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Force Strategies in Real Time Fine Motion Assembly

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Abstract

The ability to execute precise assembly by position and force measurement is governed by our understanding of constrained motion of objects. In this paper, we present object motions under position and force feedback and its implementation in an operational space hybrid scheme. The translational motion of object has a simple model. We show possible rotation motions of object due to moment control. Moments arise due to reaction forces and provide contact information. Rotation motion due to contact moment allows assembly of objects under relaxed uncertainty condition.

1 Introduction

Motion in free space, motion in contact, and transition from free space motion to motion in contact is one way of classifying the manipulator motions. Due to position uncertainty, an object usually makes contact with the surrounding and mating objects at a position not coincident with the desired goal configuration. A fine motion of the object is then required to direct it to the goal configuration. The motion under contact is along the tangent direction of contact and requires forces along the normal to maintain the contact. Other forms of constrained motion occur in situations like opening a door where some external kinematic constraint governs the motion. Such motions have been called compliant motion. Transition from free space motion to motion in contact known as *guarded move* in programming context arises while touching, grasping and mating an object.

Both, motion in contact and transition motion allow assembly to be more robust under position uncertainty. For instance, if the objective of mating a vertex of an object on a planar surface is described as a motion towards the plane until contact, the operation is robust under uncertainty in the position of the plane. Similarly, the objective of mating an edge of an object on a plane can be made robust of the uncertainty in the orientation of the plane. It is a motion towards the plane until contact and a continuing motion complying in orientation until zero moment while maintaining the contact. The zero moment condition stated more precisely is zero moment about a line passing through a point within the edge segment and lying parallel to the nominal position of the plane of contact. The point contact example is a *guarded motion*, whereas the line contact example is *compliant motion*. Force strategies like these involve selecting an appropriate point about which the desired forces and moments are specified.

The control schemes for such motions are available in two forms. Stiffness/damping control and pure force control in hybrid scheme. Stiffness/damping control strategy implements compliance by feeding sensed forces through stiffness/damping elements to the position/velocity controller. Pure force control in hybrid scheme explicitly decouples directions of motion control and force control based on motion constraints. Force control and motion control directions are orthogonal. The actuator signals are sums of control signals required for force and motion control.

This paper presents a framework of assembly under force strategies and its implementation in an operational space hybrid position/force control scheme. As illustrations, we present strategy for 3D peg-in-hole problem and planar face to planar face stacking operation. The peg-in-hole strategy refers to peg getting into the hole rather than the peg-already-in-hole problem.

1.1 Previous Work

Compliant motions and the corresponding control schemes have previously been studied by [Nevins and Whitney 1975], [Paul and Shimano 1976], [Salisbury 1980], [Khatib 1980], [Mason 1981,1983], [Raibert and Craig 1981], [Whitney 1982] and [Khatib and Burdick 1986], [Khatib 1987]. A recent paper by Whitney [Whitney 1985] traces the development of robot force control in a historical perspective.

Under automatic synthesis of fine motions, object motion under contact has been studied by [Mason 1981], [LMT 1983], [Mason 1984] and [Erdman 1986]. [Mason 1981] presented a sufficiently general framework for constrained motions. Physical Constraints on motion are natural constraints and the control strategy is orthogonal to the the natural constraints called artificial constraint. Reasonings based on this framework describe the instantaneous characteristic of object motion. In the next effort on the subject by [LMT 1983], effect of friction was accounted for during motion. The possible motions were governed by *damper model*. They proposed a recursive structure of "single motion - termination" predicate to achieve goal. [Mason 1984] in the continuing serial presented an algorithm for computing Fine Motions in bounded-complete situations. Simultaneously, [Erdman 1986] extended [LMT 1983] work by computing a pre-image from backprojections. Both extensions subsumed that motions were governed by *damper model*.

Beside these efforts on the general issue of constrained motion, specific attempts were made to understand peg-in-hole problem. [McCallion and Wong 1975] presented results on jamming, misalignment and chamfer effect of 3-D peg-in-hole problem. This paper presented an interesting idea of exploring the hole with moment application for tipping. [Ohwovoriole 1980] classified problems in insertion under jamming, partial wedging and wedging conditions. Under purely rigid body assumption, wedging was deemed theoretically imposible. Later [Whitney 1982] presented an analysis of peg insertion under RCC control structure with peg approach limited to chamfered hole.

