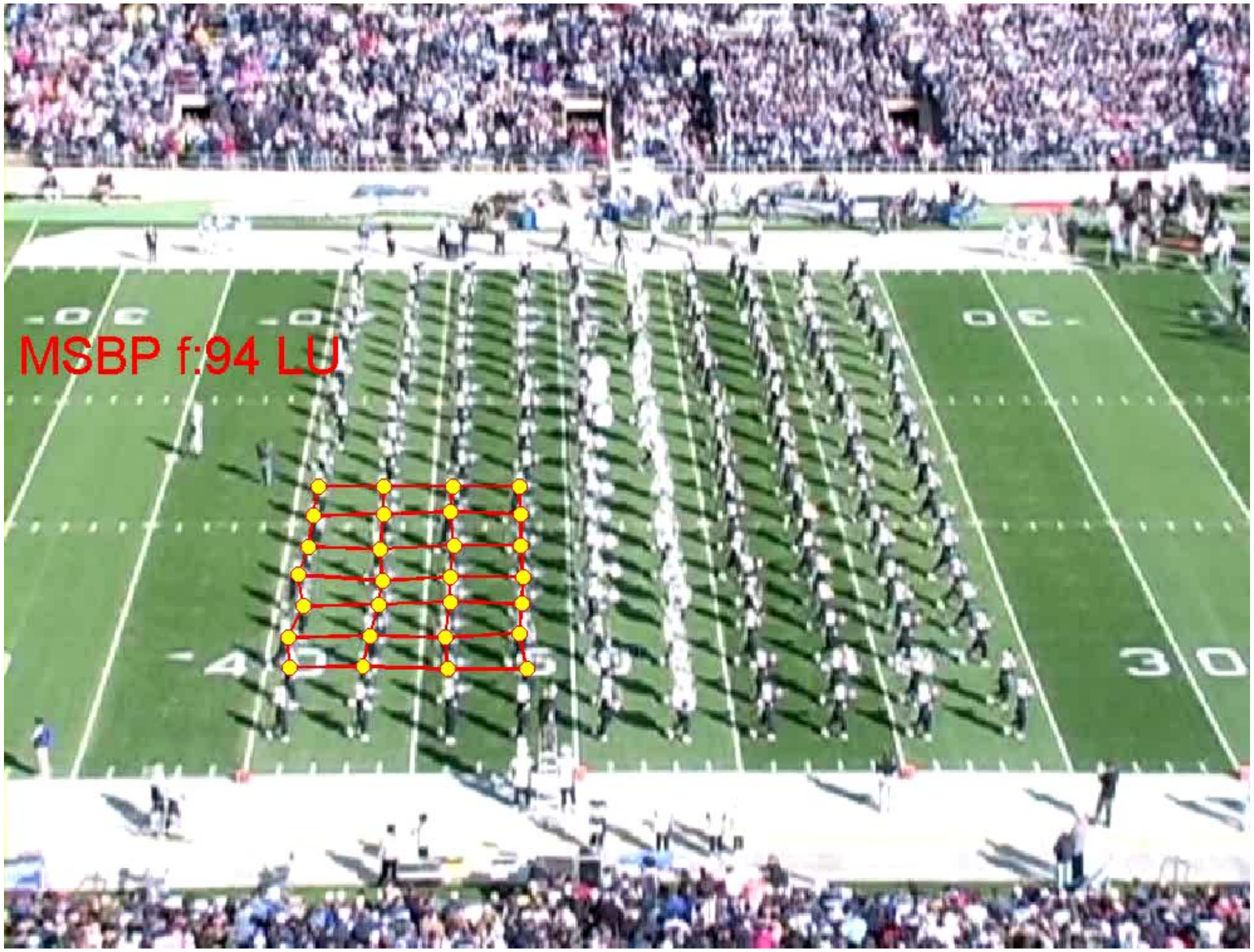
On Dynamic Near Regular Textures

-- large disturbance/distortions

-- lattice topological variations

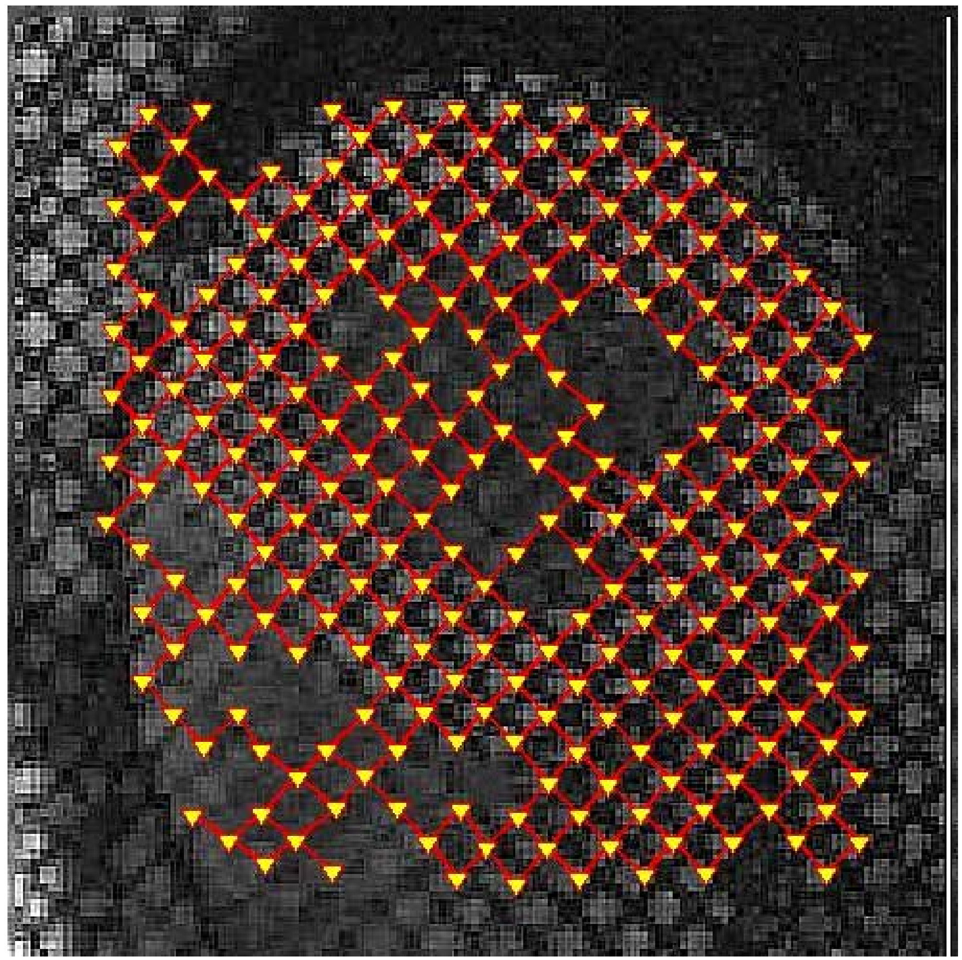
-- multiple dynamic lattices

Blue Band Tracking



Blue Band Tracking

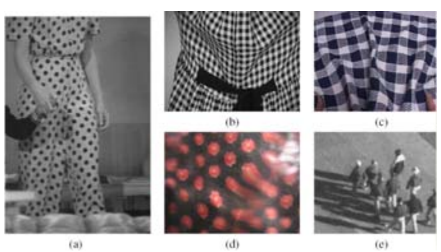
MRI Cardiology Video



Regularity Spectrum of dynamic Textures

Regular → Near-regular → … → chaotic

Stationary global lattice topology



Adaptive Local topology

Clustered local topology

Non-uniform local topology

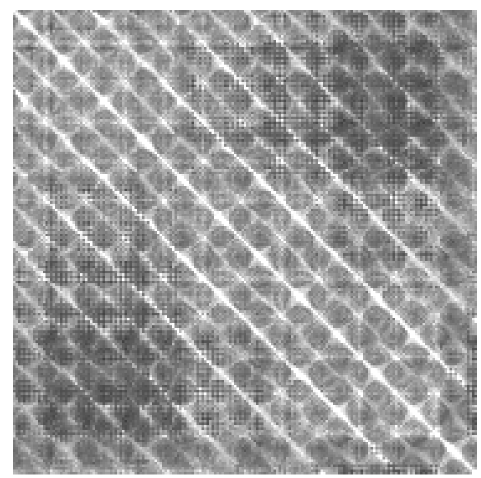
With Increasing level of difficulty
Decreasing level of regularity

Motion Texture Analysis

- Liu, Y., Collins,R.T. and Tsin, Y. ``Gait Sequence Analysis using Frieze Patterns'', European Conference on Computer Vision 2002 (ECCV'02). Copenhagen, Denmark. May 28-31, 2002.
- Liu, Y., Collins,R.T. and Tsin, Y. ``A Computational Model for Periodic Pattern Perception Based on Frieze and Wallpaper Groups'', IEEE TPAMI. March 2004.

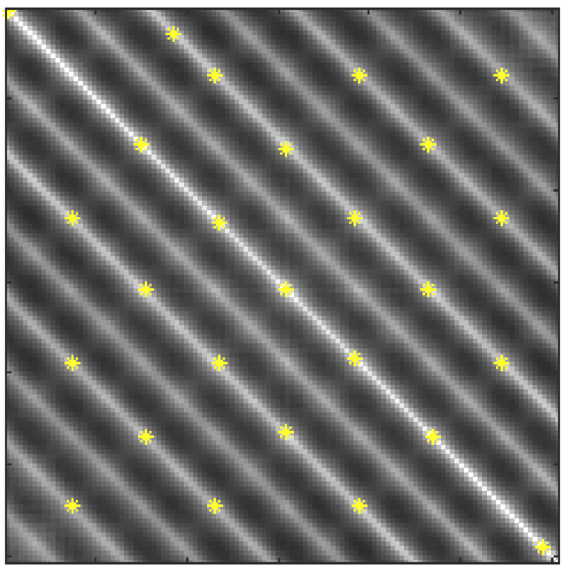
Gait Analysis using Symmetry Groups

background subtraction

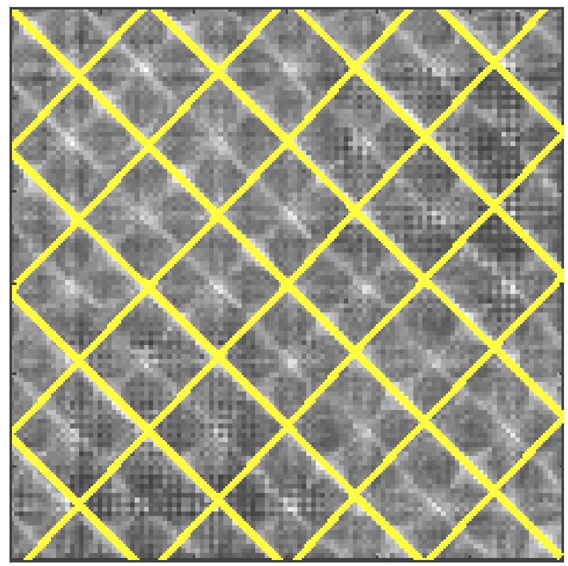


cross correlation(frame-I,frame-J)

Symmetry of A Walking Human

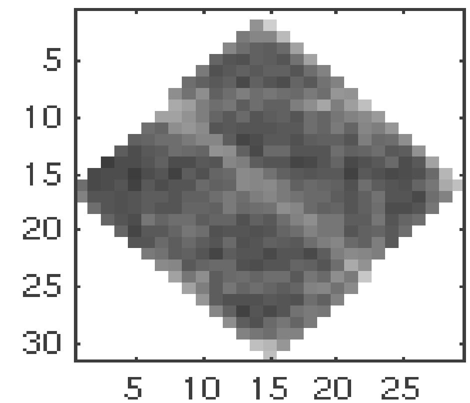


Autocorrelation peaks



Lattice

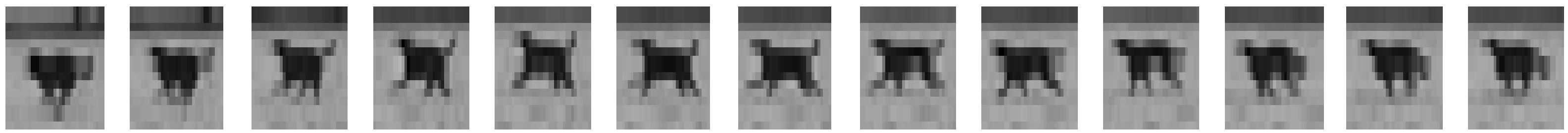
p4m

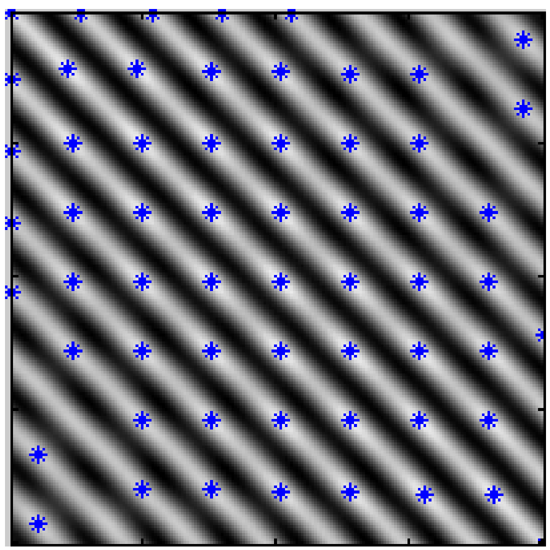


Unit tile

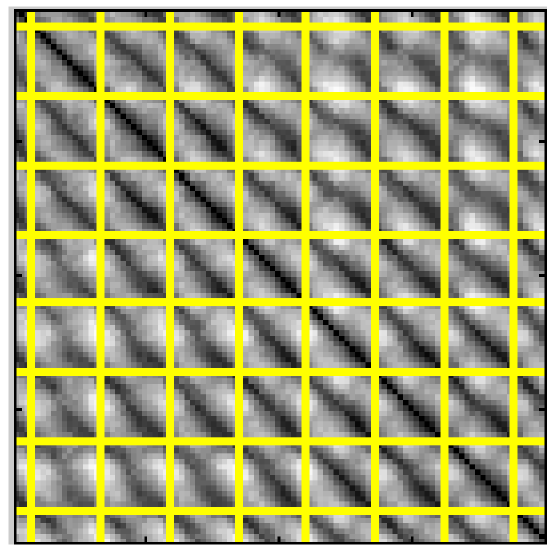
rot180	rot120	**rot90**	rot60	**flipT1**	**flipT2**	**flipD1**	**flipD2**
0.484	1.110	0.924	1.143	0.567	0.892	0.891	0.835

Symmetry of A Running Dog



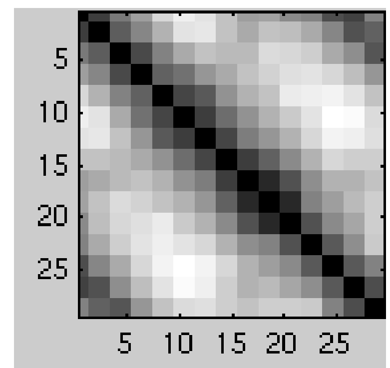


Autocorrelation peaks



Lattice

cmm

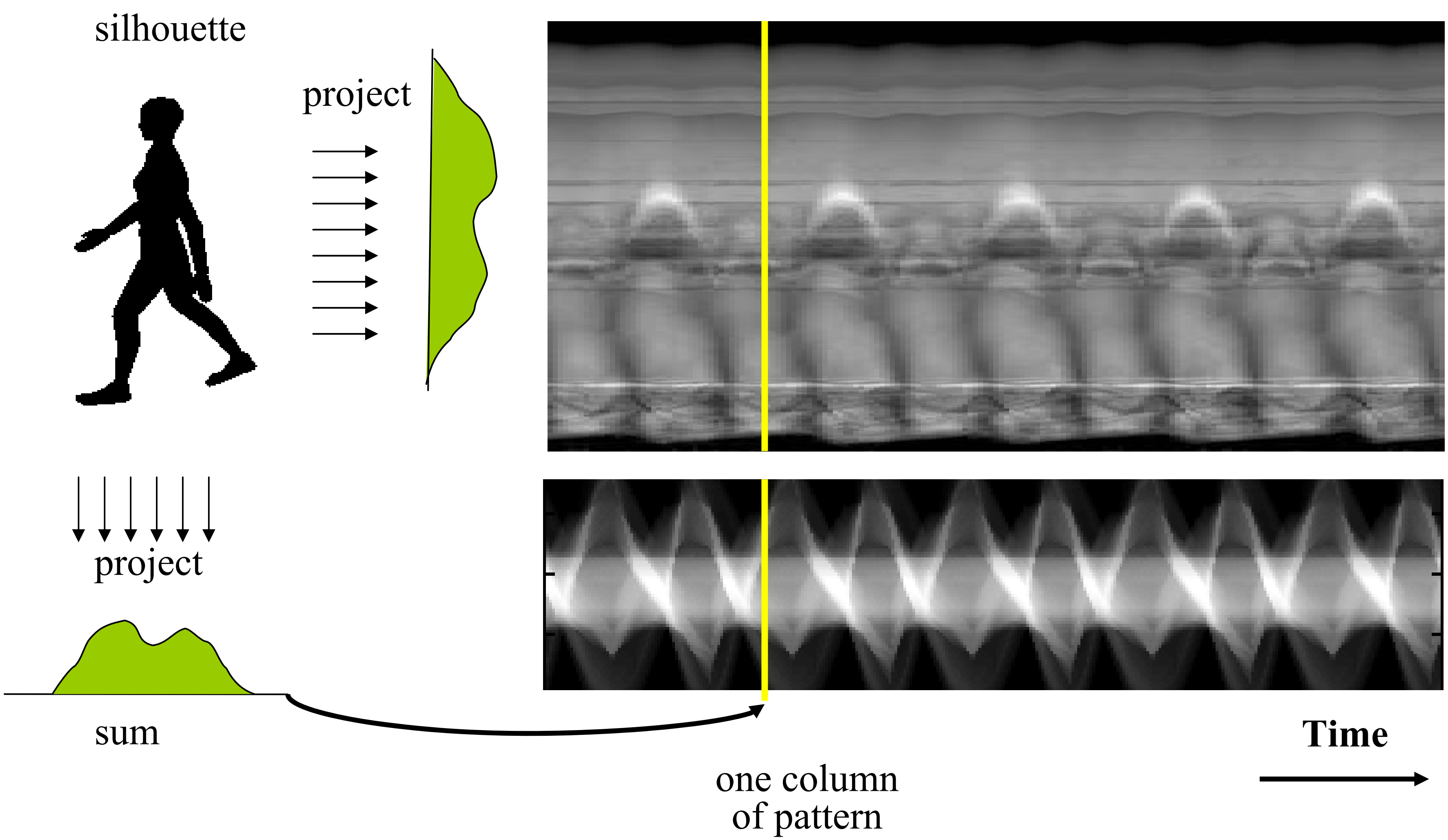


Unit tile

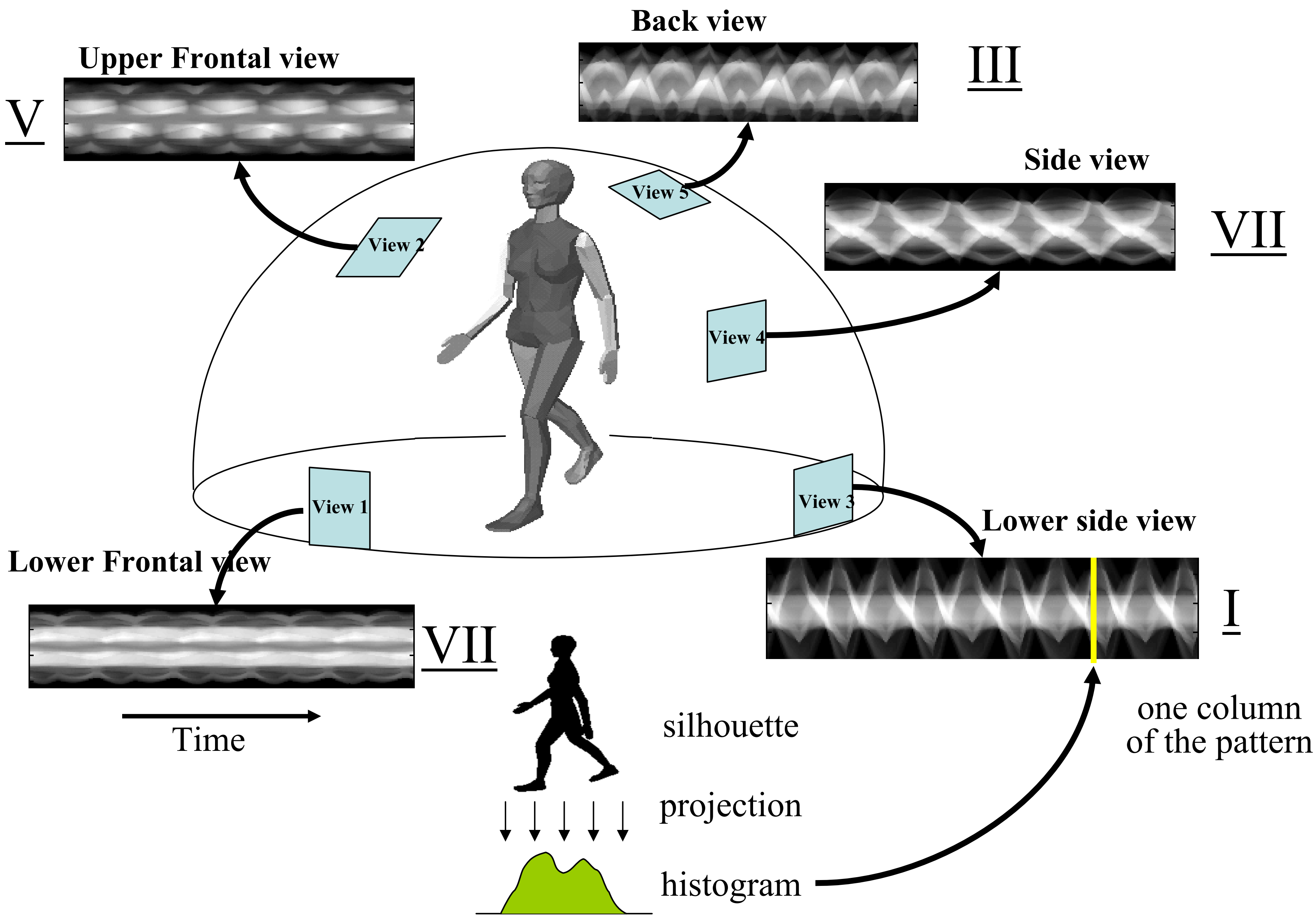
rot180	rot120	rot90	rot60	flipT1	flipT2	**flipD1**	**flipD2**
0.513	1.301	1.284	1.241	1.311	1.311	0.613	0.558

