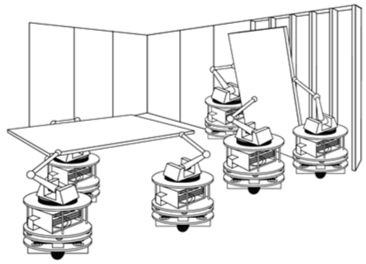
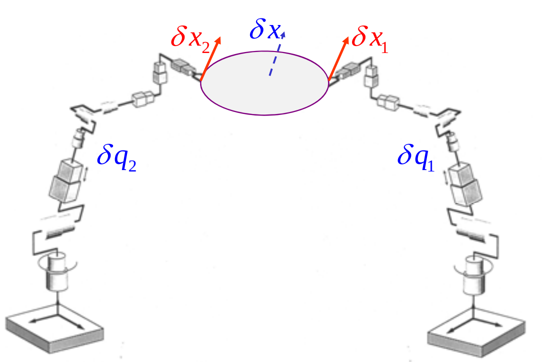


Robot Cooperation

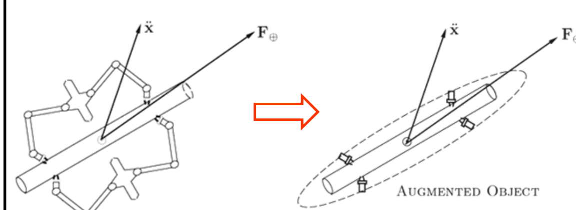
Cooperative Manipulation

- Dynamics: *Augmented Object*
- Internal Forces: *Virtual Linkage*
- Centralized Control: *Fixed-Base Manipulation*
- Decentralized Control: *Mobile-Base Manipulation*

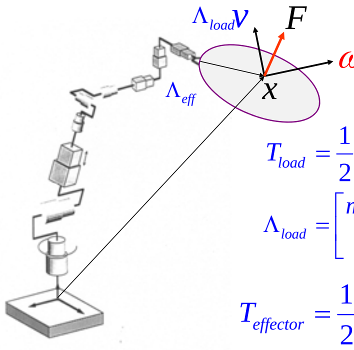
Multi-robot Control Leader/Follower



Augmented Object Model



Effect of a Load

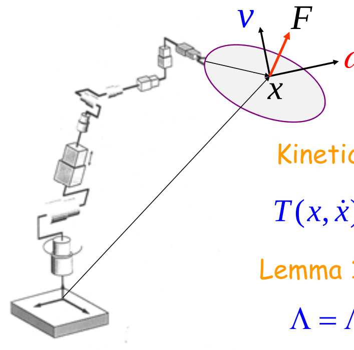


$$T_{load} = \frac{1}{2}\left(m_l v^T v + \omega^T I_l \omega\right)$$

$$\Lambda_{load} = \begin{bmatrix} m_l I_3 & 0 \\ 0 & I_l \end{bmatrix}$$

$$T_{effector} = \frac{1}{2}\dot{x}^T \Lambda_{effector}\, \dot{x}$$

Effect of a Load



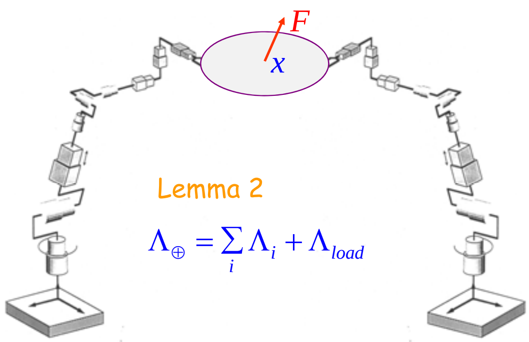
Kinetic Energy

$$T(x,\dot{x}) = T_{effector} + T_{load}$$

Lemma 1

$$\Lambda = \Lambda_{effector} + \Lambda_{load}$$

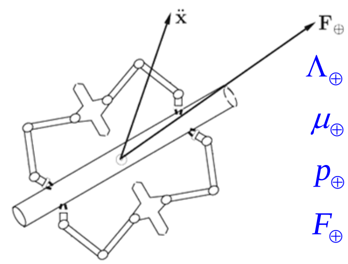
Multi-Arm Control



Lemma 2

$$\Lambda_{\oplus} = \sum_i \Lambda_i + \Lambda_{load}$$

Theorem (Augmented Object)