Finally, in a more abstract setting, Hopcroft and Wilfong [HW 1984] proved that if two objects in contact can be moved to another configuration in contact, then there is a way to move them from first configuration to the second configuration such that the objects remain in contact throughtout the motion.

2 Background

Two aspects, the specification of the force strategy and the actual scheme of controller are discussed in this section. The task specification is based on a hybrid control scheme. The controller is an operational space implementation [Khatib 1987]. This formulation is extended to incorporate change in the dynamic decoupling point. The change requires minimal additional real time computation burden.

2.1 Task Specification

Desired position and force specifications are such that the force freedom space and position freedom space are orthogonal. The position and forces can be specified in the "tool frame" or in a fixed frame. Niether of these are general enough to accommodate all motion classes. [Khatib 1987] presented a form of generalized task specification. We use the same frame specification scheme. Thus, specified with respect to the moving object is a reference point $\mathcal{O}$ either on or off the body. In a control scheme, this point is the dynamic decoupling point. This point is rigidly attached to the moving body *i.e.*they translate with the object. A global frame $\mathcal{R}_0(\mathcal{O}, \mathbf{x}_0, \mathbf{y}_0, \mathbf{z}_0)$ is defined with the origin at the reference point $\mathcal{O}$. This frame always remains parallel to a world frame irrespective of the orientation of the moving body. Two reference frames called force frame $\mathcal{R}_f(\mathcal{O}, \mathbf{x}_f, \mathbf{y}_f, \mathbf{z}_f)$ and moment frame $\mathcal{R}_\tau(\mathcal{O}, \mathbf{x}_\tau, \mathbf{y}_\tau, \mathbf{z}_\tau)$ are defined in the global frame, all with common origin at the reference point. These two frames can change their specification with respect to the global reference frame. The force reference frame $\mathcal{R}_f$ is used for specifying either position or force control along its principal directions and the moment reference frame $\mathcal{R}_\tau$ is used for specifying either orientation or moment control along its principal directions. There are two compliance selection matrices Σ_f and Σ_τ that define position control directions

$$\Sigma_f = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}; \quad \Sigma_\tau = \begin{bmatrix} \sigma_\theta & 0 & 0 \\ 0 & \sigma_\phi & 0 \\ 0 & 0 & \sigma_\psi \end{bmatrix} \tag{1}$$

where σ_i's are boolean variables; $\sigma_i = 1$ indicates position/orientation control in that direction and $\sigma_i = 0$ indicates force/moment control. The position variables and orientation variables can be measured with respect to two frames that are not necessarily coincident. Thus, independent specification of $\mathcal{R}_f$ and $\mathcal{R}_\tau$ allows specification of rotation/moment freedom about directions not necessarily aligned with the position/force directions. Since, the directions of position control and force control are orthogonal to each other

$$\overline{\Sigma}_f = I - \Sigma_f; \qquad \overline{\Sigma}_\tau = I - \Sigma_\tau \tag{2}$$

denote the force control directions.(I denotes 3×3 identity matrix).

Let S_f and S_τ represent transformations that take vectors in global frame to vectors in force frame $\mathcal{R}_f$ and moment frame $\mathcal{R}_\tau$ respectively. The force and position trajectory can be specified in the respective force/moment frames or in the global frame. If the trajectory is specified in the global frame, the following generalized selection matrices Ω for position and $\widetilde{\Omega}$ for forces are defined:

$$\Omega = \begin{bmatrix} S_f^T \, \Sigma_f \, S_f & 0 \\ 0 & S_\tau^T \, \Sigma_\tau \, S_\tau \end{bmatrix}; \tag{3}$$

$$\widetilde{\Omega} = \begin{bmatrix} S_f^T \, \overline{\Sigma}_f \, S_f & 0 \\ 0 & S_\tau^T \, \overline{\Sigma}_\tau \, S_\tau \end{bmatrix}. \tag{4}$$

The global trajectory is transformed to the respective force frame $\mathcal{R}_f$ and moment frame $\mathcal{R}_\tau$, the appropriate axes of position or forces are selected and they are brought back to the global frame. Such a scheme allows the dynamics and kinematics to remain in the global frame.