CMU Mobo Database

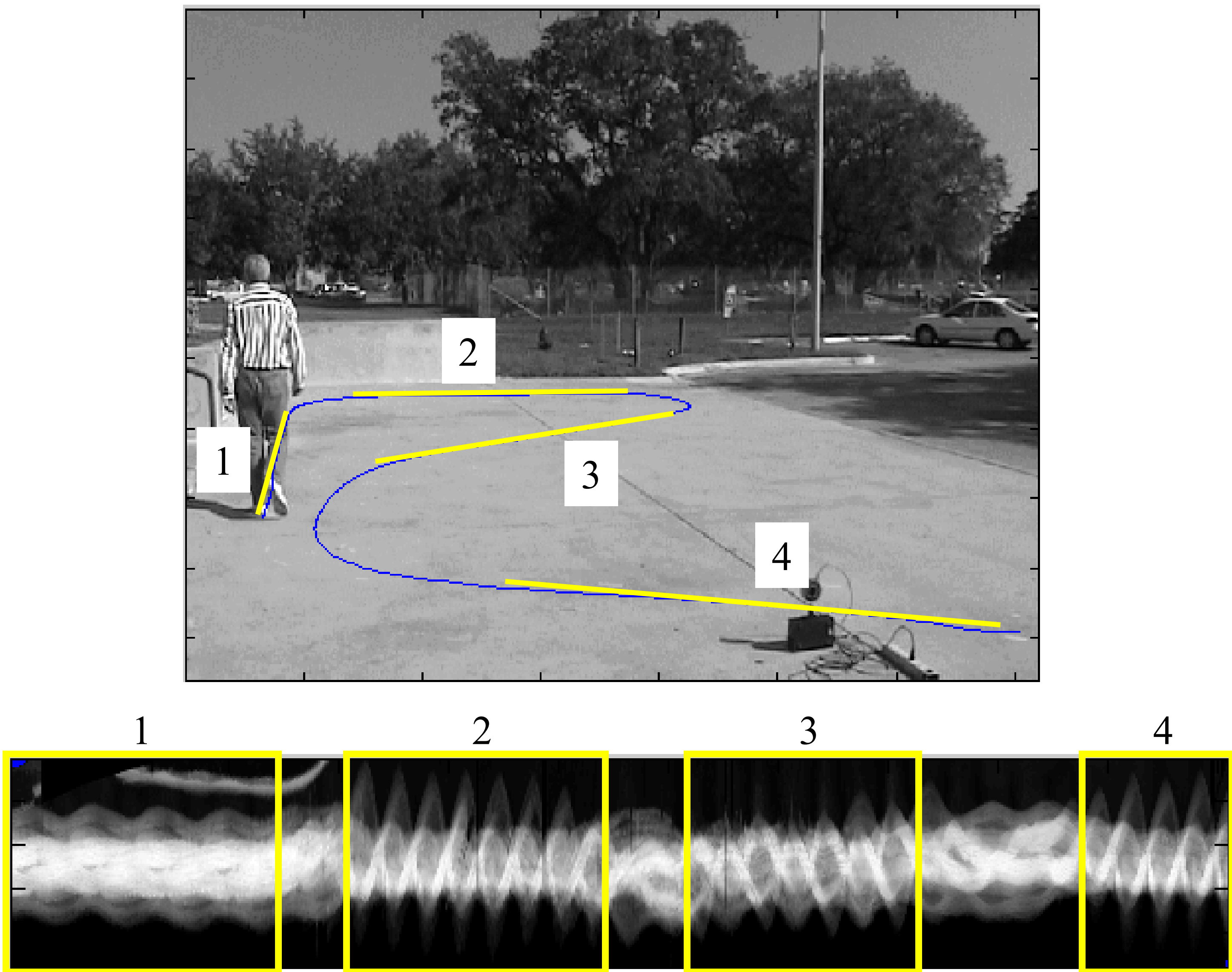
Spatiotemporal Projection into Frieze-like Patterns



Observation #1:



Gait Segments based on Frieze Patterns



Observation #2:

Camera | 'Nancy' | A male subj | A female subj

#3

#5

#7

#13

#16

#17

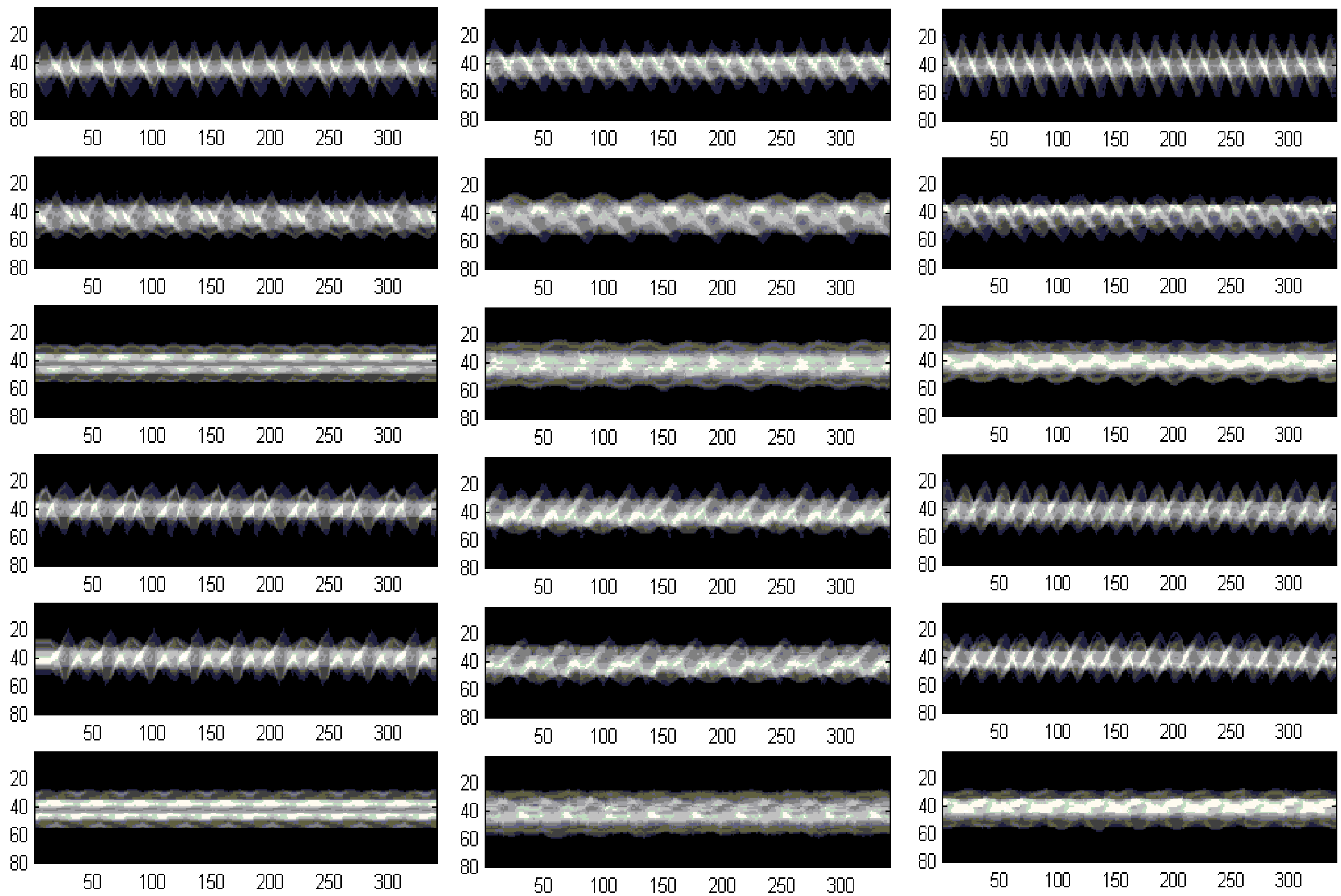
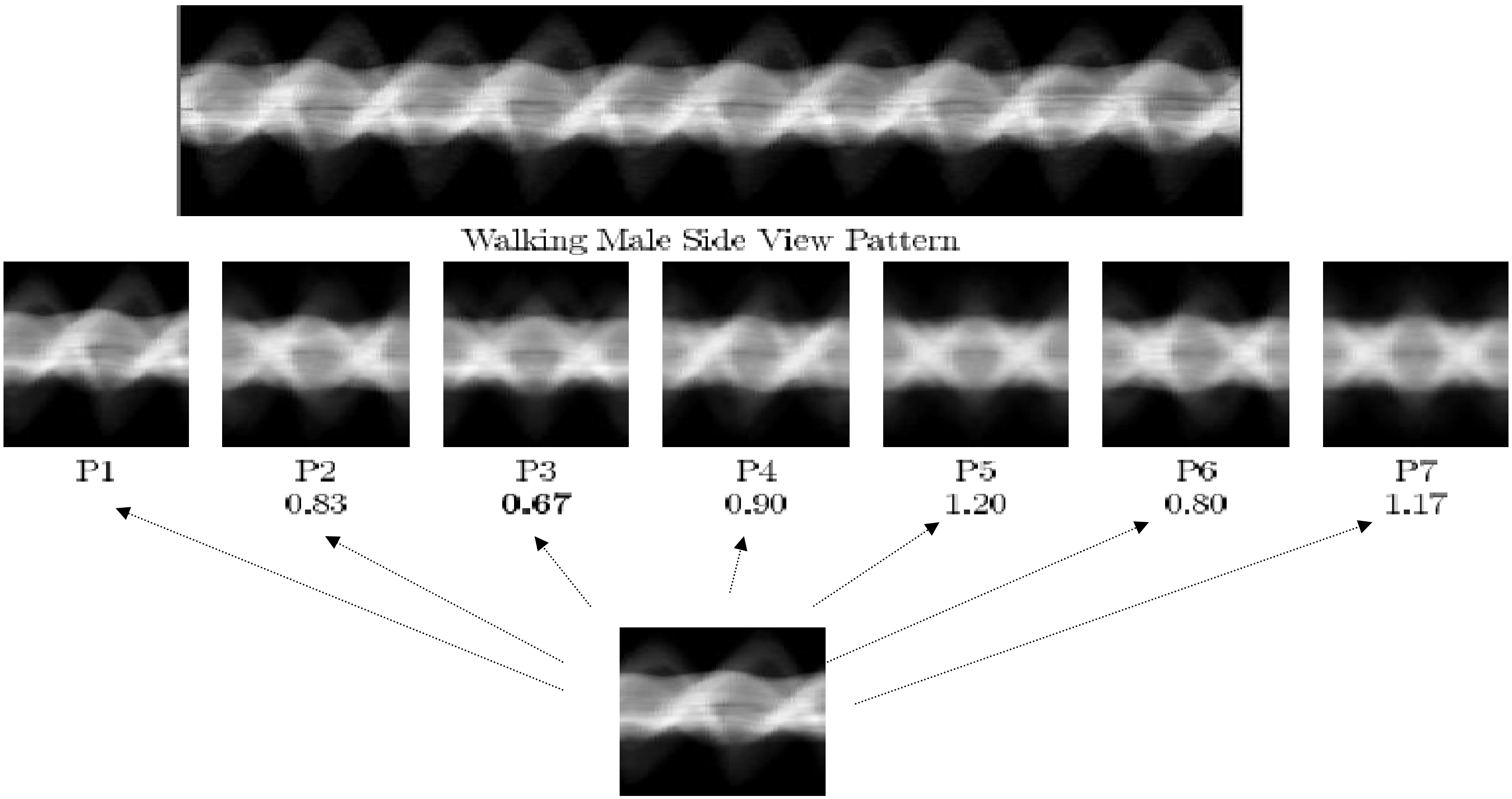


Table 1. Symmetries of frieze pattern tiles (N is number of pixels in one tile)

Symmetry Group	translation	2-fold rotation	Horizontal reflection	Vertical reflection	Glide reflection	Degrees of Freedom
F1	yes	no	no	no	no	N
F2	yes	no	no	no	yes	N/2
F3	yes	no	no	yes	no	N/2
F4	yes	yes	no	no	no	N/2
F5	yes	yes	no	yes	yes	N/4
F6	yes	no	yes	no	no	N/2
F7	yes	yes	yes	yes	no	N/4

Perils!!! The computed symmetries of a near-periodic pattern may not belong to any of these groups!

Symmetry Group As a Continuous Feature



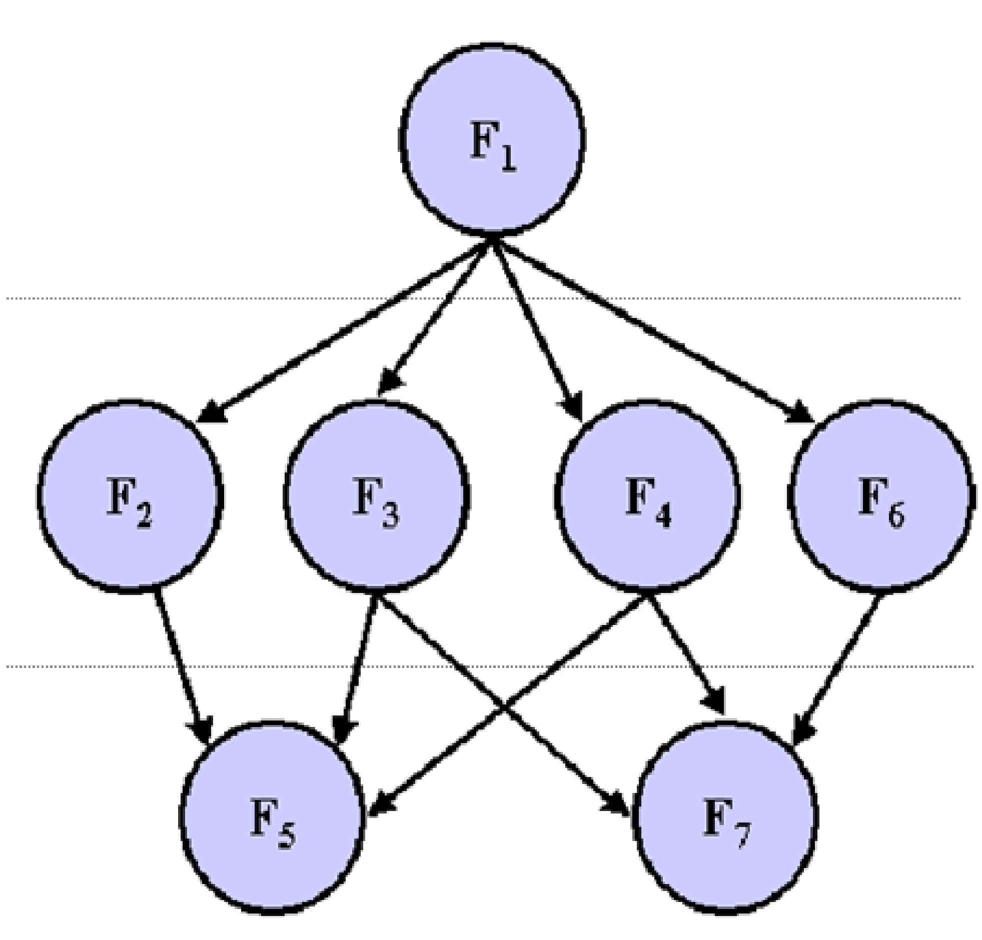
Symmetry As A Continuous Feature by Zabrodsky, Peleg, Avnir 1995

Symmetry Distance (SD)

between a near-periodic patter P and a frieze pattern Q

$$SD_n(P) = \min_{Q \in \{P_n\}} \{ \sum_{i=1}^{tN} \left(\frac{p_i - q_i}{s_i} \right)^2 \}$$

Hierarchy of Frieze patterns (groups)



MODEL Selection

Apply Geometric AIC (Kanatani 1996)

Given two frieze patterns whose symmetry groups have a subgroup relationship, G-AIC states that we prefer group F_m over F_n if

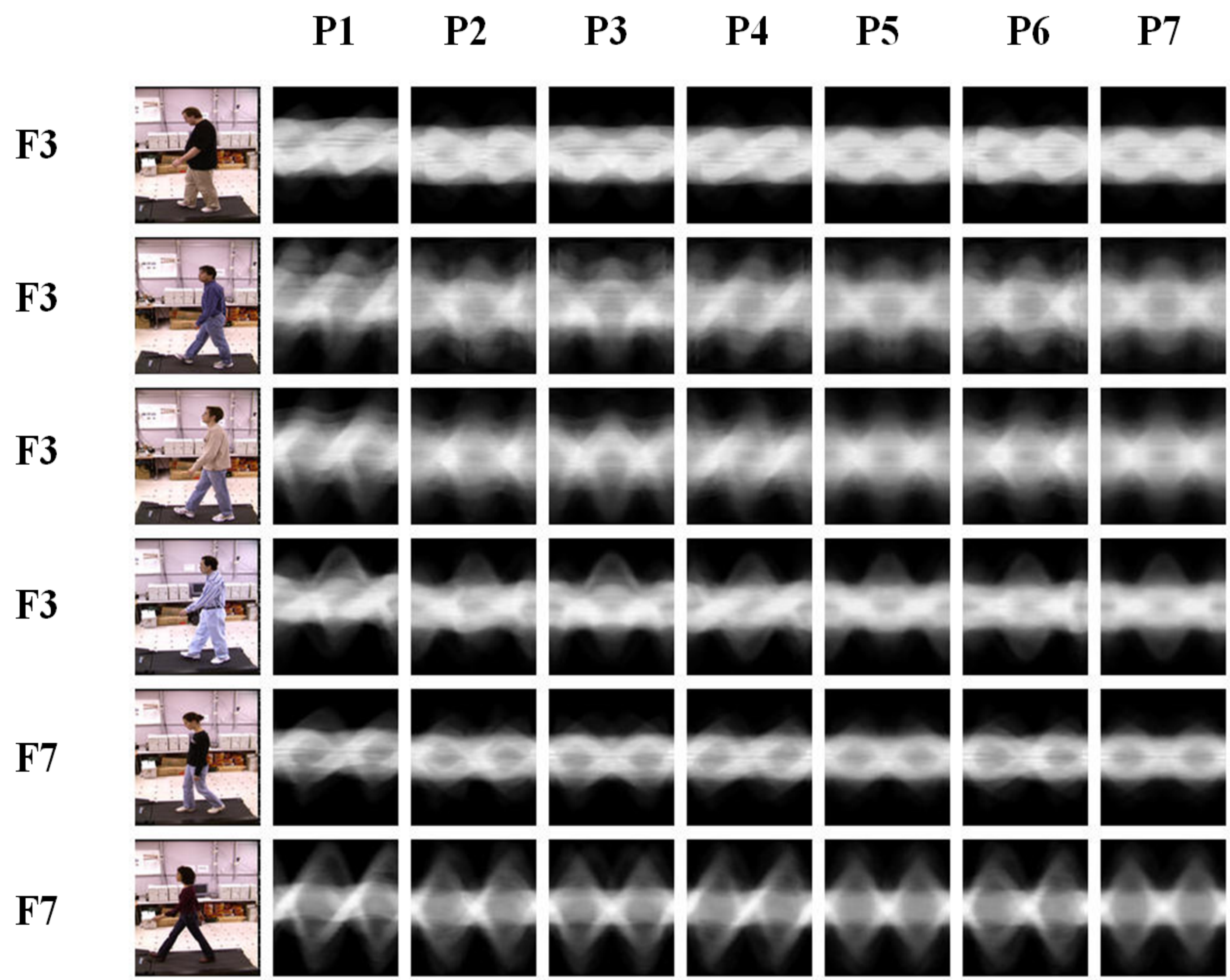
$$\frac{SD_m}{SD_n} < 1 + \frac{2(d_n - d_m)}{r(tN) - d_n} \tag{2}$$

$$\frac{SD_m(P)}{SD_n(P)} < \frac{t}{t-1}, \text{ for } m = 2,3,4,6 \text{ and } n = 1 \tag{3}$$

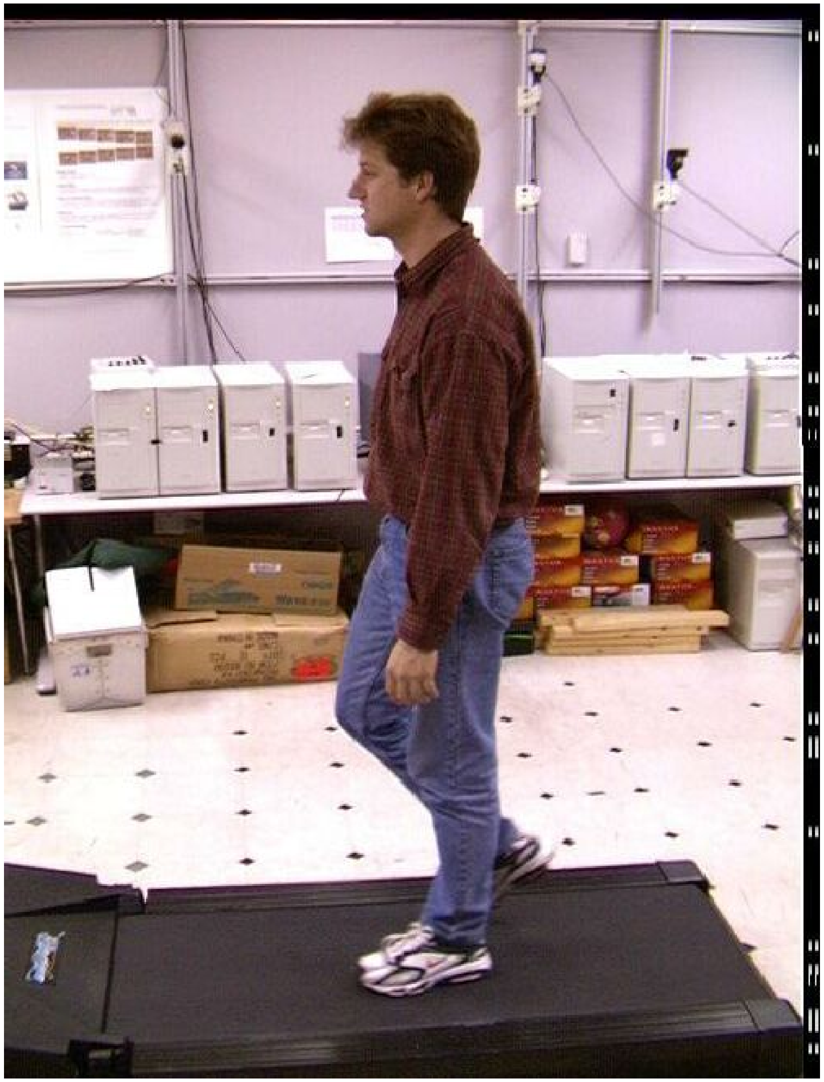
$$\frac{SD_m(P)}{SD_n(P)} < \frac{2t}{2t-1}, \text{ for } m = 5,7 \text{ and } n = 2,3,4,6 \tag{4}$$