$$\Lambda_{\oplus} = \sum_i \Lambda_i + \Lambda_{load}$$

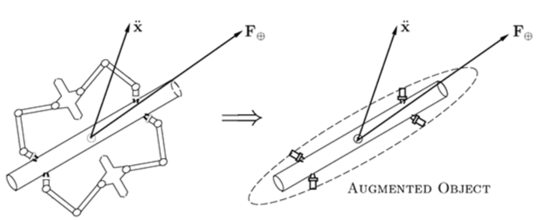
$$\mu_{\oplus} = \sum_i \mu_i + \mu_{load}$$

$$p_{\oplus} = \sum_i p_i + p_{load}$$

$$F_{\oplus} = \sum_i F_i$$

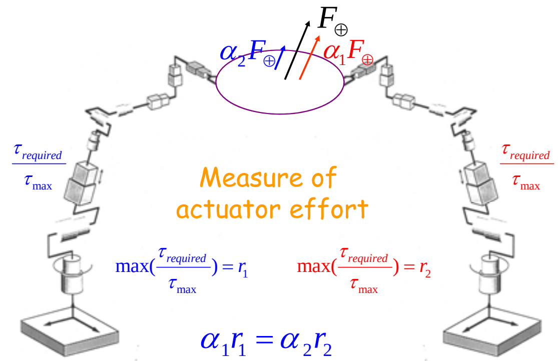
$$\Lambda_{\oplus}\ddot{x} + \mu_{\oplus}(x,\dot{x}) + p_{\oplus}(x) = F_{\oplus}$$

Augmented Object Model



$$\Lambda_{\oplus}\ddot{x} + \mu_{\oplus}(x,\dot{x}) + p_{\oplus}(x) = F_{\oplus}$$

Allocation of Forces



Measure of actuator effort

$$\max\left(\frac{\tau_{required}}{\tau_{max}}\right) = r_1 \qquad \max\left(\frac{\tau_{required}}{\tau_{max}}\right) = r_2$$

$$\alpha_1 r_1 = \alpha_2 r_2$$

Allocation of Forces

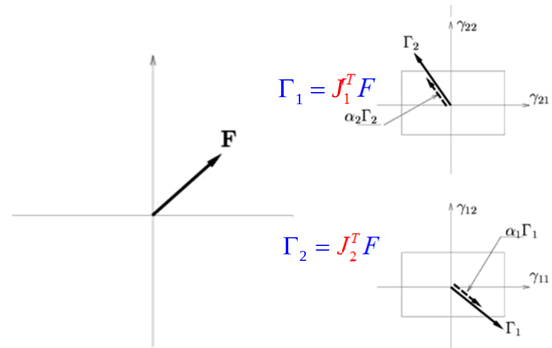
For N robots

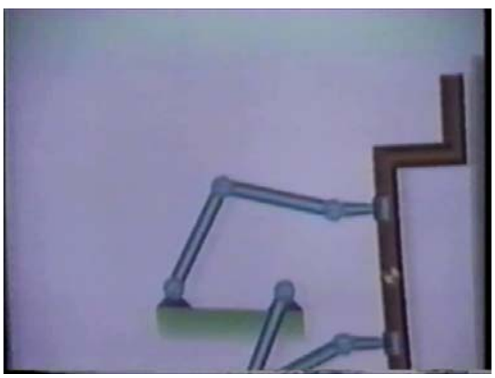
$$\alpha_1 r_1 = \alpha_2 r_2 = \cdots = \alpha_N r_N$$

$\Rightarrow \alpha_i$: Minimized overall effort

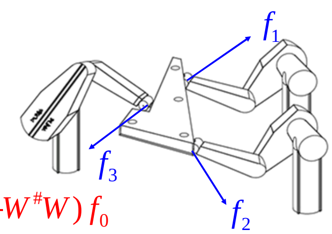
$$\alpha_i = \frac{\beta_i}{\beta_1 + \beta_2 + \cdots + \beta_N}; \text{ where } \beta_i = \frac{r_1 r_2 \cdots r_N}{r_i}$$