The force and moment frame specifications S_f and S_τ can either be constant, function of moving objects position or function of time. The reference point *i.e.* the origin of frames $\mathcal{R}_{0,f,\tau}$ can also be constant, function of moving objects position or function of time.

2.2 The Controller

2.2.1 Operational Space Formulation

The position and orientation of the reference point $\mathcal{O}$ of the end-effector is described, in the global frame $\mathcal{R}_0$, by the $m_0 \times 1$ column matrix $\mathbf{x}$ of independent configuration parameters, *i.e.*operational coordinates. Similarly, the manipulator configuration in joint space is represented by the column matrix $\mathbf{q}$ of n joint coordinates. The end-effector equations of motion in operational space can be written [Khatib 1987] in the form

$$\Lambda(\mathbf{x})\ddot{\mathbf{x}} + \mu(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{p}(\mathbf{x}) = \mathbf{F}; \qquad (5)$$

where $\Lambda(\mathbf{x})$ designates the $m_0 \times m_0$ symmetric matrix of the quadratic form, *i.e.*the kinetic energy matrix, $\mu(\mathbf{x}, \dot{\mathbf{x}})$ is the vector of end-effector centrifugal and Coriolis forces, $\mathbf{p}(\mathbf{x})$ be the vector of gravity forces and $\mathbf{F}$ is the force vector in operational space. Similarly, the manipulator equations of motion in joint space are given by

$$A(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \mathbf{\Gamma}; \qquad (6)$$

where $\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}})$, $\mathbf{g}(\mathbf{q})$, and $\mathbf{\Gamma}$ represent, respectively, the Coriolis and centrifugal, gravity, and generalized forces in joint space. $A(\mathbf{q})$ is the $n \times n$ joint space kinetic energy matrix.

The dynamic parameters in the operational space model are related to the joint space dynamic model parameters. Let the kinematic model associated with the operational co-ordinates be given by

$$\delta\mathbf{x} = J(\mathbf{q})\delta\mathbf{q}; \qquad (7)$$

The following relationships between operational space dynamic model and joint space dynamic model can be derived[Khatib 1987]

$$\Lambda(\mathbf{x}) = J^{-T}(\mathbf{q})A(\mathbf{q})J^{-1}(\mathbf{q}). \qquad (8)$$

$$\mu(\mathbf{x}, \dot{\mathbf{x}}) = J^{-T}(\mathbf{q})\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) - \Lambda(\mathbf{q})\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}); \qquad (9)$$

where

$$\mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \dot{J}(\mathbf{q})\dot{\mathbf{q}}.$$

and

$$\mathbf{p}(\mathbf{x}) = J^{-T}(\mathbf{q})\mathbf{g}(\mathbf{q}). \qquad (10)$$

2.2.2 Motion and Force Control in Operational Space

While in motion, a manipulator end-effector is subject to the inertial coupling, centrifugal, and Coriolis forces. These nonlinearities can be compensated for by dynamic decoupling in operational space using the end-effector equations of motion (5). The operational command vector for the end-effector dynamic decoupling, motion and force control is

$$\mathbf{F} = \mathbf{F_m} + \mathbf{F_a} + \mathbf{F_{ccg}}; \qquad (11)$$

where $\mathbf{F}_m$, $\mathbf{F}_a$, and $\mathbf{F}_{ccg}$ are the operational command vectors of motion, active force control, and centrifugal, Coriolis, and gravity forces. The specification of axes of rotations and applied moments in the matrices Σ_τ and $\overline{\Sigma}_\tau$ are only compatible with descriptions of the orientation using instantaneous angular rotations. The jacobian associated with the linear and angular velocity is called *the basic Jacobian.*

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = J_0(\mathbf{q})\dot{\mathbf{q}}. \qquad (12)$$