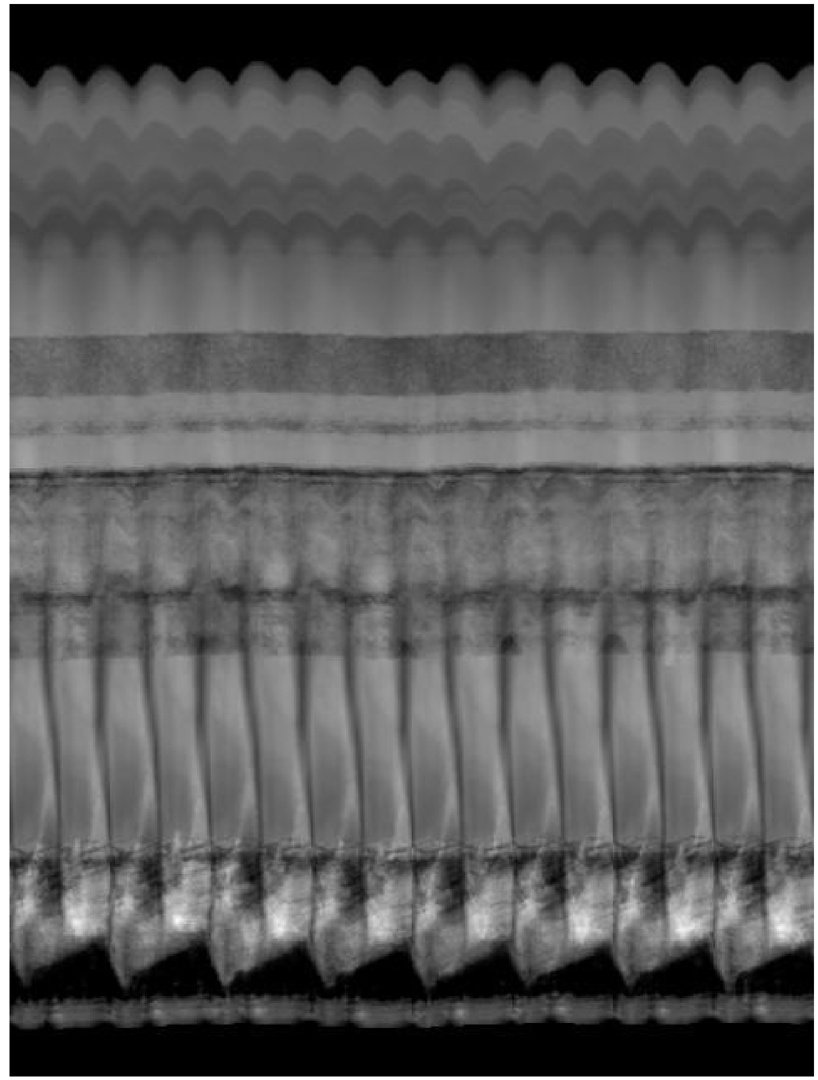
$$\frac{SD_m(P)}{SD_n(P)} < \frac{2t+1}{2t-2}, \text{ for } m = 5,7\text{and } n = 1 \tag{5}$$

Sample Estimation Results: side views (23/25)

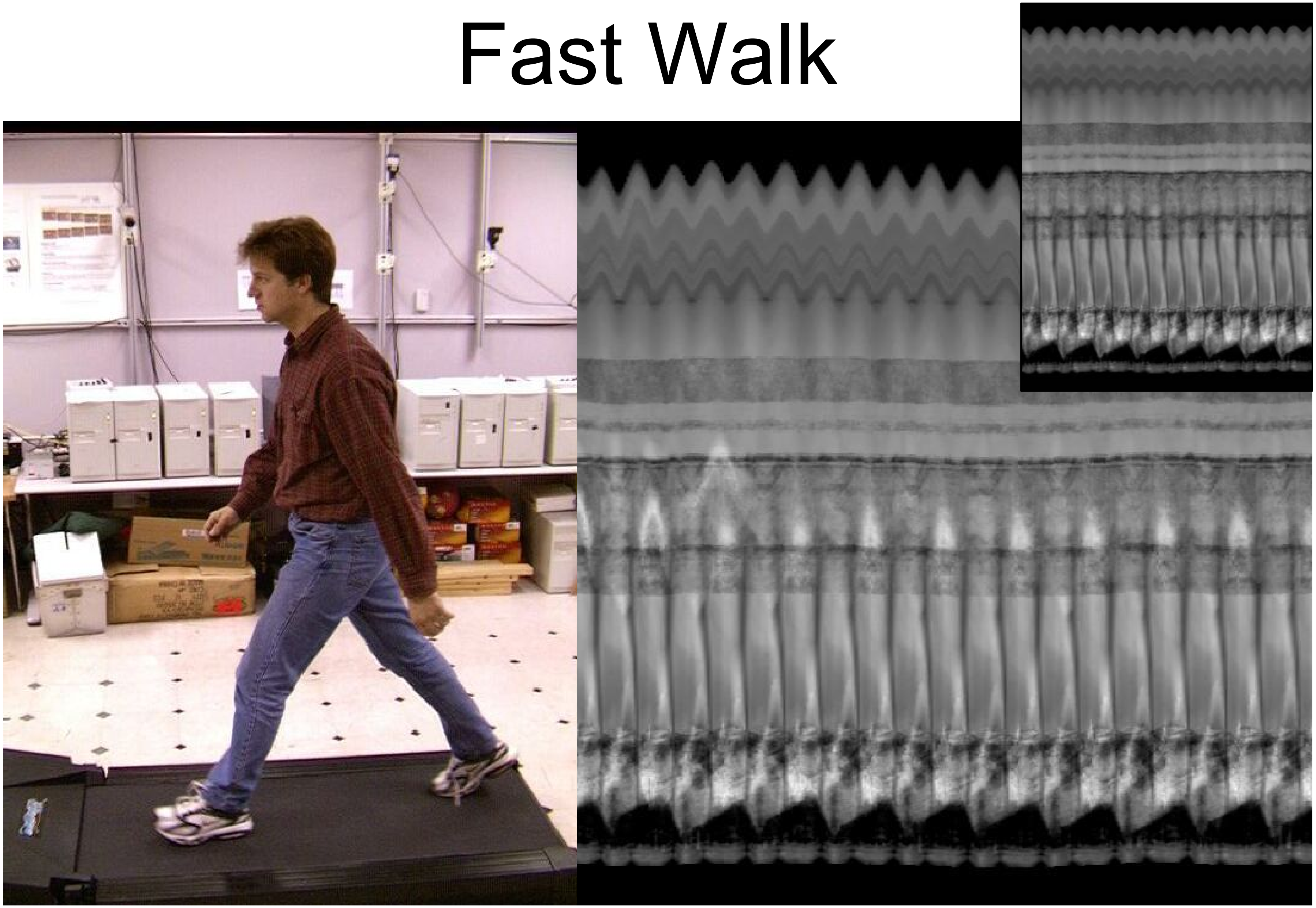


Slow Walk

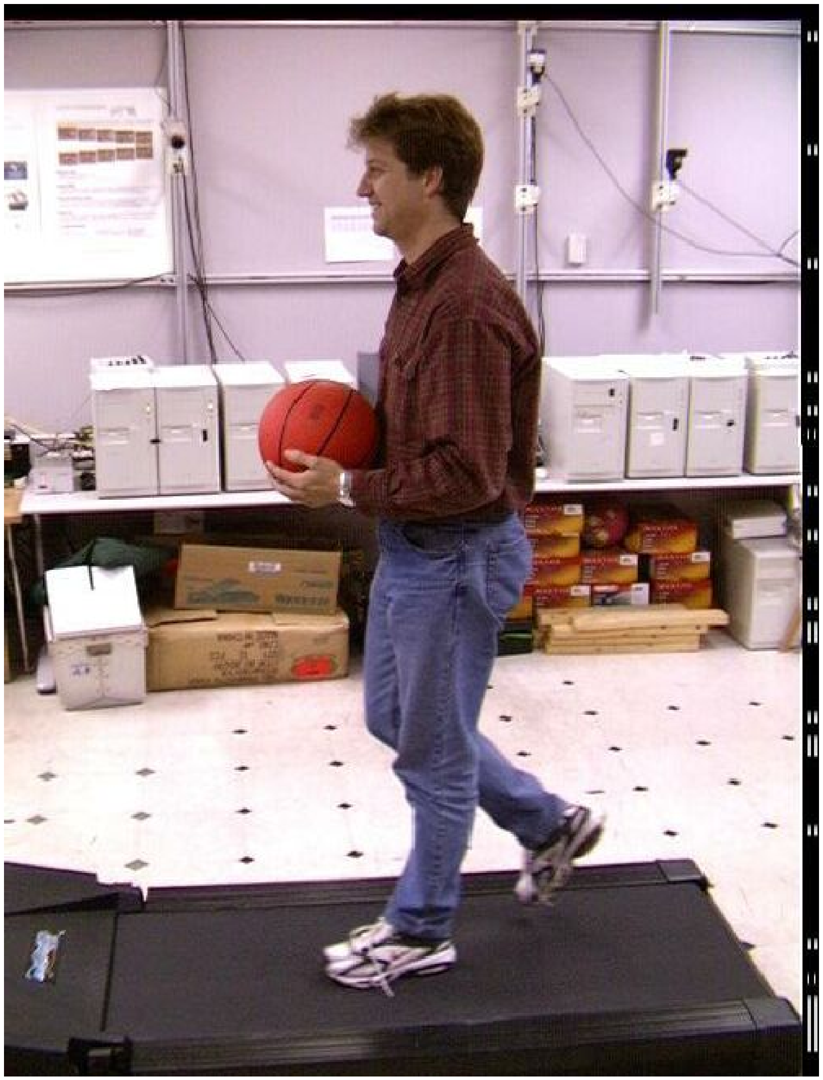


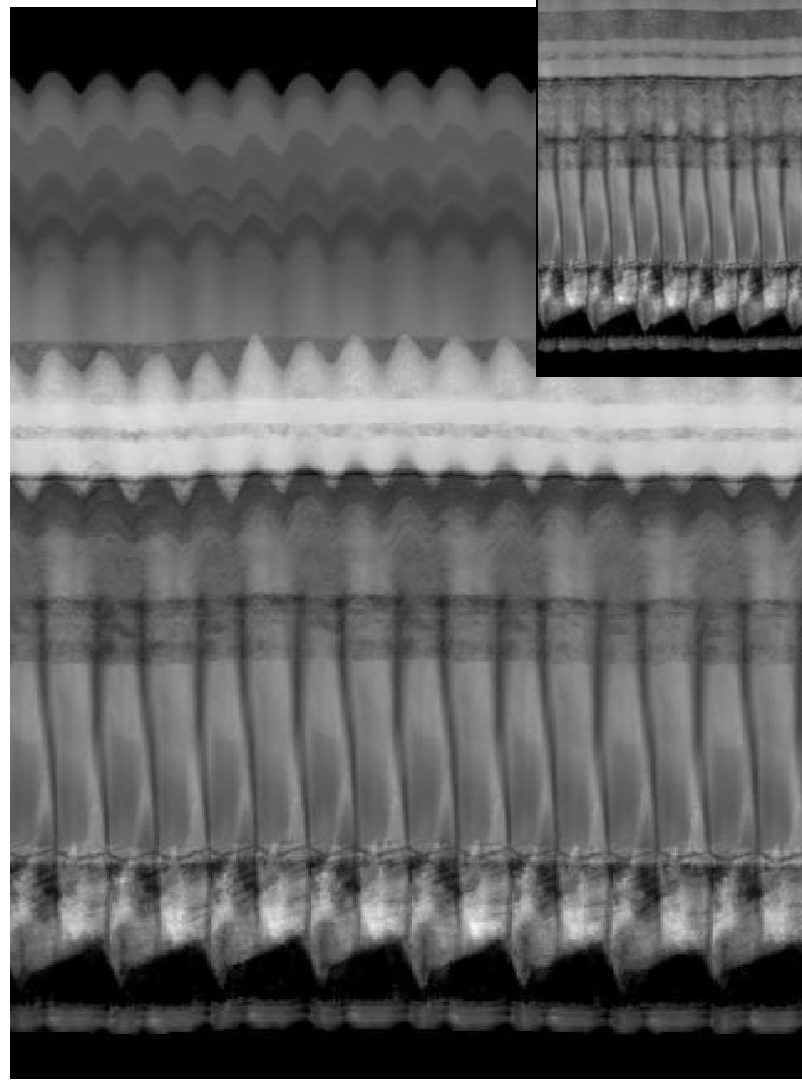


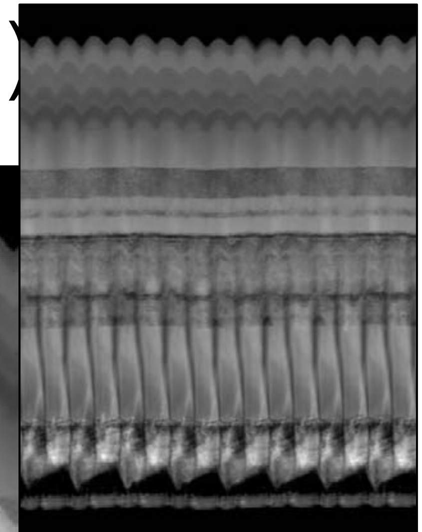
Fast Walk



Ball Carry (no arm swing)







Subject Classification Rates

Correlation of Frieze Tiles

Training	**Testing**		
Slow	Slow	Fast	Ball
	100%	100%	81%

Extracting key frames and doing direct silhouette matching

Training	**Testing**		
Slow	Slow	Fast	Ball
	100%	96%	91%

Combining two views (profile + frontal)

Training	**Testing**		
Slow	Slow	Fast	Ball
	100%	100%	100%

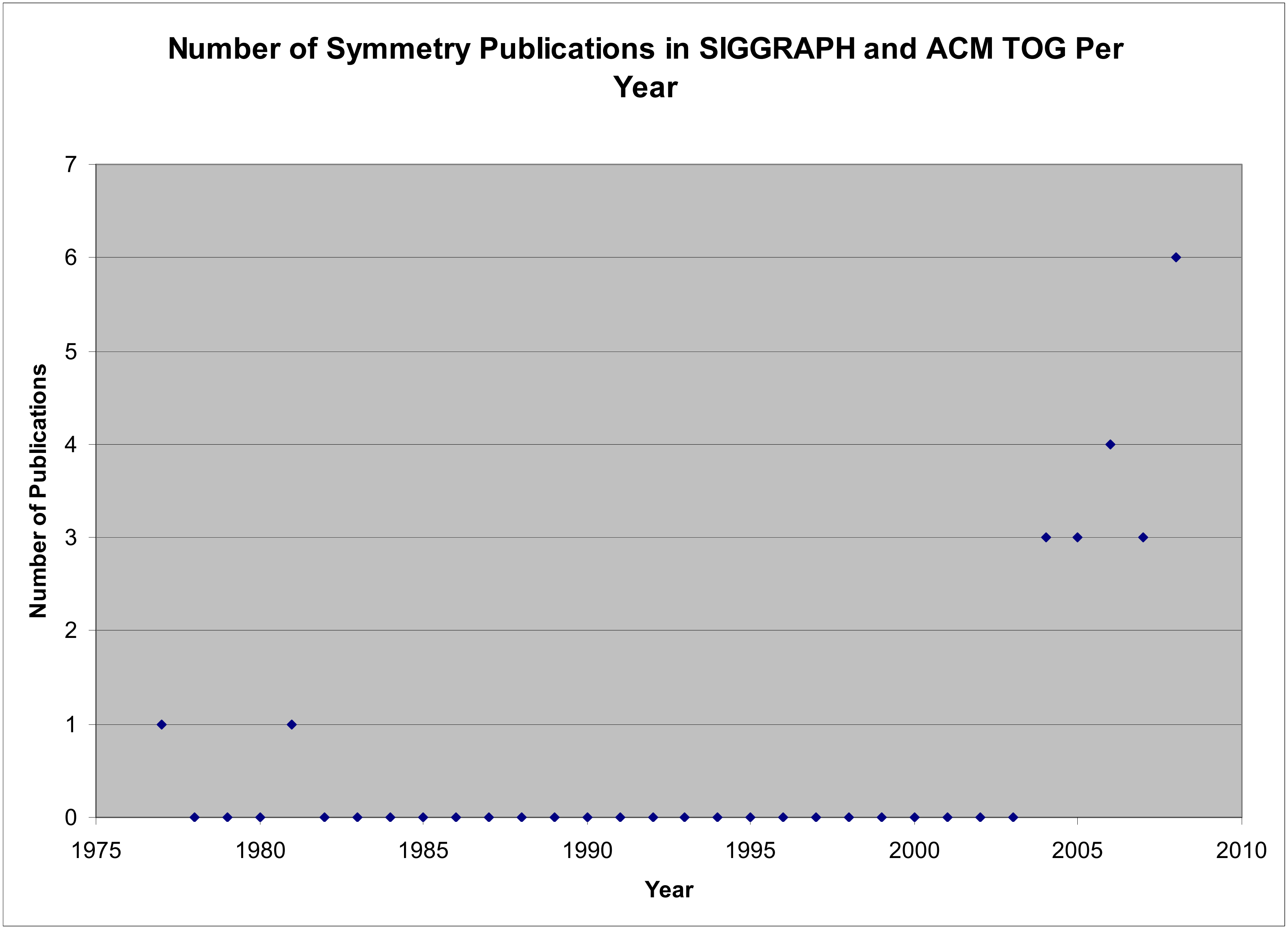
What's new

Dance Texture

See movie

Symmetry Detection Research in Computer Vision and Computer Graphics

- As early as almost 40 years ago, and never stopped since …



Number of Symmetry Publications in SIGGRAPH and ACM TOG Per Year
Number of Publications
0
1
2
3
4
5
6
7
1975
1980
1985
1990
1995
2000
2005
2010
Year

of published Symmetry Detection Papers/Year

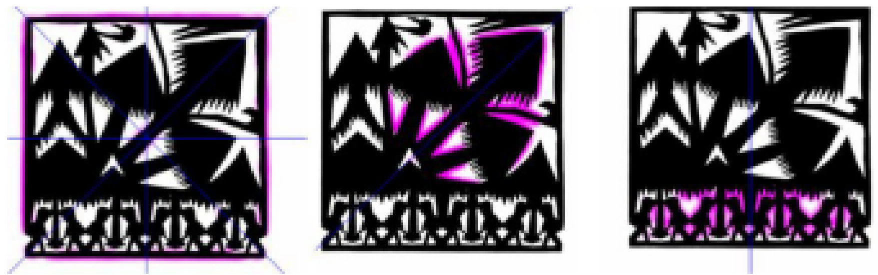
ICCV (1987~2009) [totally 18 accepted paper in 12 conferences]											
1987	1988	1990	1993	1995	1998	1999	2001	2003	2005	2007	2009
1	0	0	1	2	3	3	1	2	1	2	2

1	CVPR		
2	year	# of papers	
3	2009	3	
4	2008	7	
5	2007	4	
6	2006	2	
7	2005	0	
8	2004	2	
9	2003	0	
10	2002	0	No c
11	2001	1	
12	2000	3	
13	1999	2	
14	1998	1	
15			

43	PAMI	
44	year	# of papers
45	2009	1
46	2008	1
47	2007	0
48	2006	0
49	2005	1
50	2004	2
51	2003	3
52	2002	0
53	2001	0
54	2000	0
55	1999	1
56	1998	1
57		

Bad News (CVPR 2008)

- **Performance Evaluation of State-of-the-Art Discrete Symmetry Detection Algorithms**
  Minwoo Park, Seungkyu Lee, Po-Chun Chen, Somesh Kashyap, Asad A. Butt and Yanxi Liu
  Computer Vision and Pattern Recognition Conference (CVPR '08)

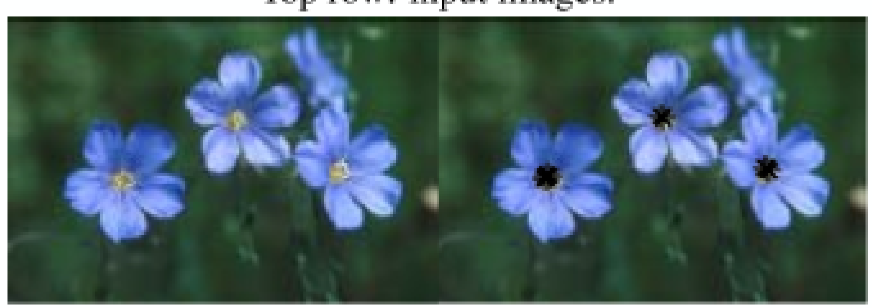


(1) reflection symmetry group detection [17]: multiple symmetry axes of local regions are detected one-by-one.

SIGGRAPH 2005
Liu, Hays, Xu, Shum

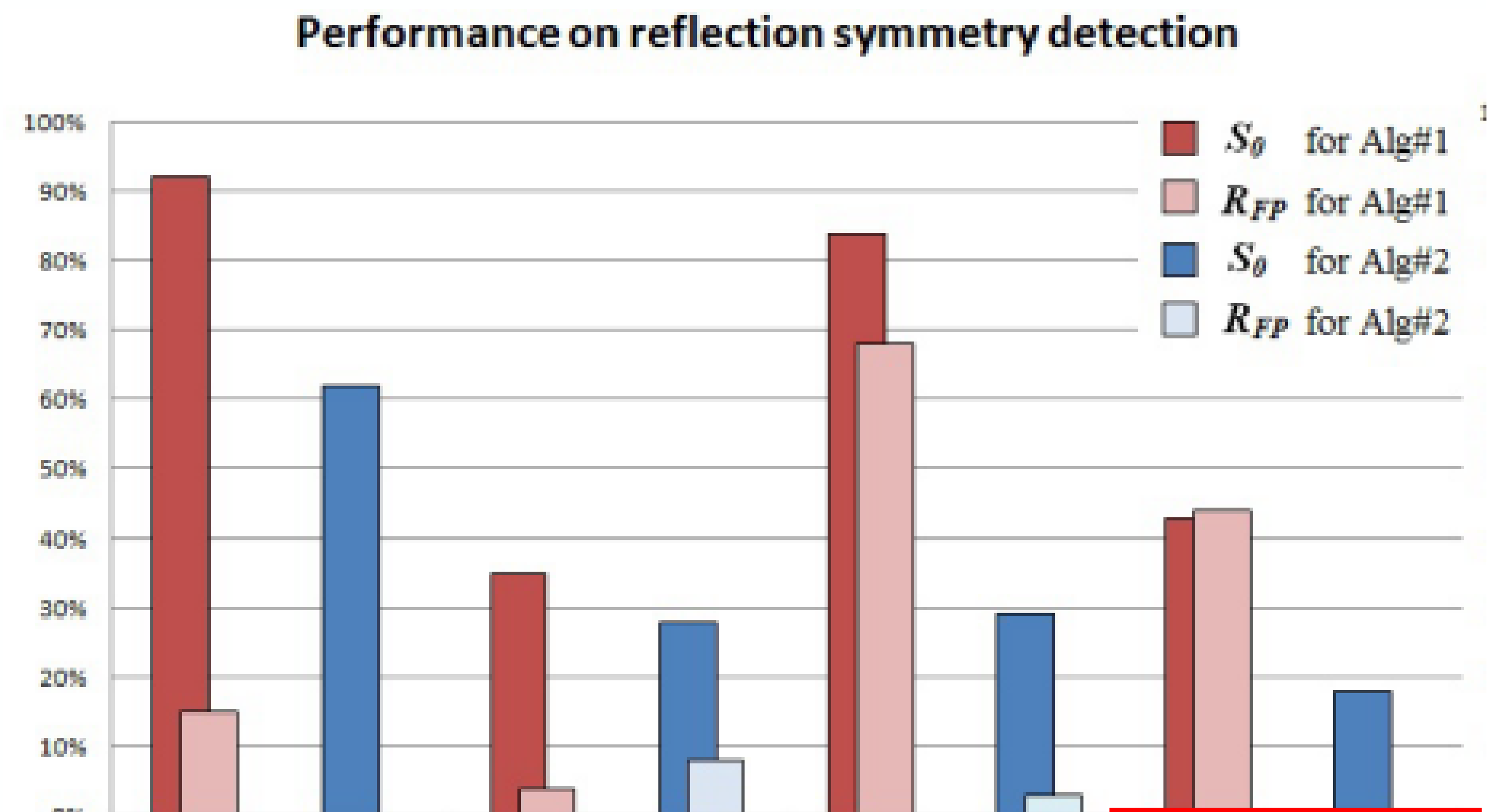
(2) reflection (left) and rotation (right) symmetry detection [19]
Top row: input images.