Allocation of Forces





Internal Forces

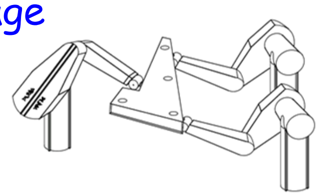


$$f_r = W f$$

$$f = W^{\#} f_r + (I - W^{\#} W) f_0$$

$$f_{\text{internal}} = (I - W^{\#} W) f$$

Virtual Linkage

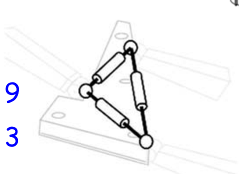


Actuator DOF: 18

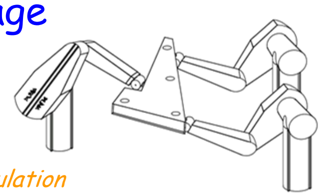
Resultant Force: 6

Actuator Redundancy: 12

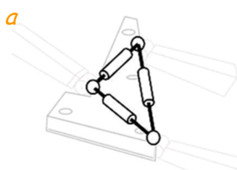
- Internal Moments (3N): 9
- Internal Forces (3N-6): 3



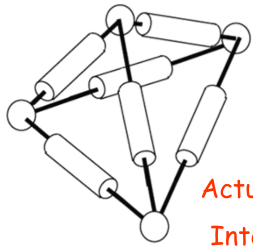
Virtual Linkage



For an N-grasp manipulation task, the virtual linkage is a 6(N-1) DOF mechanism, whose actuated joints characterize the internal forces and moments.



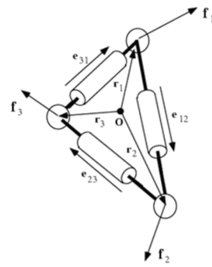
A Four-Grasp Virtual Linkage



Actuation: 24DOF

Internal: 18DOF

Virtual Linkage Resultant Forces



$$\begin{bmatrix} f_r \\ m_r \end{bmatrix} = W_{(6\times18)} \begin{bmatrix} f \\ m \end{bmatrix}$$

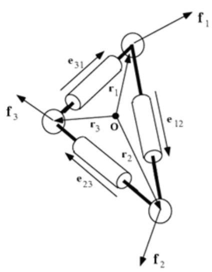
$$W_{(6\times18)} = \begin{bmatrix} W_{f\,(6\times9)} & W_{m(6\times9)} \end{bmatrix}$$

$$W_f = \begin{bmatrix} I_3 & I_3 & I_3 \\ \hat{r}_1 & \hat{r}_2 & \hat{r}_3 \end{bmatrix}$$

$$W_m = \begin{bmatrix} 0 & 0 & 0 \\ I_3 & I_3 & I_3 \end{bmatrix}$$

Virtual Linkage Internal Forces

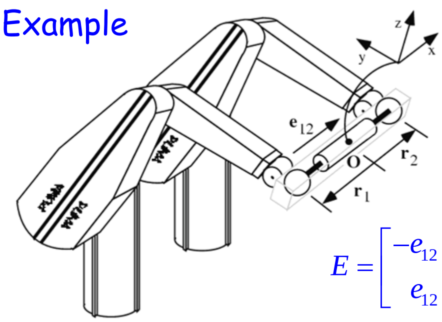
$$f = E_{(9\times3)}\, t$$



$$E = \begin{bmatrix} -e_{12} & 0 & e_{31} \\ e_{12} & -e_{23} & 0 \\ 0 & e_{23} & -e_{31} \end{bmatrix}$$

$$t = \bar{E}_{(3\times9)}\, f$$

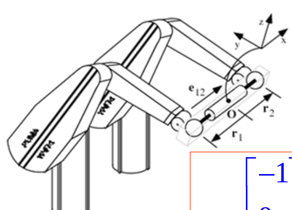
Example



$$E = \begin{bmatrix} -e_{12} \\ e_{12} \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{E} = \frac{1}{2}\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$



$$f = E\;t$$

$$t = \bar{E}\;f$$

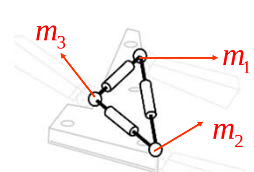
$$t = 1$$

$$f = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\bar{E} = \frac{1}{2}\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Virtual Linkage Internal Moments

$$\tau = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix}$$



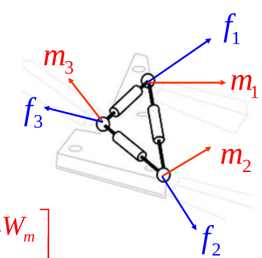
Grasp Description Matrix

$$\begin{bmatrix} f_r \\ m_r \\ t \\ \tau \end{bmatrix} = G_{(18\times18)} \begin{bmatrix} f \\ m \end{bmatrix}$$