ECCV06 Loy & Eklundh



(3) rotation symmetry detection [25].

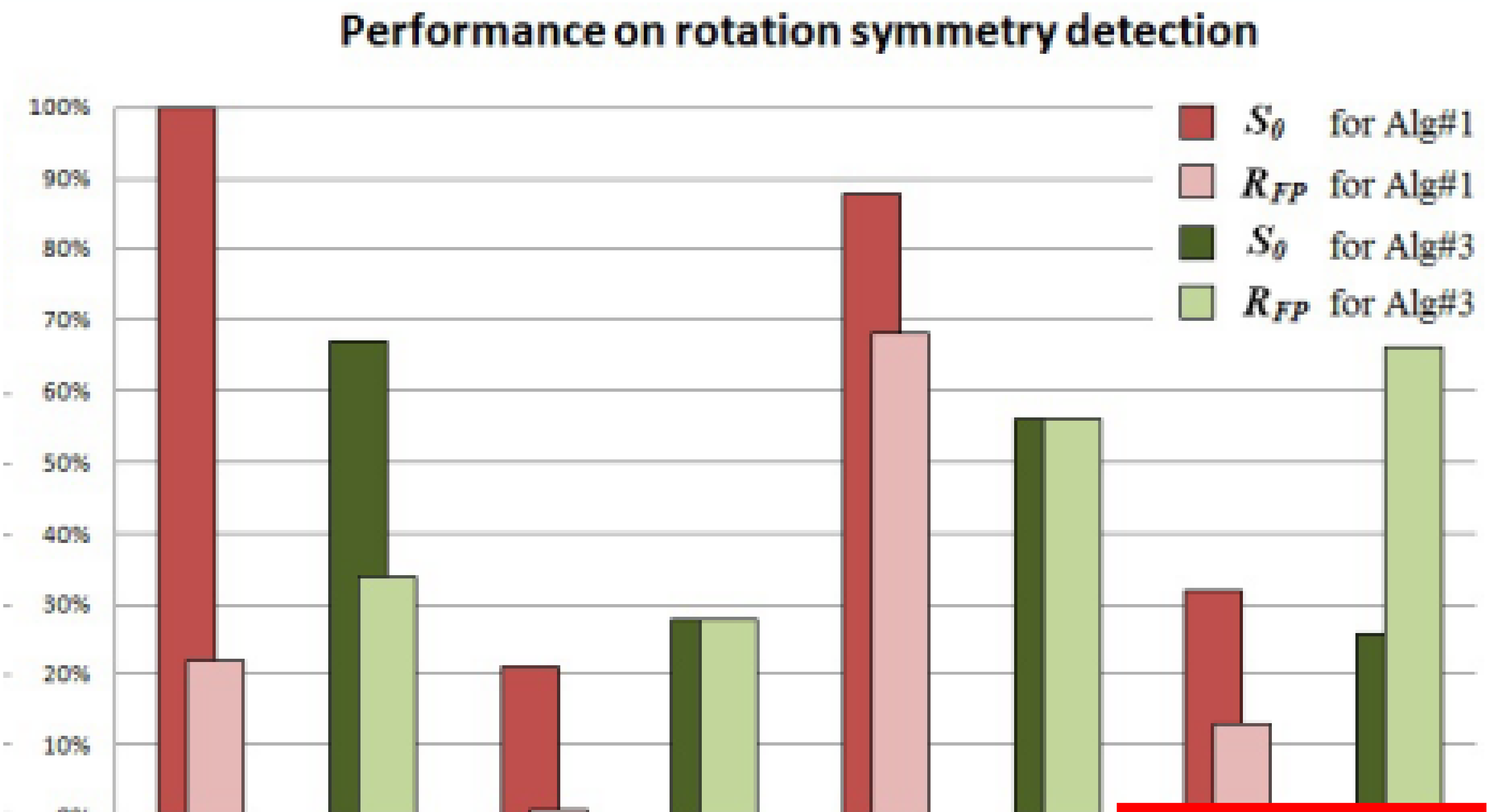
ICCV05 Prasad & Davis



	Syn. Single		Syn. Multiple		Real Single		Real Multiple	
	Alg#1	Alg#2	Alg#1	Alg#2	Alg#1	Alg#2	Alg#1	Alg#2
S_θ	92%	62%	35%	28%	84%	29%	43%	18%
R_{FP}	15%	0 %	4%	8%	68%	3%	44%	0%

S_θ :Sensitivity R_{FP} :False positive rate **Real**: Real Image **Syn**:Synthetic Image **Al**

Figure 4. The pairwise reflection and rotation symmetry detection algo



	Syn. Single		Syn. Multiple		Real Single		Real Multiple	
	Alg#1	Alg#3	Alg#1	Alg#3	Alg#1	Alg#3	Alg#1	Alg#3
S_0	100%	67%	21%	28%	88%	56%	32%	26%
R_{FP}	22%	34%	1%	28%	68%	56%	13%	66%

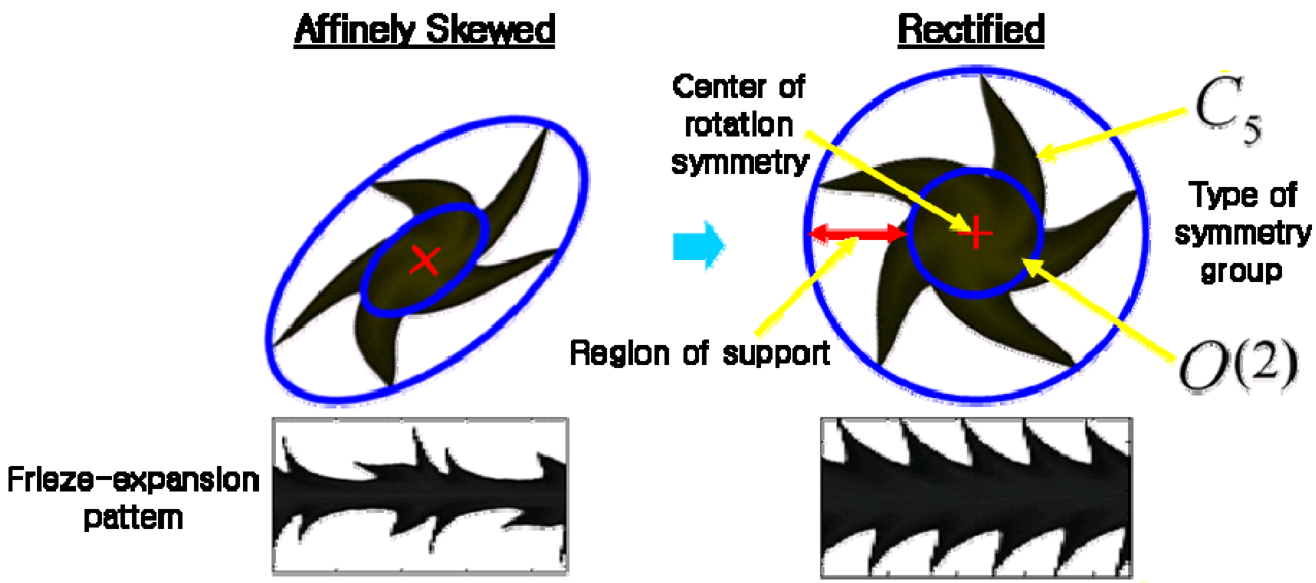
Alg#1: Loy and Eklundh 2006 **Alg#2**: Liu.et al. 2005 **Alg#3**: Prasad and Davis 2005

New Advance

- **Rotation Symmetry Group Detection Via Frequency Analysis of Frieze-Expansions**
  Seungkyu Lee, Robert T. Collins and Yanxi Liu
  Computer Vision and Pattern Recognition Conference (CVPR '08)

- movie

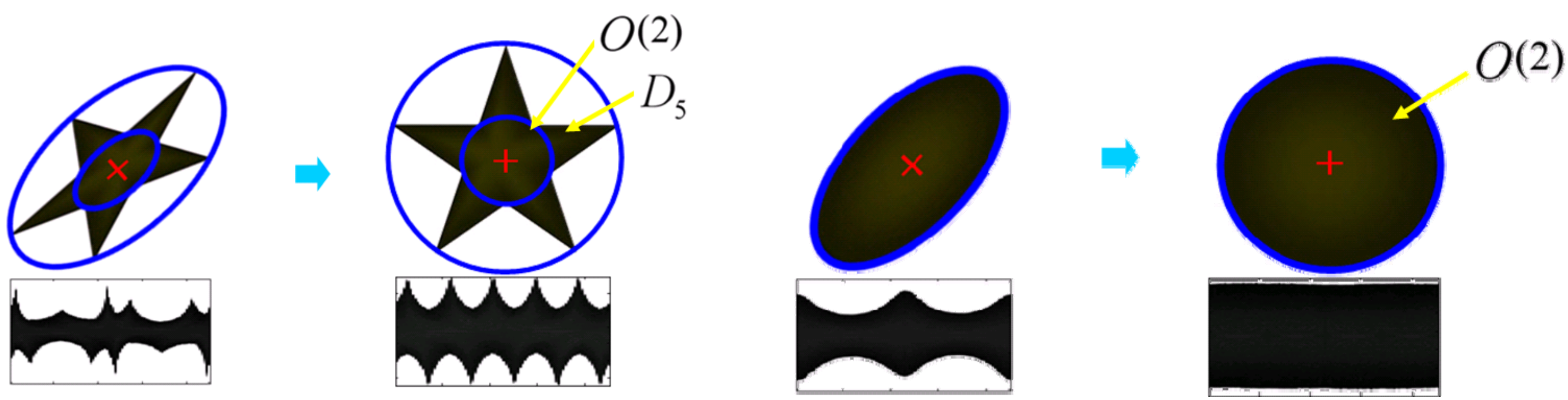
Skewed Rotation Symmetry Groups



Cyclic group

[5 properties]

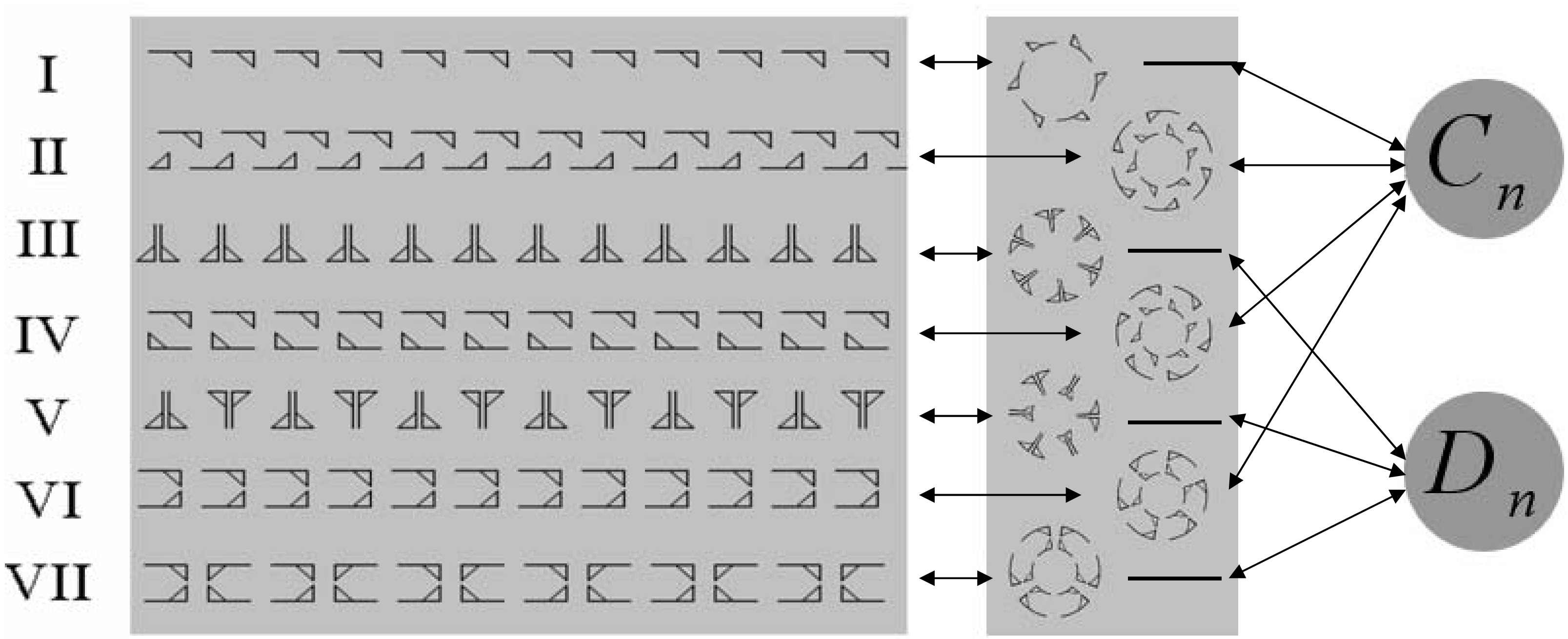
1. Center of rotation
2. Affine deformation
3. Symmetry type
4. Cardinality
5. Supporting region



Dihedral group

Orthogonal group

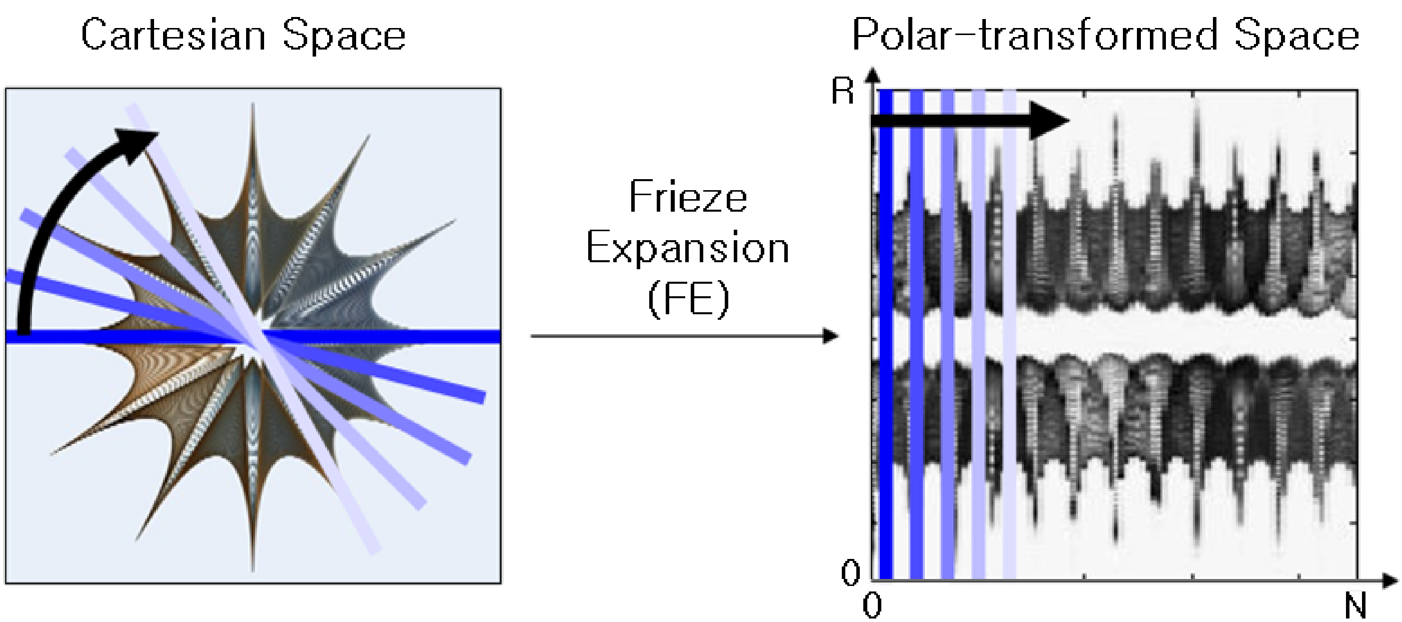
Mappings between Translation and Rotation Groups



Polar–transformed space
(Translation symmetry groups)

Cartesian space
(Rotation symmetry groups)

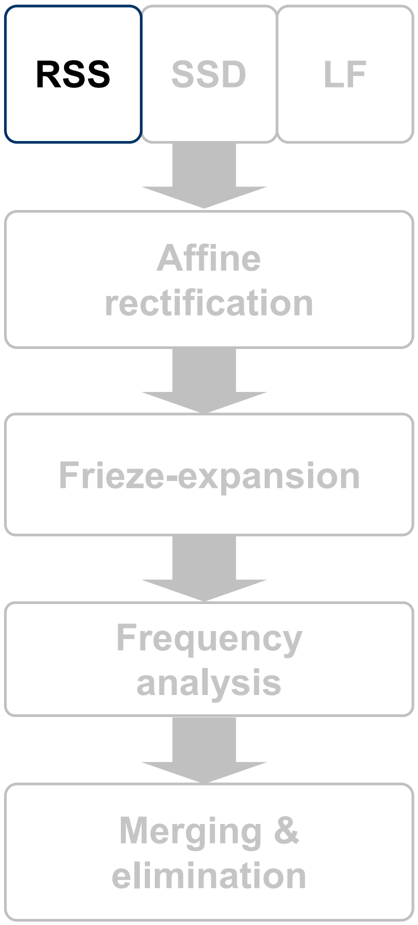
Frieze-Expansion

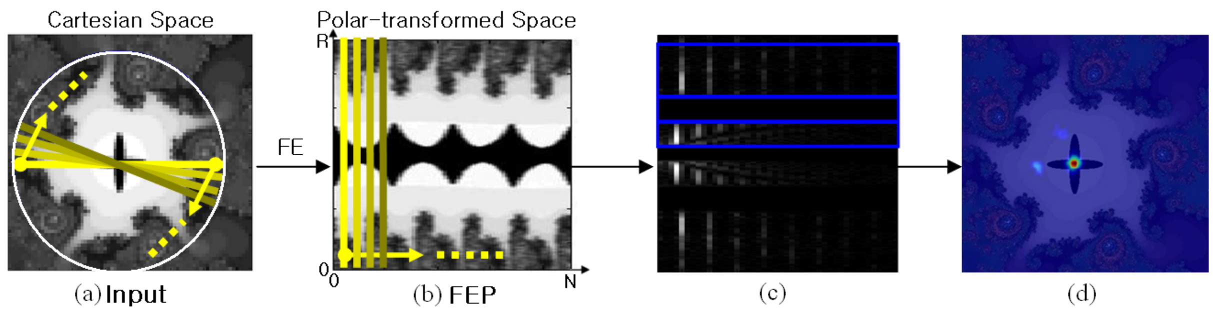


Input pattern

FEP

Saliency Map #1:
Rotation Symmetry Strength (RSS)



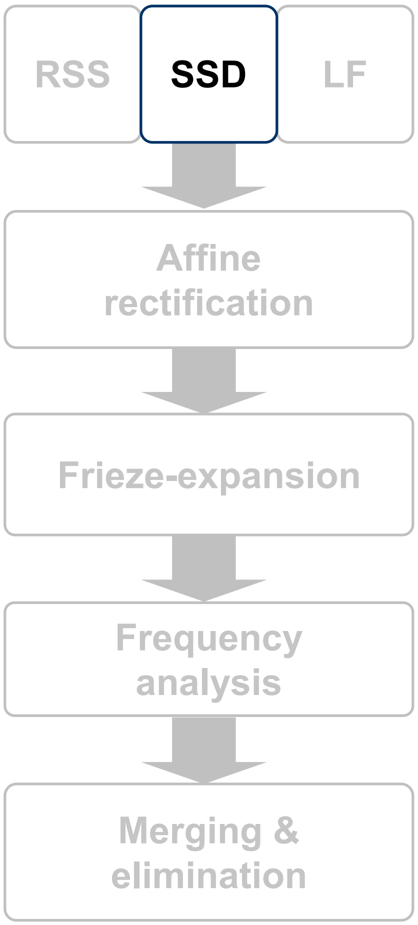


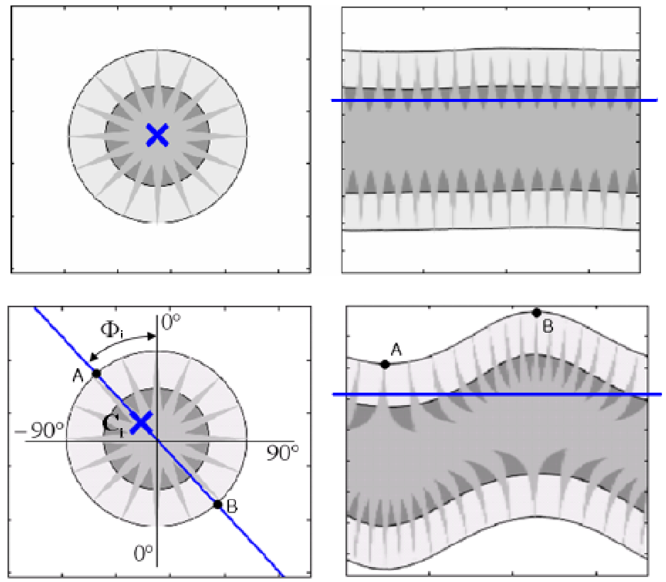
$$p_{x,y}(r,n) \longrightarrow S_{x,y}(r,k)$$

spectral density

$$RSS(x,y) = \sum_{r=1}^{R} \rho_r \frac{mean(S_{x,y}(r, i_{peak}))}{mean(S_{x,y}(r,j))}$$