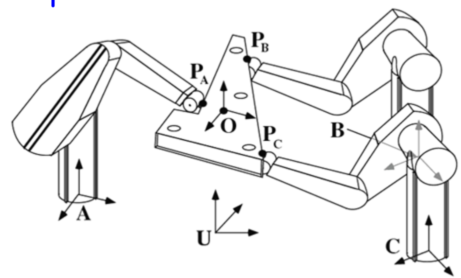
$$G = \begin{bmatrix} W_f & W_m \\ \bar{E} & 0_{3\times9} \\ 0_9 & I_9 \end{bmatrix}$$

full rank

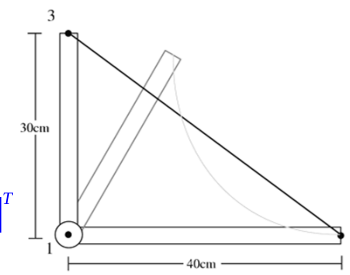
$$G^{-1} = \begin{bmatrix} \bar{W}_f & E & -\bar{W}_f W_m \\ 0_{9\times6} & 0_{9\times3} & I_9 \end{bmatrix}$$



Example



Example



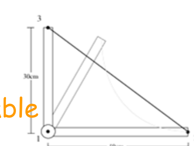
$$e_{12} = [1 \;\; 0 \;\; 0]^T$$

$$e_{23} = [-0.8 \;\; 0.6 \;\; 0]^T$$

$$e_{31} = [0 \;\; -1 \;\; 0]^T$$

$$r_1 = [0 \;\; 0 \;\; 0]^T \quad r_2 = [0.4 \;\; 0 \;\; 0]^T \quad r_3 = [0 \;\; 0.3 \;\; 0]^T$$

Requirements



- 10 Newtons tension in cable
- 5 Newtons compression in horizontal member
- *no tension in vertical member*
- *zero internal moments*
- *no resultant forces or moments*

$$f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -5.00 \\ 10.00 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$W_f =$

1.00	0	0	1.00	0	0	1.00	0	0
0	1.00	0	0	1.00	0	0	1.00	0
0	0	1.00	0	0	1.00	0	0	1.00
0	0	0	0	0	0	0	0	0.30
0	0	0	0	0	-0.40	0	0	0
0	0	0	0	0.40	0	-0.30	0	0

$\bar{W}_f =$

0.39	-0.08	0	0	0	0.60
-0.08	0.44	0	0	0	-0.80
0	0	1.00	-3.33	2.50	0
0.39	-0.08	0	0	0	0.60
0.16	0.12	0	0	0	1.60
0	0	0.00	0	-2.50	0
0.21	0.16	0	0	0	-1.20
-0.08	0.44	0	0	0	-0.80
0	0	0	3.33	0	0

$W_m =$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
1.00	0	0	1.00	0	0	1.00	0	0
0	1.00	0	0	1.00	0	0	1.00	0
0	0	1.00	0	0	1.00	0	0	1.00

$E =$

-1.00	0	0
0	0	-1.00
0	0	0
1.00	0.80	0
0	-0.60	0
0	0	0
0	-0.80	0
0	0.60	1.00
0	0	0

$\bar{E} =$

-0.61	-0.08	0	0.39	0.16	0	0.21	-0.08	0
0.27	0.20	0	0.27	-0.40	0	-0.53	0.20	0
-0.08	-0.56	0	-0.08	0.12	0	0.16	0.44	0

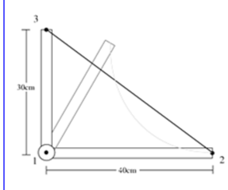
$G =$

1.00	0	0	1.00	0	0	1.00	0	0	0	0	0	0	0	0	0	0	0
0	1.00	0	0	1.00	0	0	1.00	0	0	0	0	0	0	0	0	0	0
0	0	1.00	0	0	1.00	0	0	1.00	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0.30	1.00	0	0	1.00	0	0	1.00	0	0
0	0	0	0	0	-0.40	0	0	0	0	1.00	0	0	1.00	0	0	1.00	0
0	0	0	0	0.40	0	-0.30	0	0	0	0	1.00	0	0	1.00	0	0	1.00
-0.61	-0.08	0	0.39	0.16	0	0.21	-0.08	0	0	0	0	0	0	0	0	0	0
0.27	0.20	0	0.27	-0.40	0	-0.53	0.20	0	0	0	0	0	0	0	0	0	0
-0.08	-0.56	0	-0.08	0.12	0	0.16	0.44	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1.00	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1.00	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1.00	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1.00	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1.00	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00