Bidirectional Flow

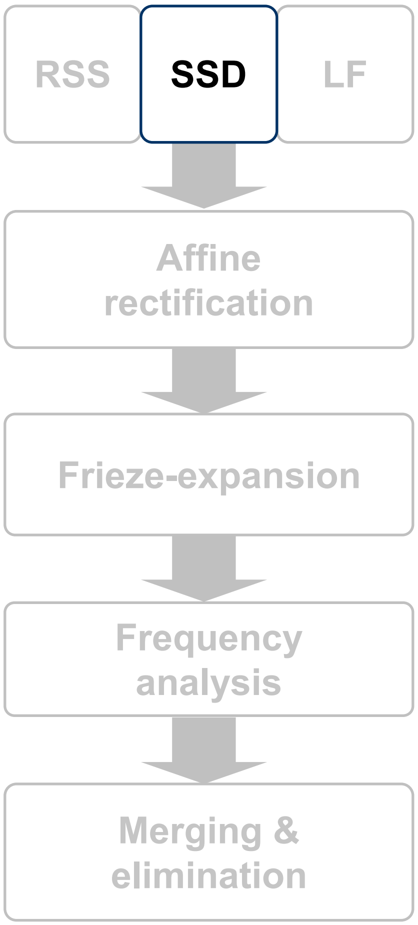


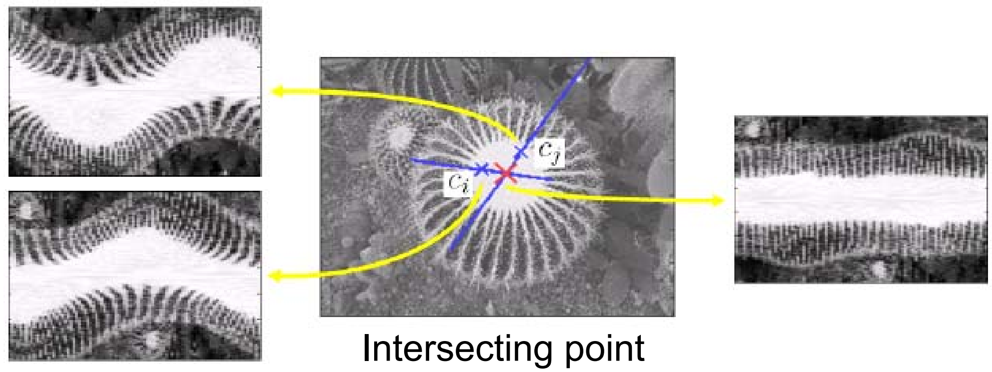


$$\phi_i(r) = arctan(\frac{Re(P_{x_i,y_i}(r,2))}{Im(P_{x_i,y_i}(r,2))})$$

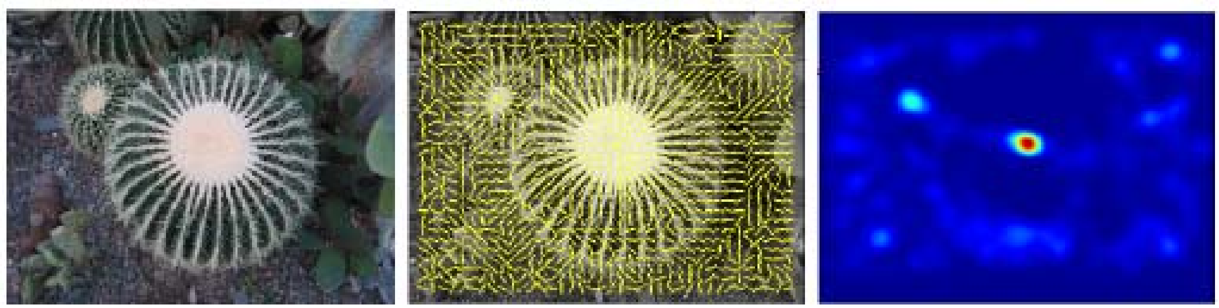
Bidirectional flow: $\frac{tan\Phi_i}{x_i + y_i tan\Phi_i} y + \frac{1}{x_i + y_i tan\Phi_i} x = 1$

Saliency Map #2: Symmetry Shape Density (SSD)





Intersecting point

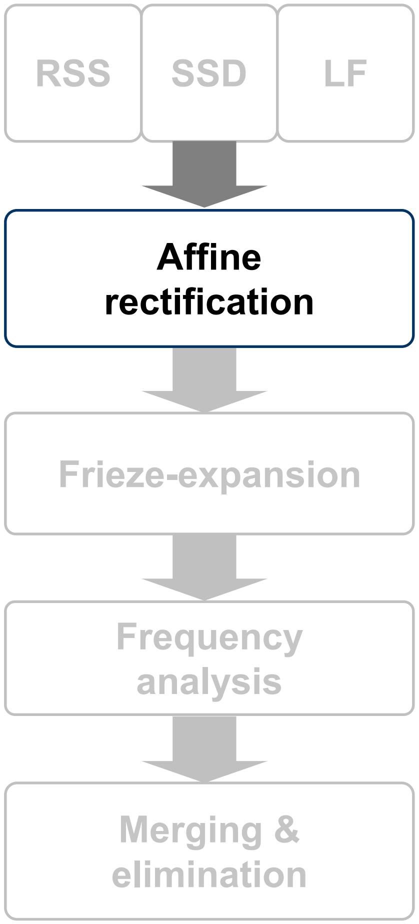


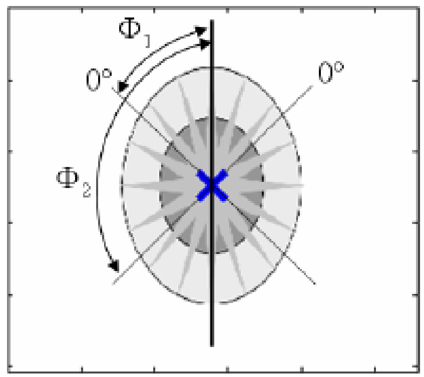
Input | Bidirectional flows | SSD

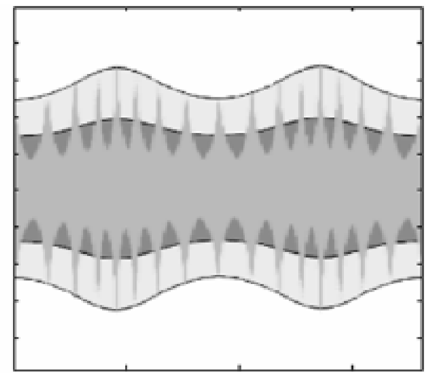
$$SSD(x,y) = D(x,y) \bullet G(l,l)$$

Intersecting point distribution ($D(x,y)$) | Gaussian kernel ($G(l,l)$)

Affine Rectification





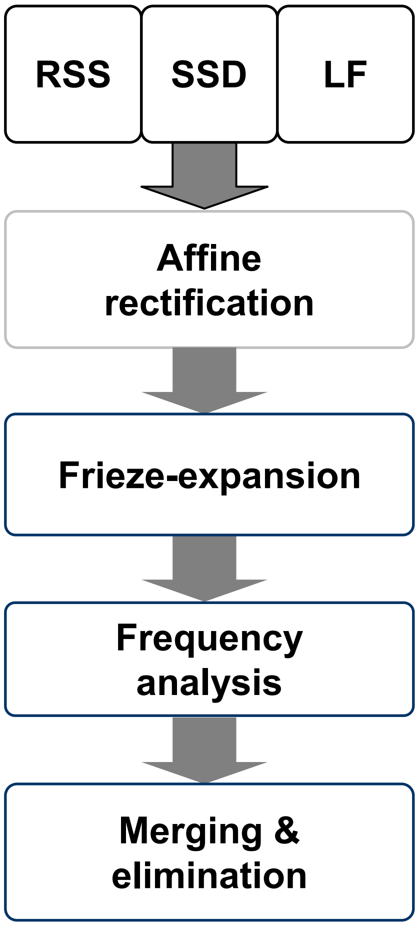


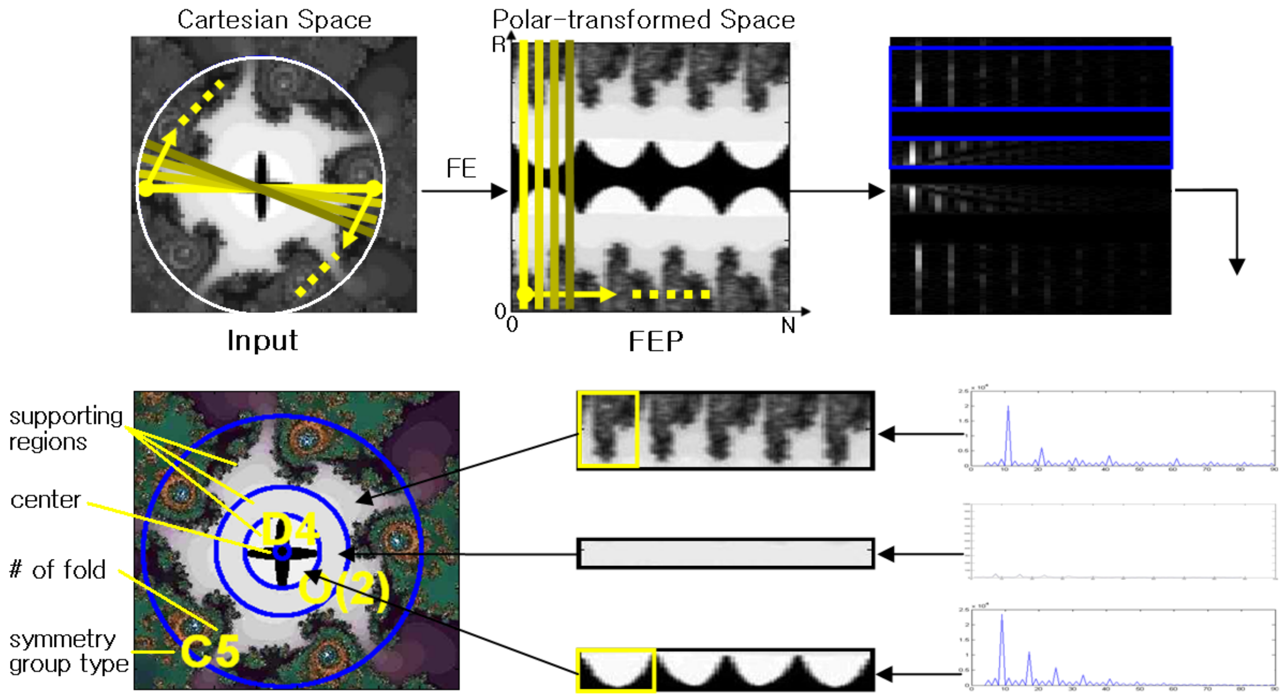
Affine transformation

$$T = \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\frac{\hat{\Phi}-90}{2}) & -sin(\frac{\hat{\Phi}-90}{2}) \\ sin(\frac{\hat{\Phi}-90}{2}) & cos(\frac{\hat{\Phi}-90}{2}) \end{bmatrix} = \begin{bmatrix} \alpha cos(\frac{\hat{\Phi}-90}{2}) & -\alpha sin(\frac{\hat{\Phi}-90}{2}) \\ sin(\frac{\hat{\Phi}-90}{2}) & cos(\frac{\hat{\Phi}-90}{2}) \end{bmatrix}$$

$$(0 < \sigma < 1)$$

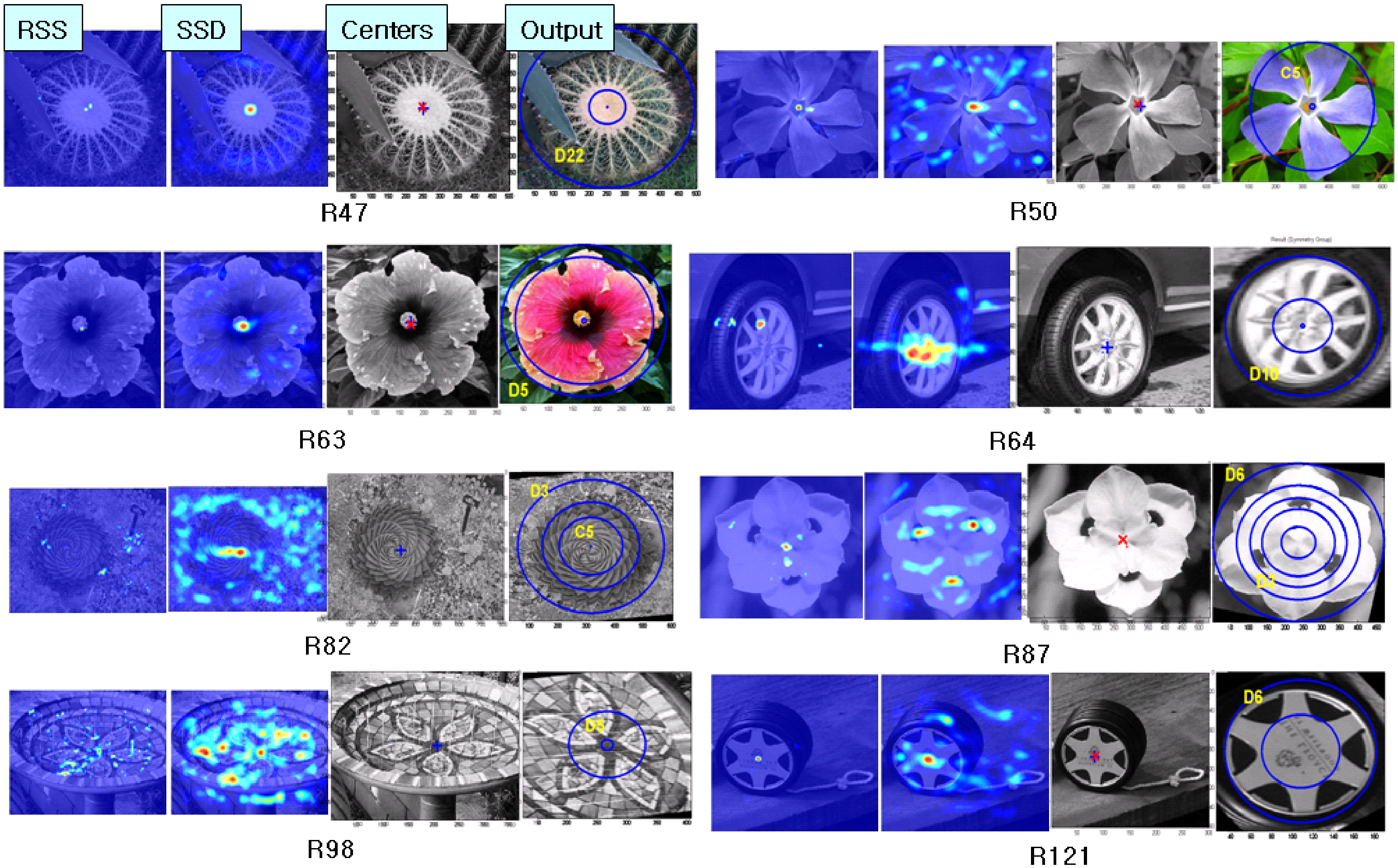
Frequency Analysis





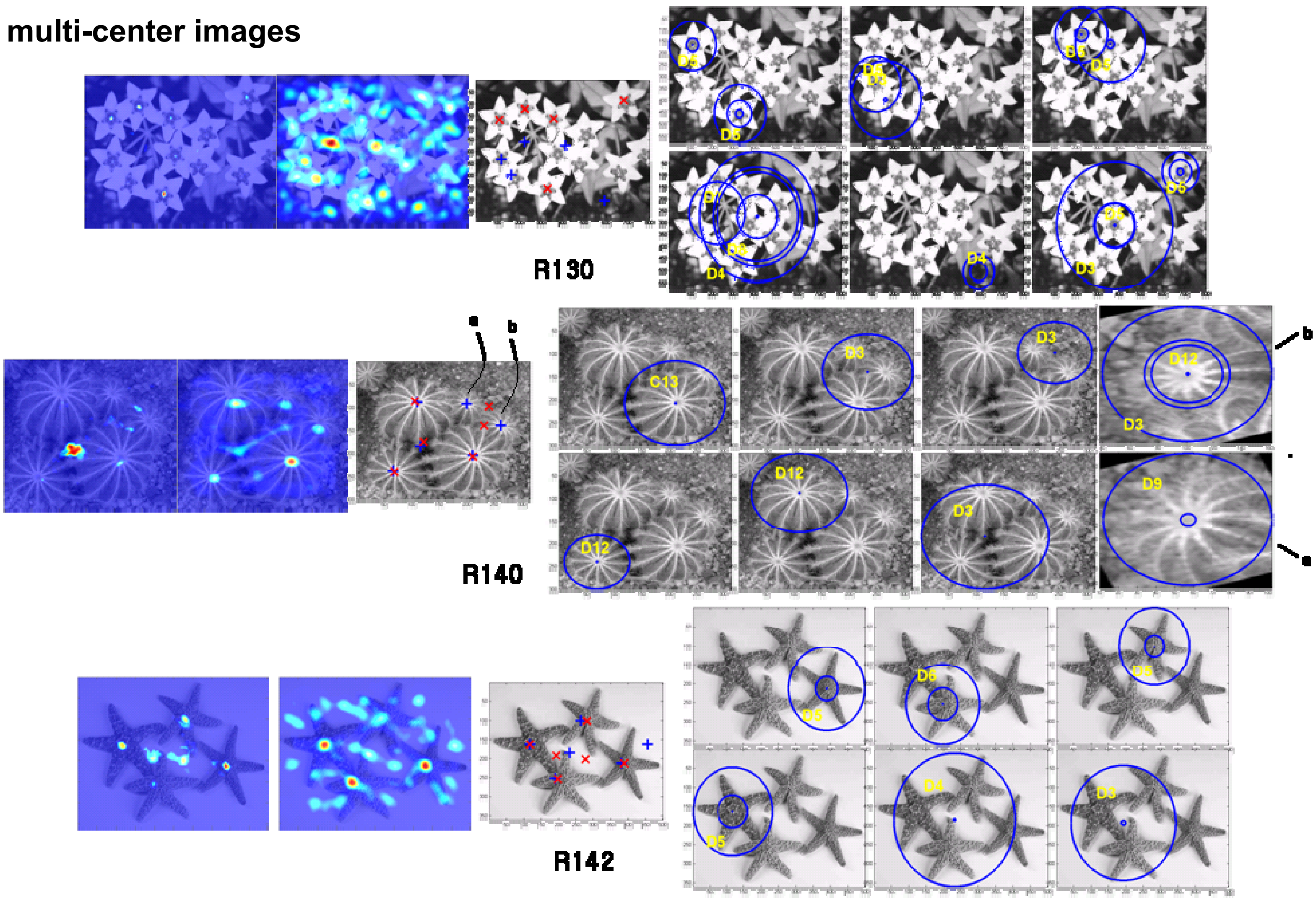
Experimental Results

Real single-center images



Experimental Results

Real multi-center images



Experimental Results

Quantitative evaluation on 170 images for object-level ground truth

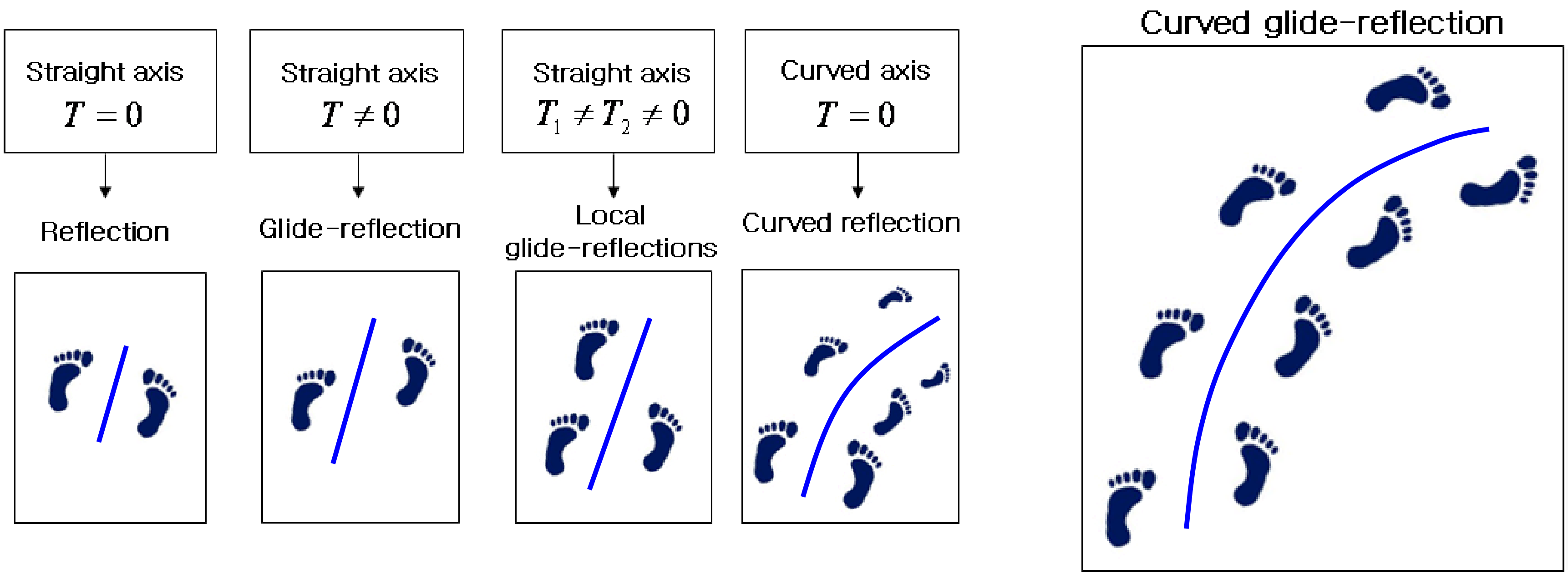
Detection Accuracy	Loy and Eklundh [58] ECCV2006			
	TP Center	FP Center	# of fold	$C_n/D_n/O(2)$
Synthetic	31/48 = 65%	4/48 = 8%	22/49 = 45%	N/A
Real-Single	50/58 = 86%	41/58 = 71%	16/64 = 25%	N/A
Real-Multi	32/78 = 41%	6/78 = 8%	12/42 = 29%	N/A
Overall	113/184 = 61%	51/184 = 28%	50/155 = 32%	N/A

Detection Accuracy	Proposed Method #2 (RSS+SSD+LF)			
	TP Center	FP Center	# of fold	$C_n/D_n/O(2)$
Synthetic	43/48 = 90%	12/48 = 25%	44/62 = 71%	51/62 = 82%
Real-Single	54/58 = 93%	31/58 = 53%	35/66 = 53%	54/66 = 82%
Real-Multi	55/78 = 71%	22/78 = 28%	40/70 = 57%	53/70 = 76%
Overall	152/184 = 83%	65/184 = 35%	119/198 = 60%	158/198 = 80%

New Advance

- Curved Glide Reflection Symmetry Detection
  Seungkyu Lee and Yanxi Liu
  Computer Vision and Pattern Recognition Conference (CVPR '09) oral paper

What is a curved glide-reflection?