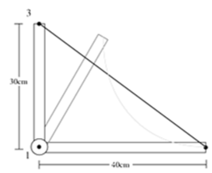
$G^{-1} =$

0.39	-0.08	0	0	0	0.60	-1.00	0	0	0	0	0.60	0	0	0.60	0	0	0.60
-0.08	0.44	0	0	0	-0.80	0	0	-1.00	0	0	-0.80	0	0	-0.80	0	0	-0.80
0	0	1.00	-3.33	2.50	0	0	0	0	-3.33	2.50	0	-3.33	2.50	0	-3.33	2.50	0
0.39	-0.08	0	0	0	0.60	1.00	0.80	0	0	0	0.60	0	0	0.60	0	0	0.60
0.16	0.12	0	0	0	1.60	0	-0.60	0	0	0	1.60	0	0	1.60	0	0	1.60
0	0	0.00	0	-2.50	0	0	0	0	0	-2.50	0	0	-2.50	0	0	-2.50	0
0.21	0.16	0	0	0	-1.20	0	-0.80	0	0	0	-1.20	0	0	-1.20	0	0	-1.20
-0.08	0.44	0	0	0	-0.80	0	0.60	1.00	0	0	-0.80	0	0	-0.80	0	0	-0.80
0	0	0	3.33	0	0	0	0	0	3.33	0	0	3.33	0	0	3.33	0	0
0	0	0	0	0	0	0	0	0	1.00	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1.00	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1.00	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1.00	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1.00	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.00

$f =$	$G^{-1} f =$
0	5.00
0	0
0	0
0	3.00
0	-6.00
0	0
-5.00	-8.00
10.00	6.00
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0
0	0



$$f_{applied} = \begin{bmatrix} -3.00 \\ -3.00 \\ 0 \\ 3.00 \\ -3.00 \\ 0 \\ -3.00 \\ 3.00 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad G\ f_{applied} = \begin{bmatrix} -3.00 \\ -3.00 \\ 0 \\ 0 \\ 0 \\ -0.30 \\ 1.88 \\ 2.80 \\ 2.16 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



Augmented Object

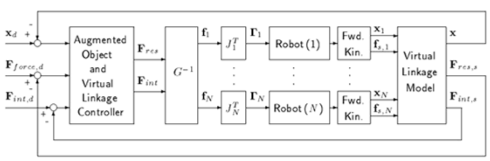
$$\Lambda_{\oplus}\ddot{x} + \mu_{\oplus}(x,\dot{x}) + p_{\oplus}(x) = F_{\oplus}$$

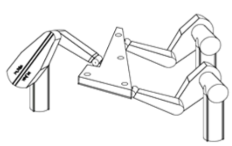
Virtual Linkage: Grasp Matrix

$$\begin{bmatrix} f_{res} \\ f_{\text{int}} \end{bmatrix} = G_{(6N\times 6N)} \begin{bmatrix} f_1 \\ \vdots \\ f_N \end{bmatrix}$$

$$\begin{bmatrix} f_1 \\ \vdots \\ f_N \end{bmatrix} = G^{-1}_{(6N\times 6N)} \begin{bmatrix} f_{res} \\ f_{\text{int}} \end{bmatrix}$$

Centralized Control Structure





Example

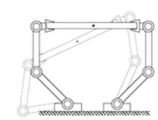
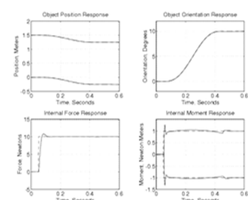
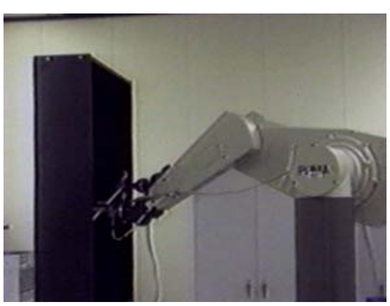