Glide-reflection: $P_i = T + R(P_j)$

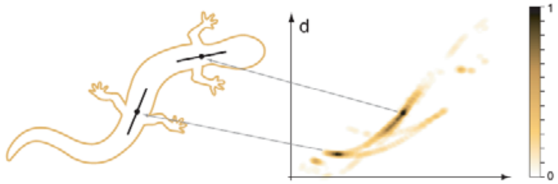
translation (T) **reflection** (R)

Real world examples

Previous Work

1 Reflection axis detection

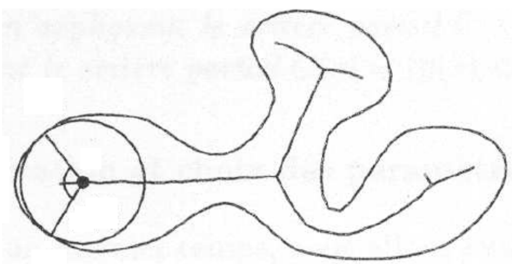
[Loy and Eklundh, ECCV 06]



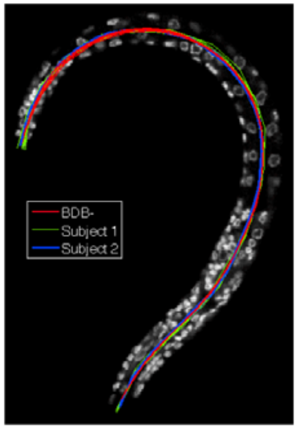
[Mitra et. al., SIGGRAPH 06]

They detect only straight reflection symmetry axes.

2 Medial axis detection



[Bonnassie et. al., ICIP 01]



[Peng et. al., Bioinformatics 08]

Medial axis cannot find (glide) reflection symmetries and depends on closed contours

Feature Point Detection

Feature point detection

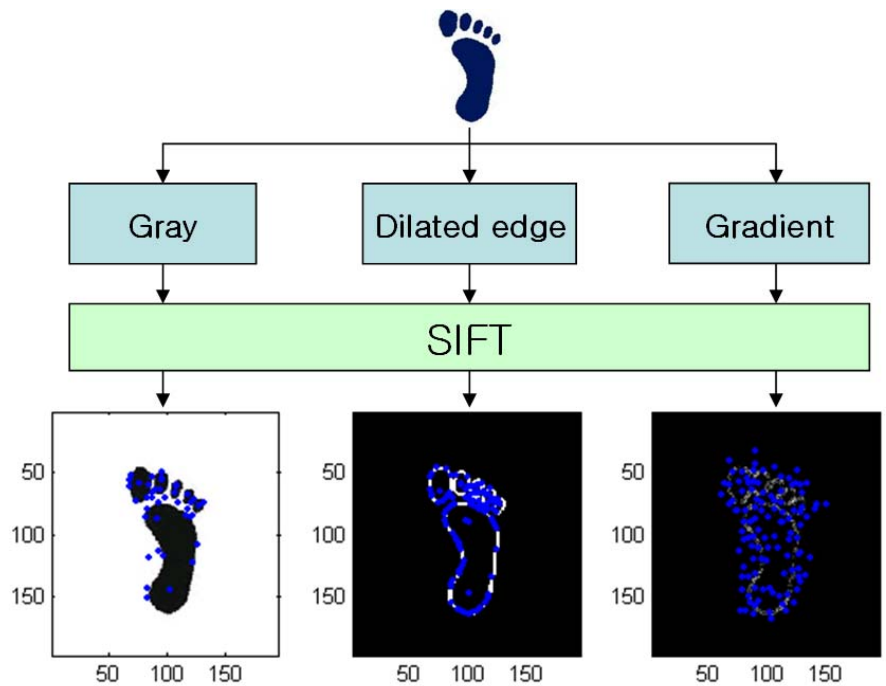
Matched pair selection

3D parameter space

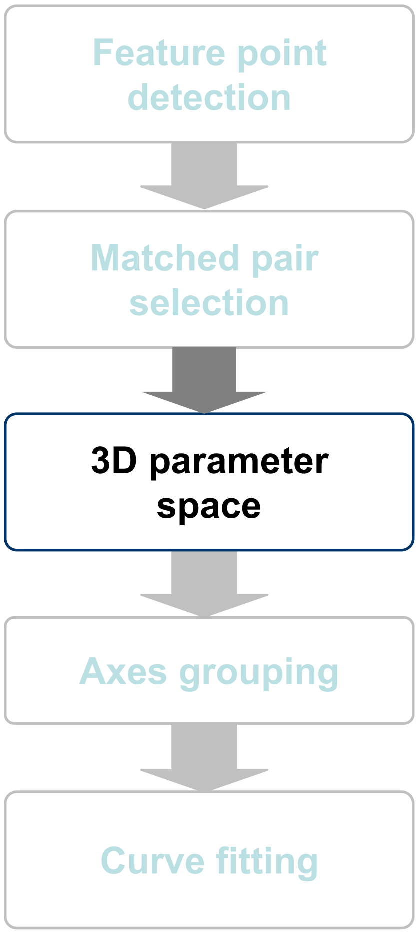
Axes grouping

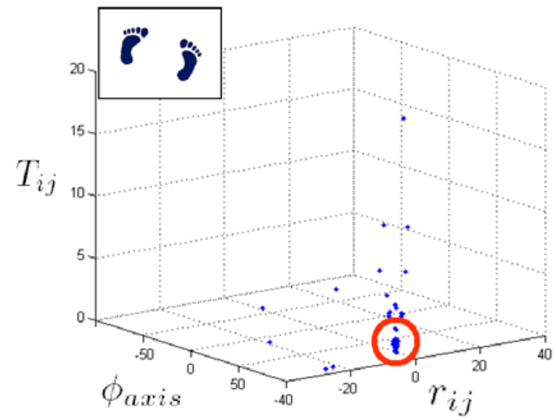
Curve fitting

SIFT key points detection from a set of filtered images

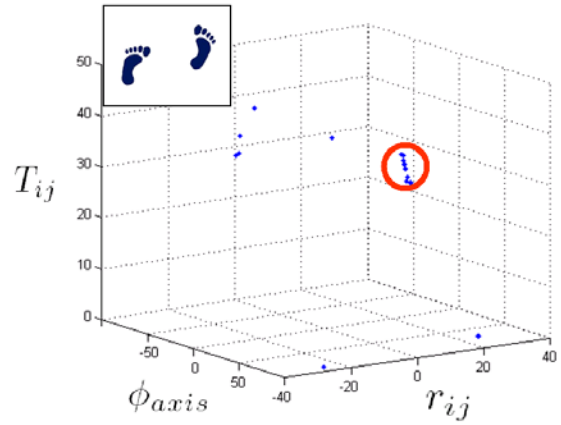


3D Parameter Space

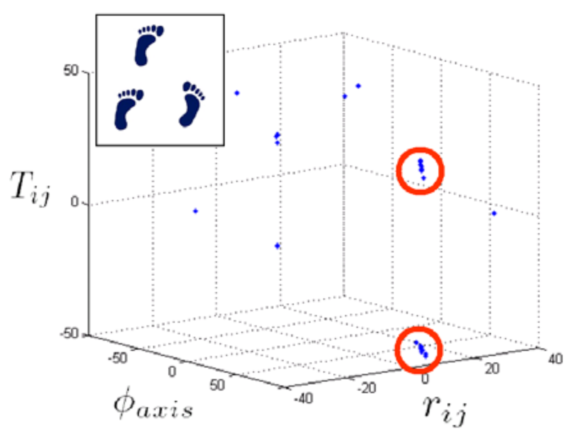




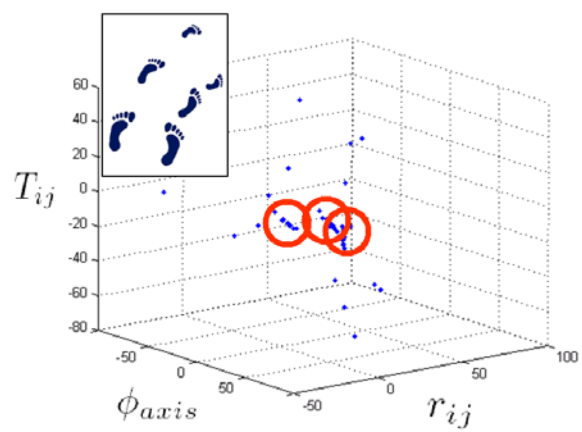
Reflection



Glide-reflection



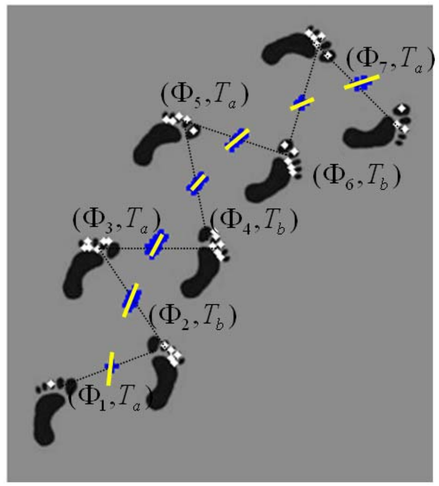
Local glide-reflections

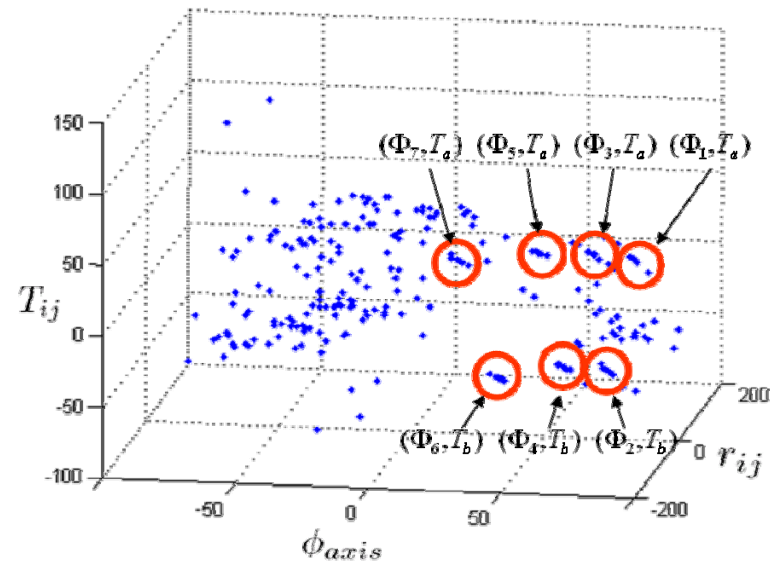


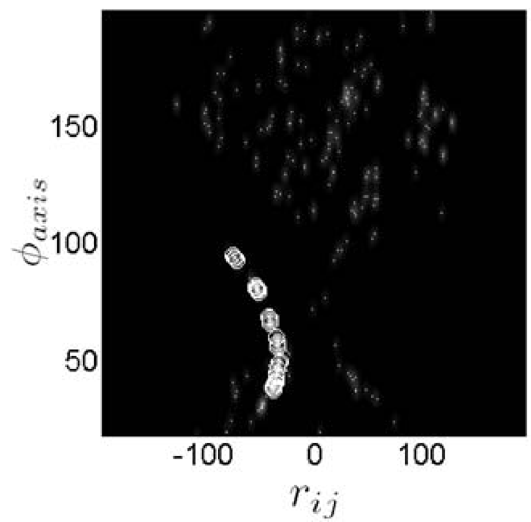
Curved reflection

3D Parameter Space

- Feature point detection
- Matched pair selection
- 3D parameter space
- **Axes grouping**
- Curve fitting

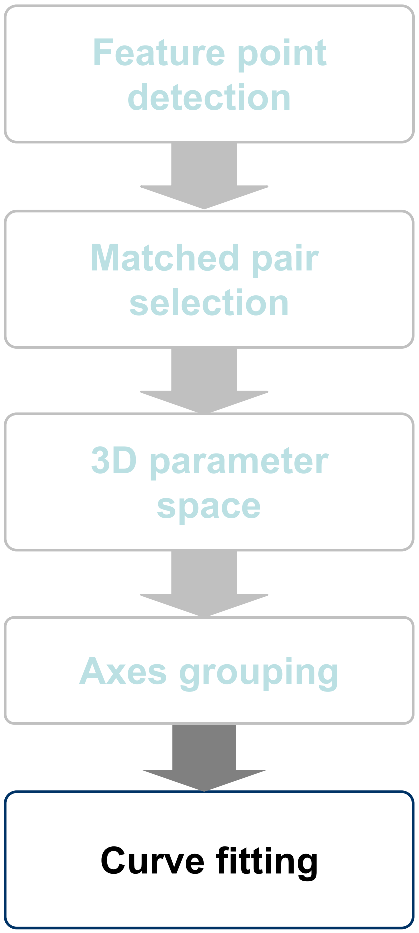


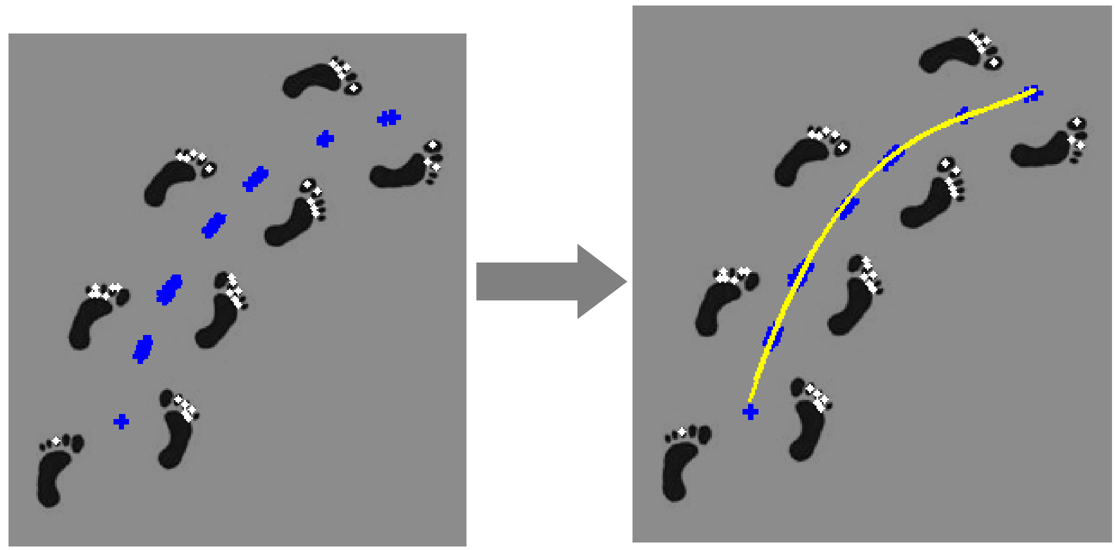




Axes grouping based on the distance in 2D density space

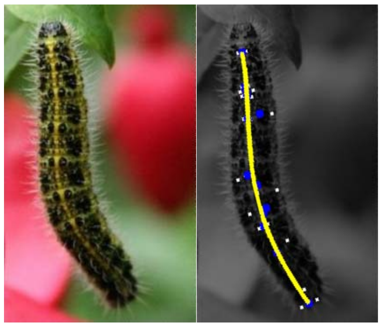
Curve Fitting



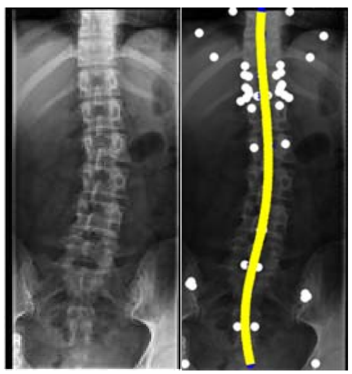


-Fit polynomial curves of degree between 1 to 5 to the middle points (blue)
-Choose the curve with shortest sum of squared distance from all middle points

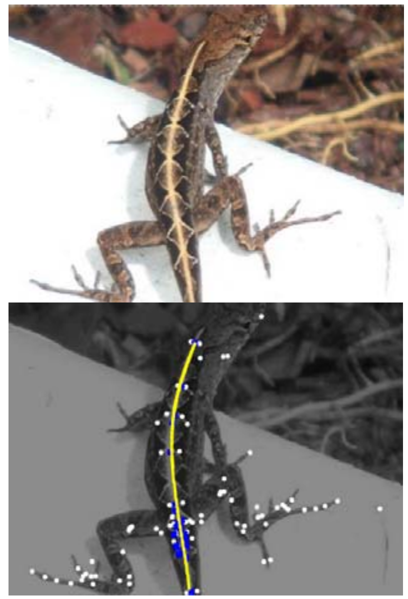
Experimental Results



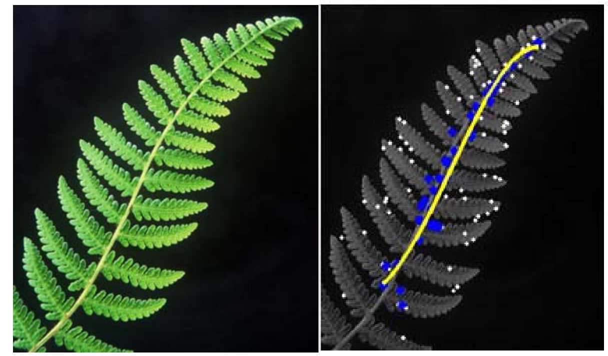
Caterpillar

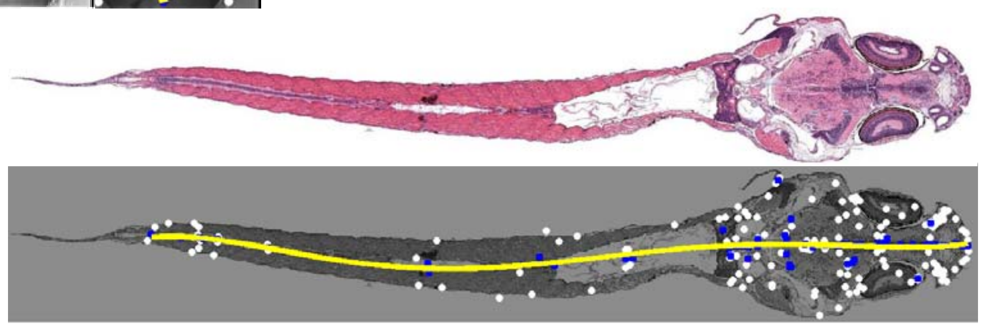


Spine

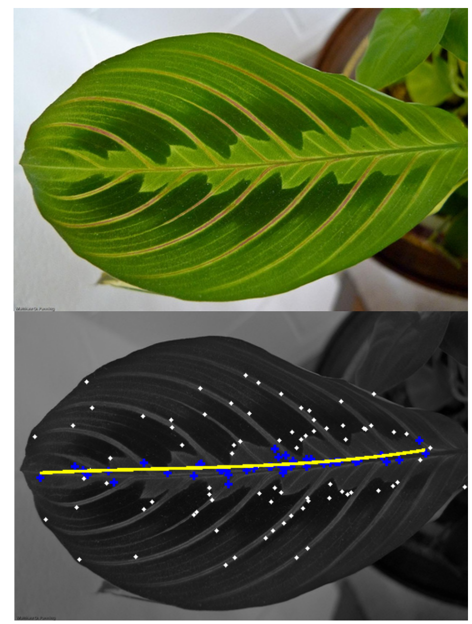


Lizard



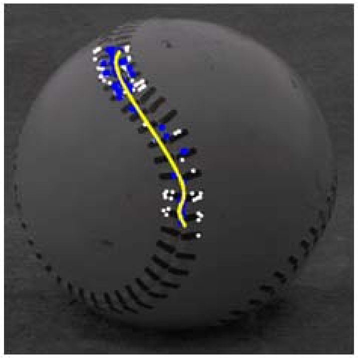
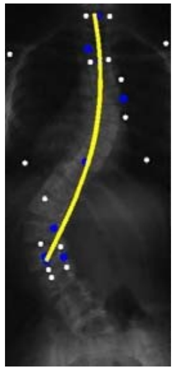


Zebra fish

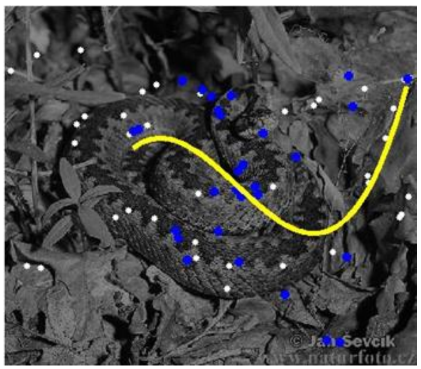


Leaves

Experimental Results (Failures)

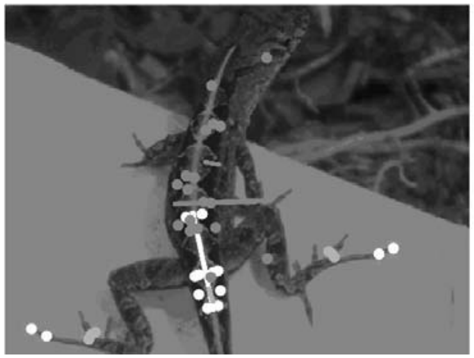
: Not enough matched pairs are found



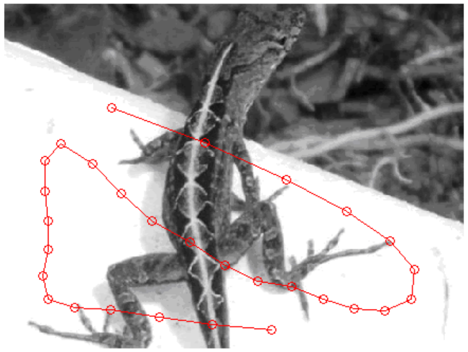
: Background clutter

Experimental Result Comparison

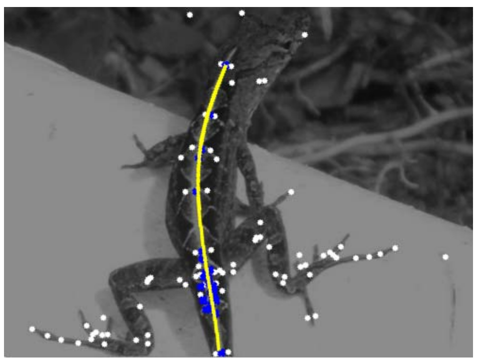
Method	Detection rate	Processing time* (sec)
Loy and Eklundh [58]	7.5%	6.6(9.8)
Peng et. al. [72]	0.0%	75.0(220.5)
Lee and Liu [42]	70.0%	9.9(11.5)



[Loy and Eklundh, ECCV 06]



[Peng et. al., Bioinformatics 08]



[Lee and Liu, CVPR 09]

Curved Axis Straightening

Feature point detection

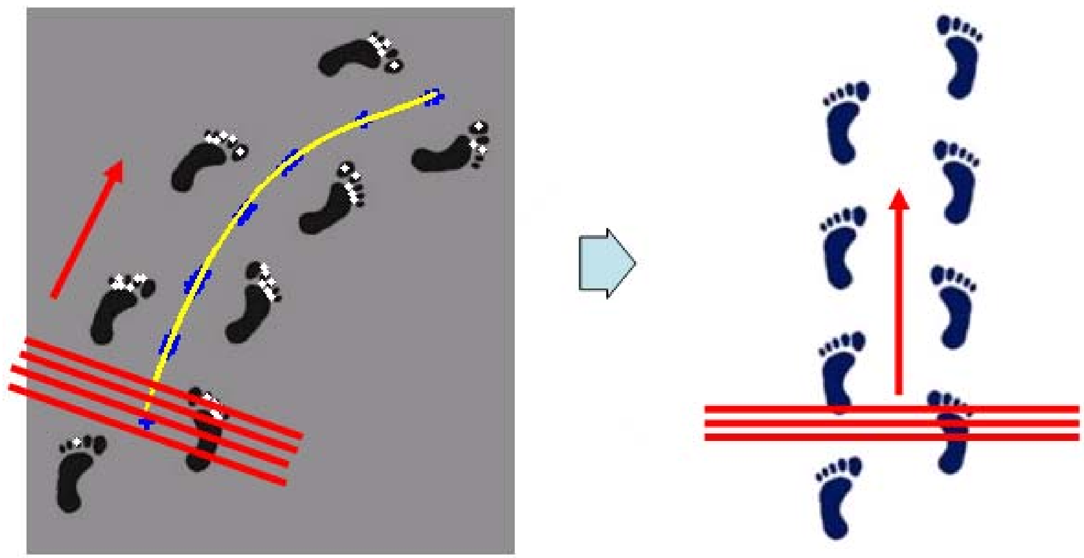
Matched pair selection

3D parameter space

Axes grouping

Curve fitting

Straightening

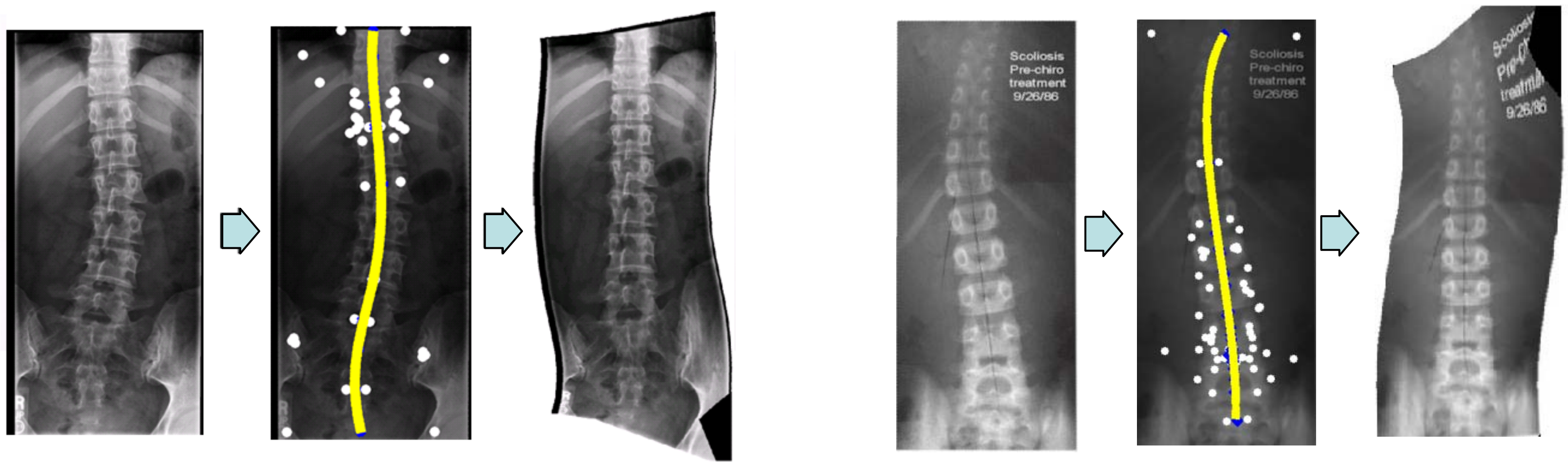


Application: Image Registration

Swedish leaves

- straighten curved leaves to improve a recognition accuracy
- curvature has to be removed from leaves

Application: Medical Diagnosis



Scoliosis spines

- curvature can be detected and quantified

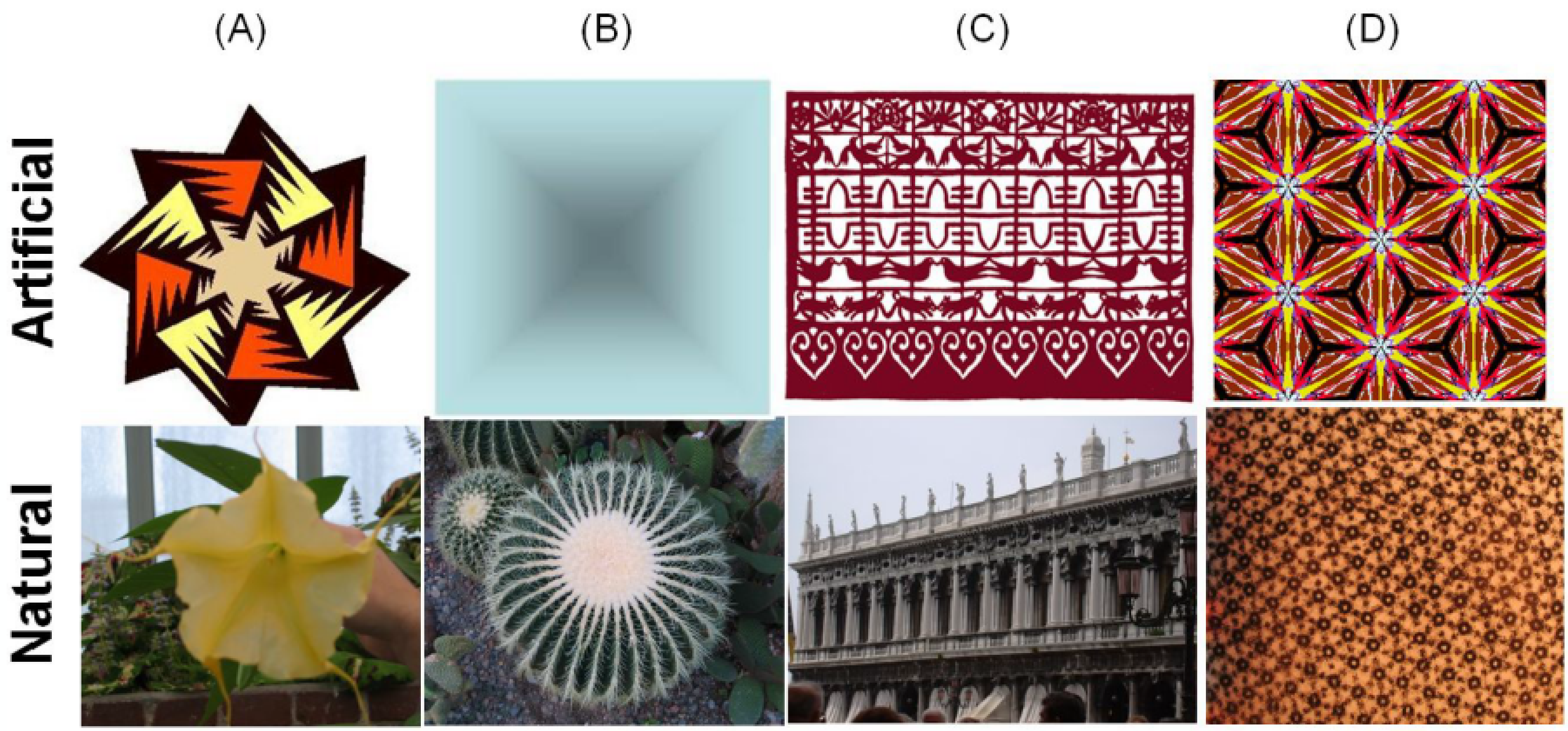
Symmetry Group Detection Algorithms

- Introduction
- **Symmetry Group Detection Algorithms**
  - Curved glide-reflection symmetry detection
  - Skewed rotation symmetry group detection
- Applications
  - Shape
    - Symmetry-driven shape matching
    - Symmetry-driven object recognition
  - Motion
    - Gait recognition
    - Symmetry of dance
- Conclusion & Plan

Good News (CVPR 2008)

- **[Performance Evaluation of State-of-the-Art Discrete Symmetry Detection Algorithms]**
  Minwoo Park, Seungkyu Lee, Po-Chun Chen, Somesh Kashyap, Asad A. Butt and Yanxi Liu Computer Vision and Pattern Recognition Conference (CVPR '08)
- We have a quantified starting point
- We have existence proof that “group theory + statistical learning theory” works for MANY real world problems (e.g. deformed lattice detection and tracking) that cannot be handled before
- We have a Bayesian formulation …

2D Euclidean Space



Cyclic Symmetry Group (rotation)

Dihedral Symmetry Group (rotation+ reflection)

Frieze symmetry Group (translation + reflection)

Wallpaper symmetry Group (translations + rotation + Reflection + glide-reflection)

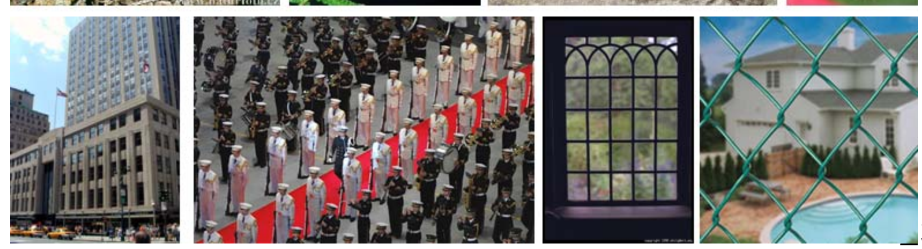
Detection of Basic Symmetries

Rotation symmetry (CVPR08, PAMI)

Reflection symmetry (CVPR09)

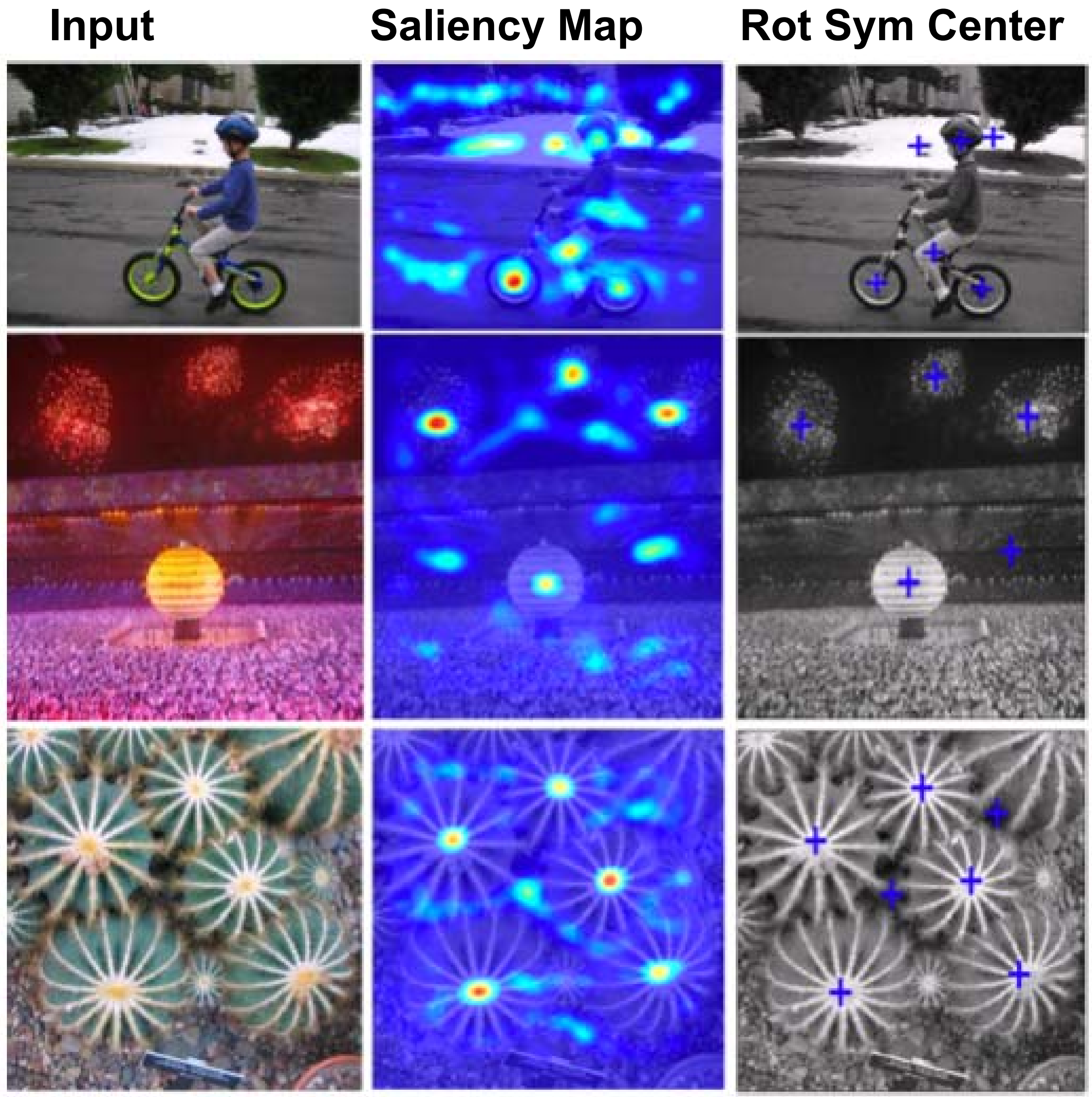
Curved glide-reflection Symmetry (CVPR09)

Translation symmetry (PAMI09)

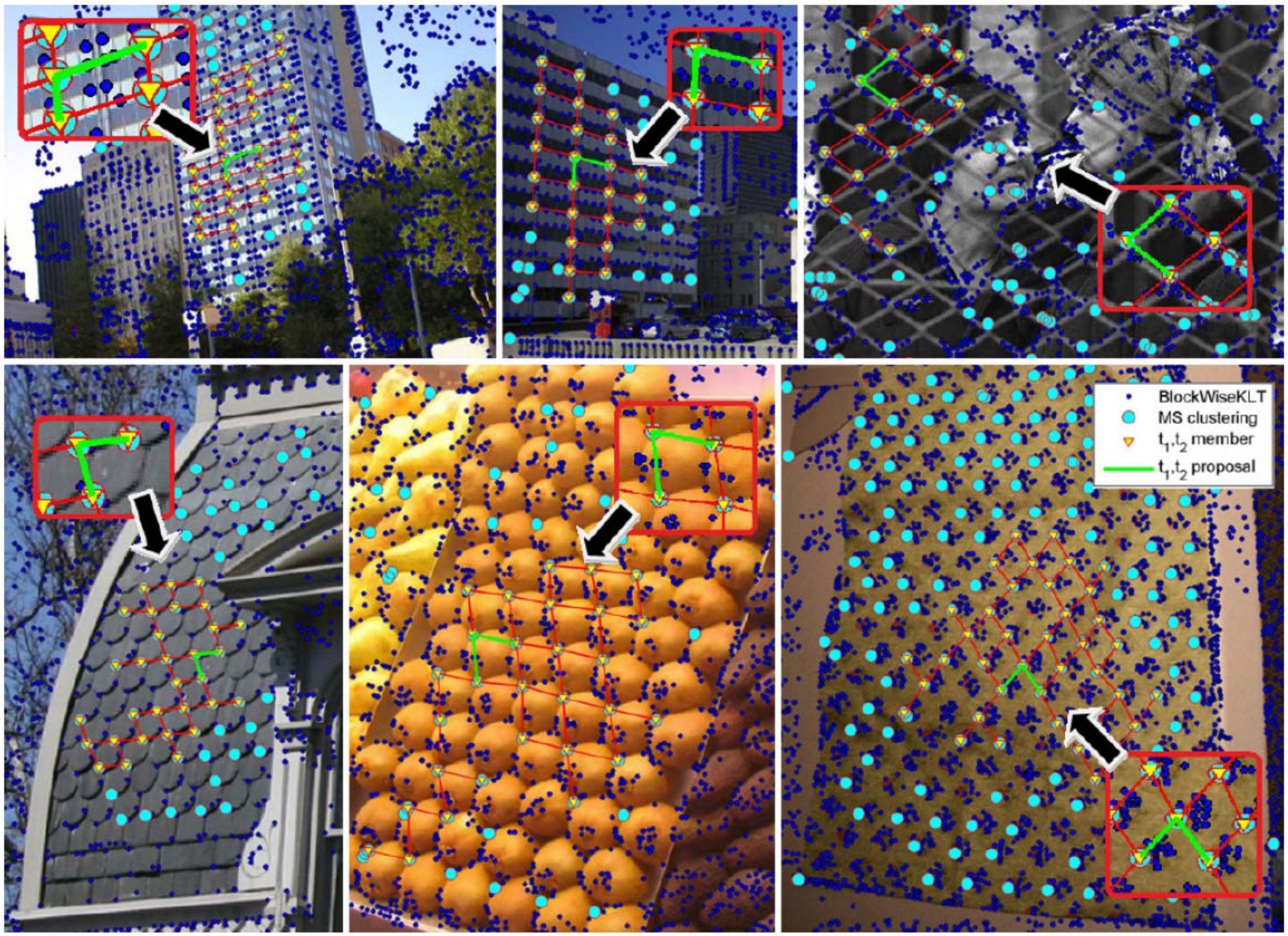


Regularity-based Saliency

Scene Saliency



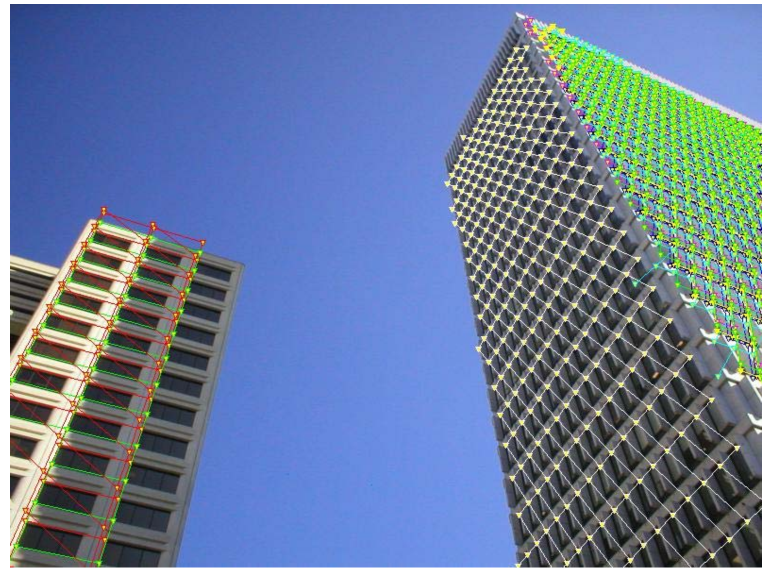
Translation Symmetries

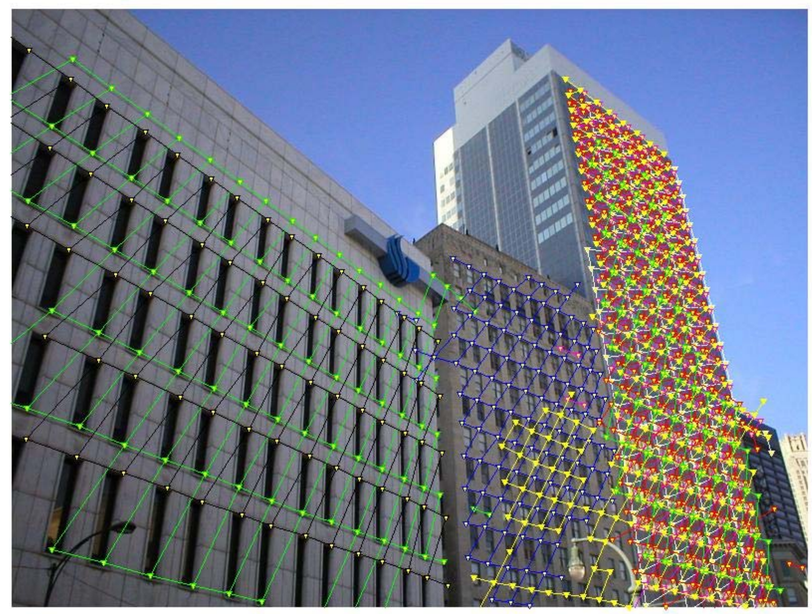


The green L-shape inside the red enlarged rectangular window is the proposed (t_1, t_2) vectors pair, and the red L-shapes are its supporting members (inlier votes).