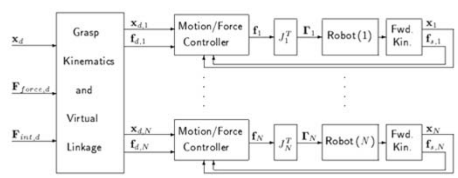
Figure 4.3: A Planar Manipulation Task

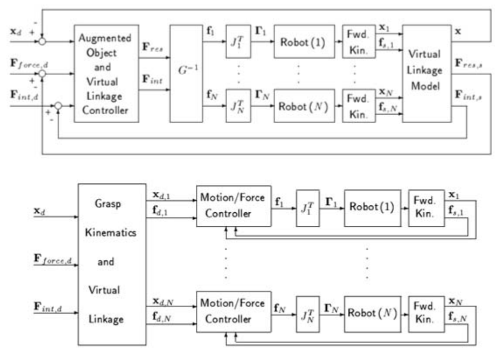
Item	Length (m)	Mass (kg)	Inertia (kg m²)
Link 1	1.0	1.0	0.08333
Link 2	1.0	1.0	0.08333
Link 3	0.2	0.2	0.00067
Object	1.0	1.0	1.0

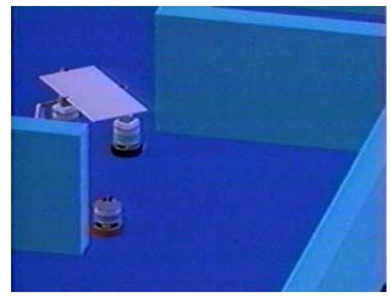




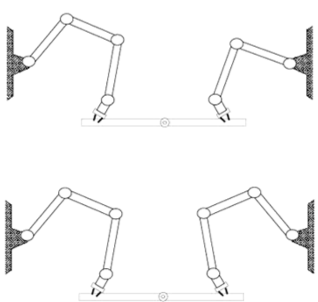
Decentralized Control Structure







Redundancy



Grubler formula

$$n_S = n_0\left(n_{link} - 1\right) - \left(n_0 - 1\right) n_{joint}$$

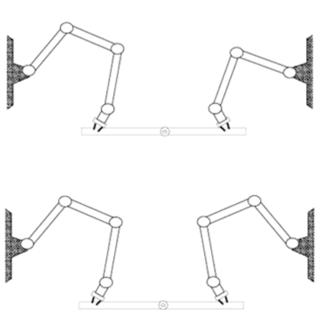
with

$n_0 = 3$ planar

$n_0 = 6$ spatial

Redundancy

$$n_{system} = 3\left(n_{link} - 1\right) - 2n_{joint}$$



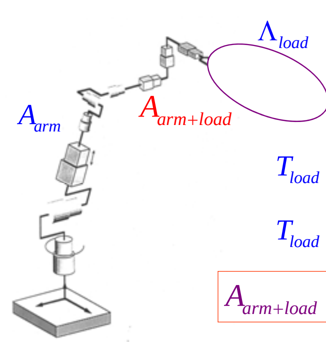
$n_{system} = 4$

$n_{object} = 3$

$n_{system} = 5$

$n_{object} = 3$

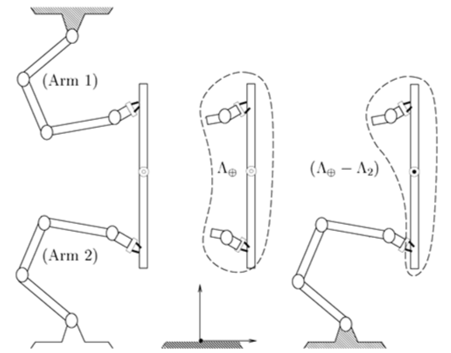
Reflected Load



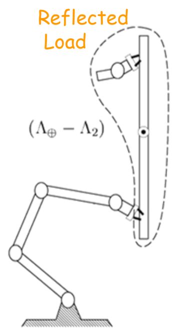
$$T_{load} = \frac{1}{2}\dot{x}^T \Lambda_{load}\, \dot{x}$$

$$T_{load} = \frac{1}{2}\dot{q}^T (J^T \Lambda_{load}\, J)\dot{q}$$

$$A_{arm+load} = A_{arm} + J^T \Lambda_{load} J$$



Multiple Redundant Robots



$$A_{+_i} = A_i + J_i^T \left(\Lambda_\oplus - \Lambda_i\right) J_i$$

$$\bar{J}_i = A_{+_i}^{-1} J_i^T \left(J_i A_{+_i}^{-1} J_i^T\right)^{-1}$$

$$\Gamma_i = \alpha J_i^T F + \left(I - J_i^T \bar{J}_i^T\right)\Gamma_{i0}$$