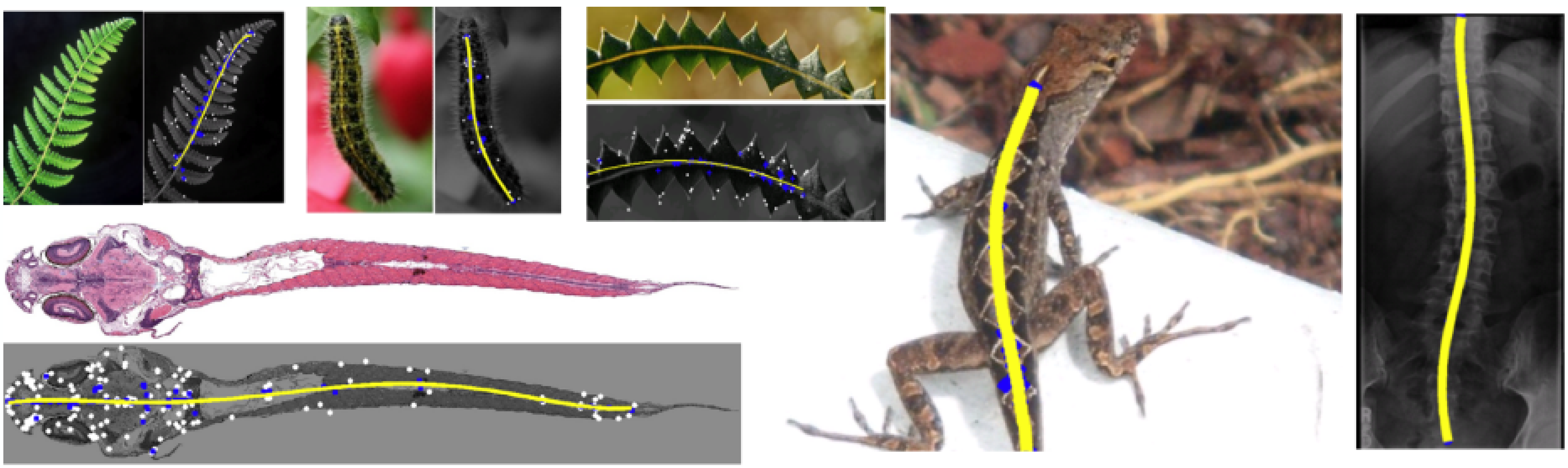
Current Research

- Regularity-based
  - Segmentation
  - 3D reconstruction
  - Super-resolution
  - Compression
  - Indexing/retrieval
  - Saliency





Curved Glide **Reflection** Symmetry

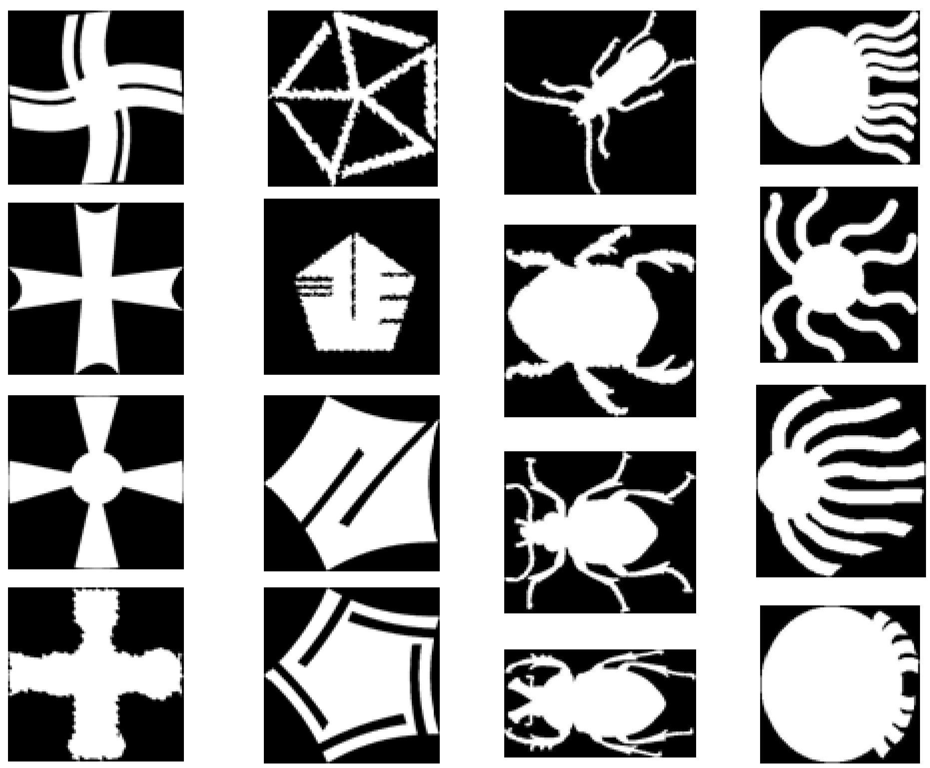


New Advance

- Symmetry-driven Shape Matching
  Seungkyu Lee and Yanxi Liu

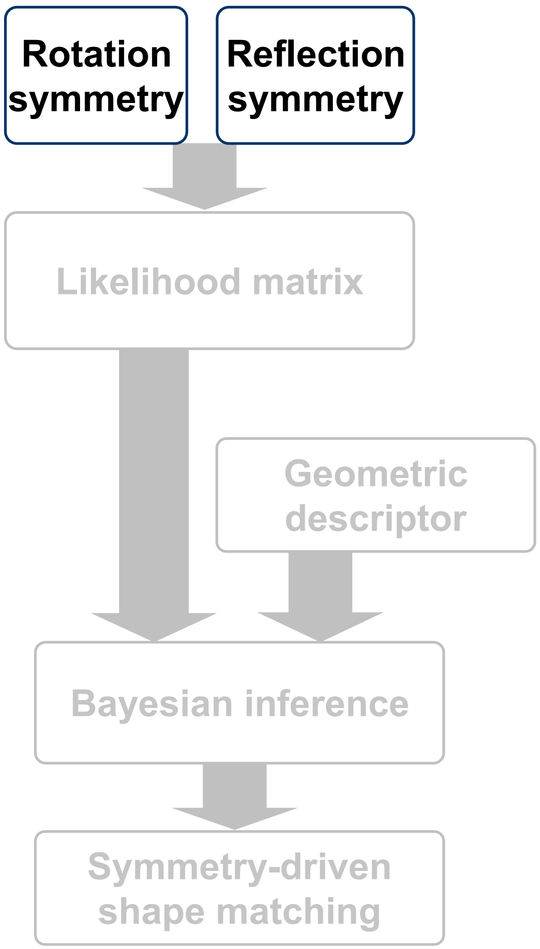
Symmetry of Shapes

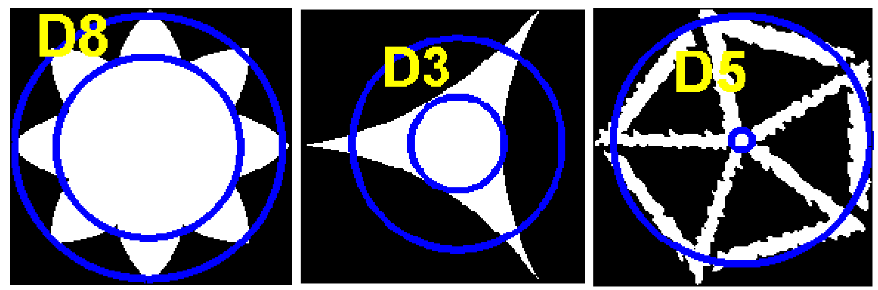
- Symmetry provides a clear discrimination among geometrical shape classes despite their high intra class variance.



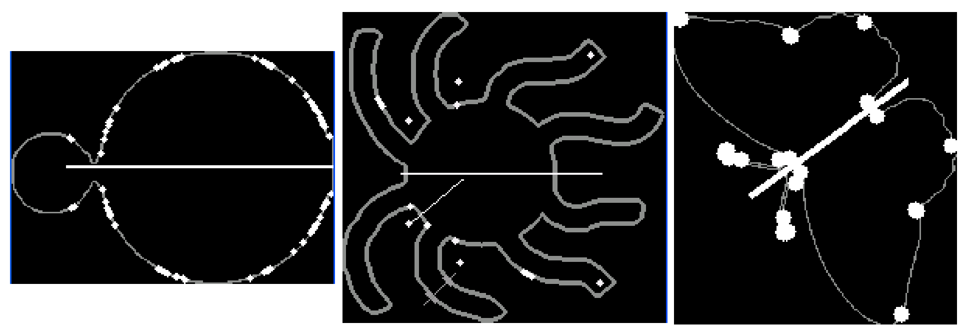
D4 & C4	D5	D1	D1

Symmetry Detection





Our proposed rotation symmetry detection method



[G. Loy and J. Eklundh, ECCV06]

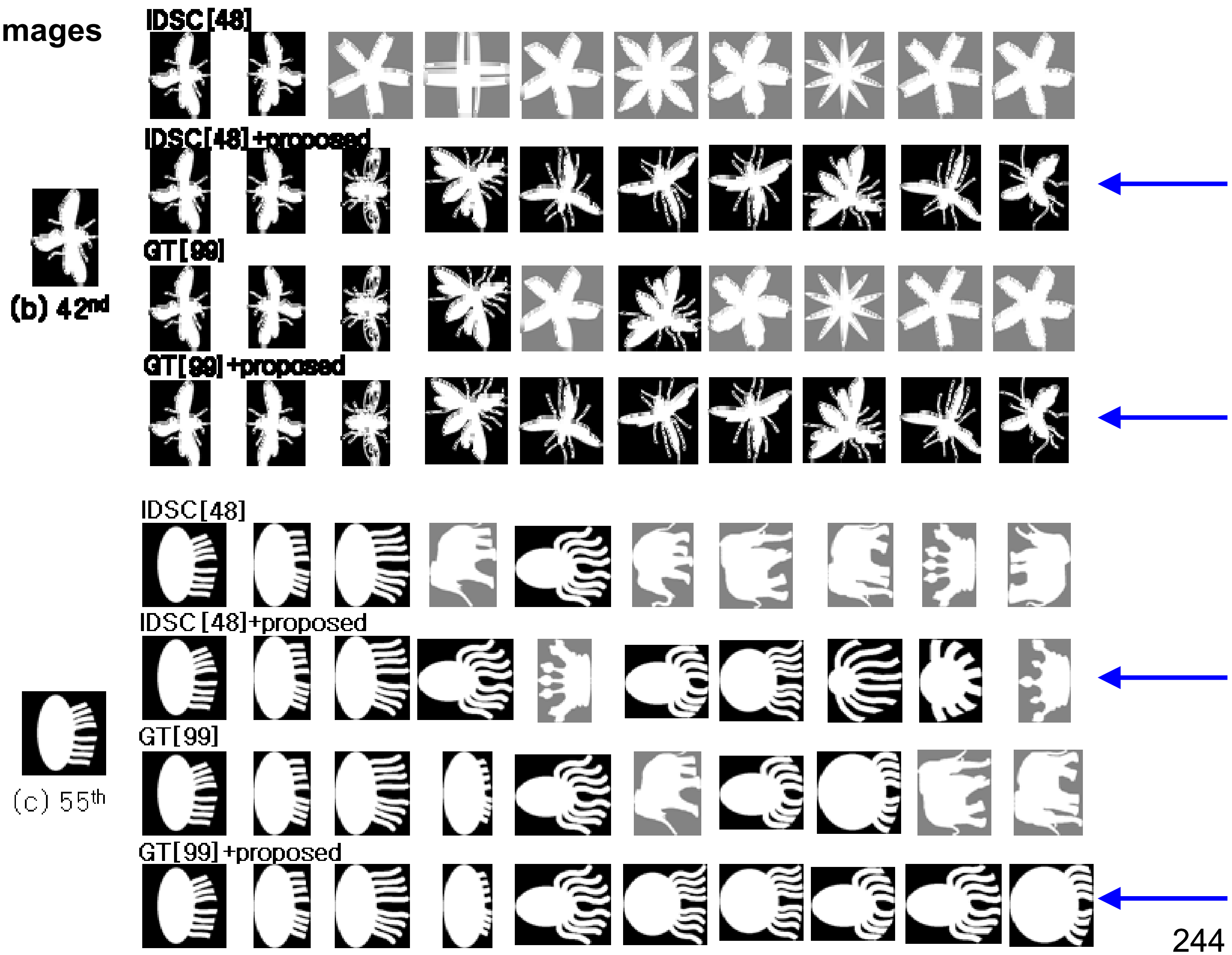
Experimental Results

Retrieval accuracy on the MPEG-7 data set

Method	year	Retrieval accuracy Avg (std) %
CSS[66]	1997	75.44(N/A)%
Visual Parts[38] [39]	2000	76.45(N/A)%
SC+TPS[3]	2002	76.51(N/A)%
Curve Edit[82]	2003	78.14(N/A)%
Generative Models[93]	2004	80.03(N/A)%
IDSC[48]	2007	85.40(18.37)%
Hierarchichal Procrustes[63]	2006	86.35(N/A)%
Shape-tree[17]	2007	87.70(N/A)%
GT[99]	2008	90.89(17.20)%
IDSC[48]+Proposed	2009	**89.34(14.53)%**
GT[99]+Proposed	2009	**91.71(14.49)%**

Experimental results

Top 10 retrieved images

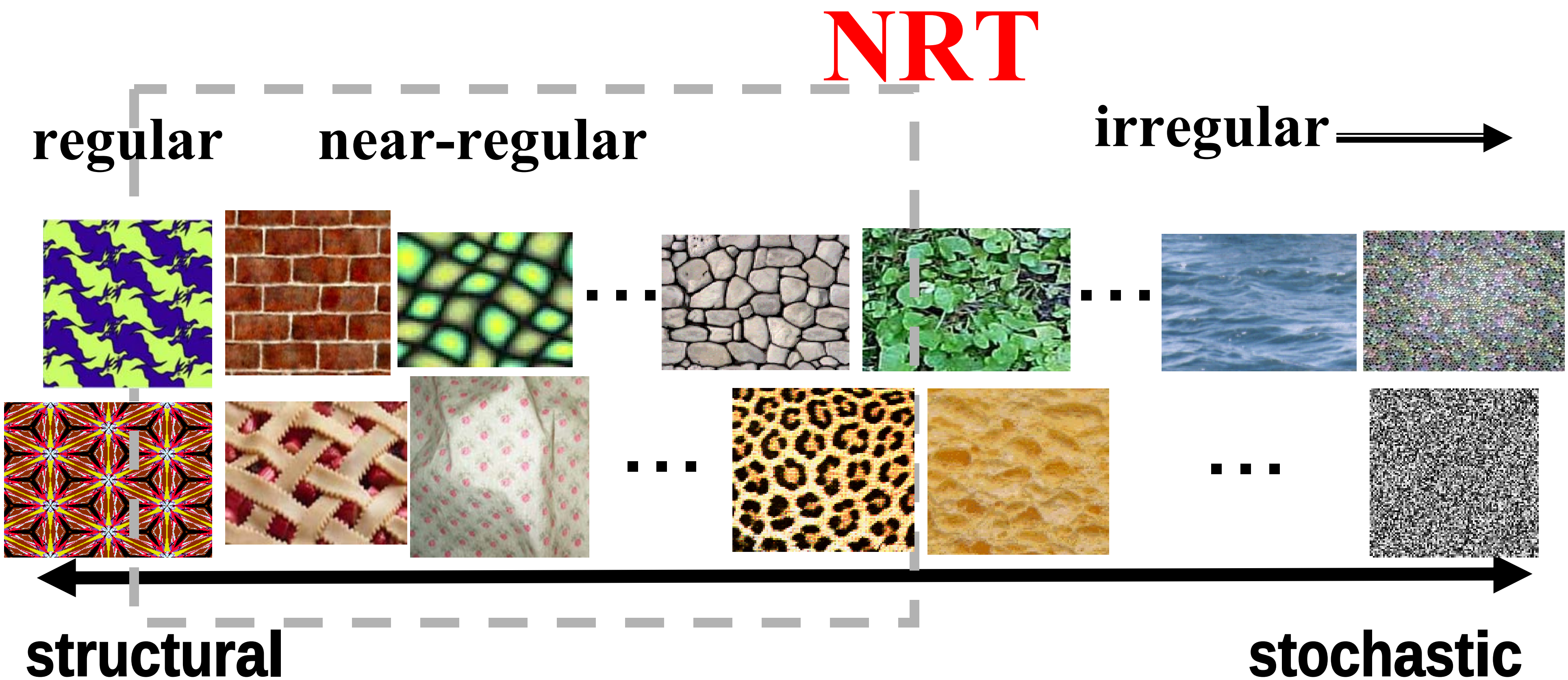


Biomedical Image Analysis

- Face
- Brain

Challenging Open Problems in Computational Regularity and Saliency

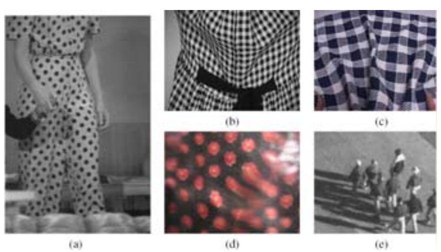
A Texture Spectrum in terms of texture REGULARITY



Regularity Spectrum of dynamic Textures

Regular → Near-regular → … → chaotic

Stationary global lattice topology



Adaptive Local topology

Clustered local topology

Non-uniform local topology

With Increasing level of difficulty
Decreasing level of regularity

- Layered Regularities

Courtesy of David Martin

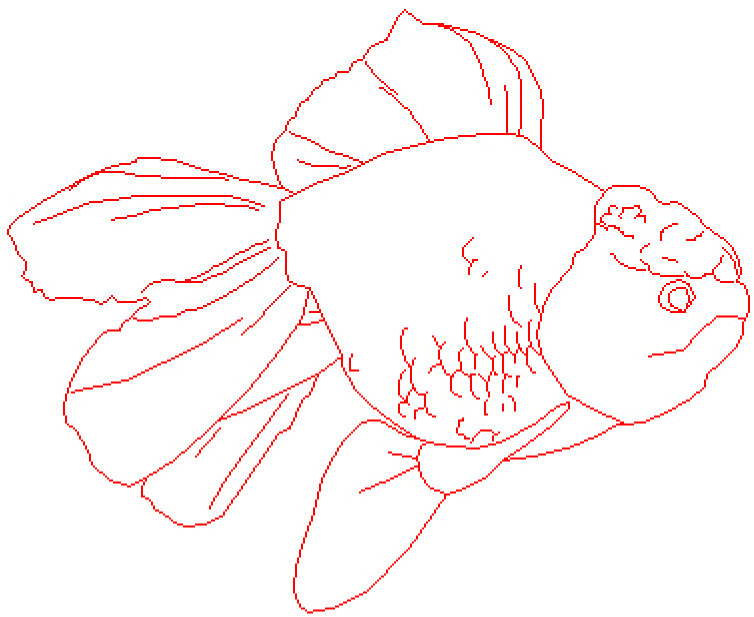
- Disguised regularities/symmetries (symmetry groups)

May we label the net just as a region at high resolution?

Courtesy of Songchun Zhu

Courtesy of Songchun Zhu

Labelers don’t know how to describe the scales on body and striae on fin of the fish (page 13).



Courtesy of Songchun Zhu

Resources

- Our lab: http://vision.cse.psu.edu/index.html
- Data:
- http://vision.cse.psu.edu/data.html

- Code:
- Lattice detection (2 methods), rotation symmetry detection, reflection symmetry detection

Resources

- A survey paper available upon request:

Computational Symmetry for Computer graphics and Computer Vision: A Survey

- Liu et al
- To appear at
- Foundations and Trends® in Computer Graphics and Vision (FTCGV) Published by Now Publishers and marketed by World Scientific

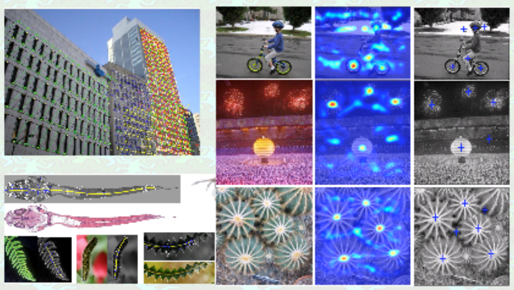
Course:

Machine Learning for Computational Regularity

Yanxi Liu yanxi@cse.psu.edu, **CSE and EE of PSU**

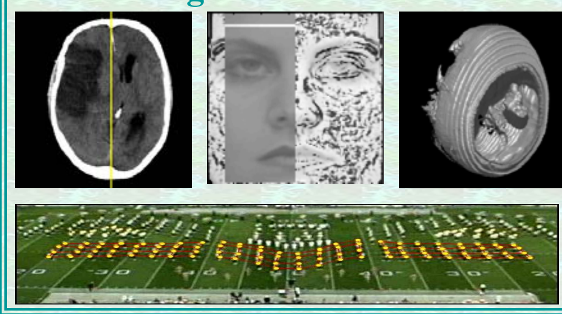
affiliated Radiology Dept. of UPMC, SCS of CMU

Saliency-based Segmentation

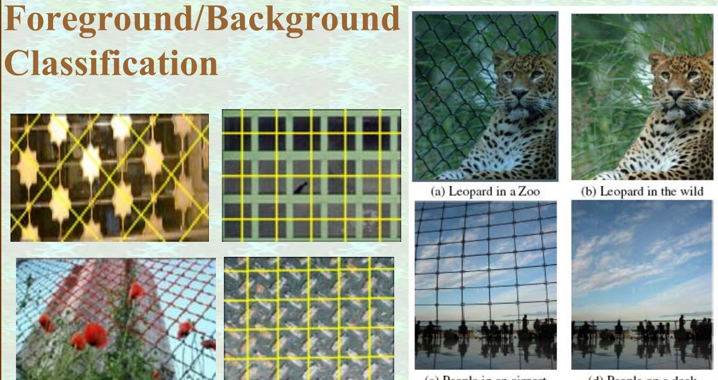


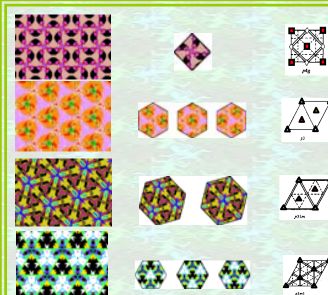
Skewed Symmetry Groups for Automatic Geo-tagging and Indexing

Analysis of Statistically Regular Structure



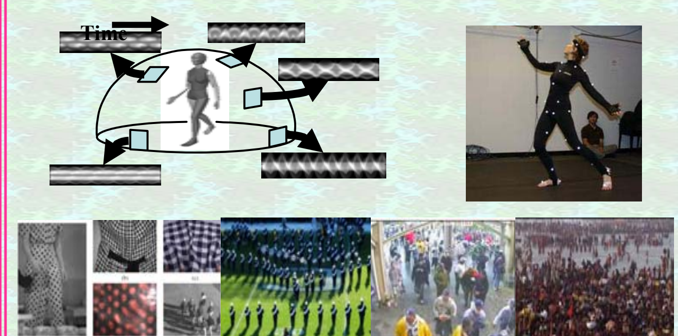
Foreground/Background Classification





Automatic Motif Extraction

Gait, Dance and Crowd Pattern Tracking



The Perils

Tyger! Tyger! burning bright,
In the forest of the night,
What immortal hand or eye
Could frame thy fearful symmetry?

…

William Blake 1794

The Promise:

… beyond absolute regularity there are nuances of decreasing regularities, of reductive invariance: that there is **an order to so-called disorder**.

Ordering Disorder after K.L. Wolf by **W.S. Huff**