The Jacobian matrix $J(\mathbf{q})$ associated with a given representation of the end-effector orientation x_r can then be expressed in the form

$$J(\mathbf{q}) = E_{x_r} J_0(\mathbf{q}); \qquad (13)$$

where the matrix E_{x_r} is simply given as a function of x_r. Thus, $\Lambda_0(\mathbf{q})$, μ_0 and $\mathbf{p}_0$ can be defined similar to $\Lambda(\mathbf{q})$ and μ and $\mathbf{p}$ with $\mathbf{J}(\mathbf{q})$ being replaced by $J_0(\mathbf{q})$. The command vectors of motion, active force control and centrifugal, coriolis and gravity forces are thus given by

$$\begin{aligned} \mathbf{F}_m &= \widehat{\Lambda}_0(\mathbf{q})\Omega\mathbf{F}^*_m; \\ \mathbf{F}_a &= \widetilde{\Omega}\mathbf{F}^*_a + \widetilde{\Omega}\widehat{\Lambda}_0(\mathbf{q})\mathbf{F}^*_s; \\ \mathbf{F}_{ccg} &= \widehat{\mu_0}(\mathbf{x}, \dot{\mathbf{x}}) + \widehat{\mathbf{p}_0}(\mathbf{x}); \end{aligned} \qquad (14)$$

where $\Lambda(\mathbf{q})$ here is expressed as function of joint parameters rather than the operational space coordinates. $\widehat{\Lambda}$, $\widehat{\mu}$, and $\widehat{\mathbf{p}}$ are estimates of the respective quantities. But the hat has been dropped later for the sake of symbol simplicity. $\mathbf{F}^*_s$ represents the vector of end-effector velocity damping that acts in the direction of force control. If end-effector motions are specified in terms of cartesian coordinates and instantaneous angular rotations

$$\mathbf{F}^*_m = I_{m_0}\ddot{\mathbf{x}}_d - k_p(\mathbf{x} - \mathbf{x}_d) - k_v(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d); \qquad (15)$$

where $\mathbf{x}_d$, $\dot{\mathbf{x}}_d$ and $\ddot{\mathbf{x}}_d$ are the desired position, velocity and acceleration, respectively, of the end-effector. I_{m_0} is the $m_0 \times m_0$ identity matrix. k_p and k_v are the position and velocity gain matrices.
The joint force vector corresponding to $\mathbf{F}$ in (11), is

$$\Gamma = J_0^T(\mathrm{q})\mathbf{F}; \qquad (16)$$

Thus,

$$\Gamma = J_0^T(\mathrm{q})[\Lambda_0(\mathrm{q})\Omega\mathbf{F}^*_m + \tilde{\Omega}\Lambda_0(\mathrm{q})\mathbf{F}^*_s + \tilde{\Omega}\mathbf{F}^*_a + \mathbf{F}_{ccg}];$$

With a perfect nonlinear dynamic decoupling, the end-effector becomes equivalent to a *single unit mass*, I_{m_0}, moving in the m_0-dimensional space.

2.2.3 Change in Dynamic Decoupling Point

In a conventional control scheme, the dynamic decoupling point is some point of the end-effector chosen a-priori. The dynamic parameters are precomputed at this reference point W. In order to extend the control scheme so that the same dynamic model can be used with changing reference point $\mathcal{O}$, the following simple observation is used.
The jacobian $\mathbf{J}_w$ at the point where the dynamic parameters are precomputed and the jacobian $\mathbf{J}$ at the desired reference point are related by

$$\mathbf{J}(\mathrm{q}) = \mathbf{M}(\mathrm{q})\mathbf{J}_w(\mathrm{q}) \qquad (17)$$

where

$$\mathbf{M}(\mathrm{q}) = \begin{bmatrix} \mathbf{I}_{3\times3} & -\widehat{\mathbf{e}}_{3\times3} \\ 0_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix};$$

and

$$\mathbf{e} = \mathbf{r} - \mathbf{r}_w$$

where r and $\mathbf{r}_w$ are position vectors for points $\mathcal{O}$ and W in global frame and $\widehat{\mathbf{e}}$ is outer product operator. The subscript w in parameters refer to the values evaluated at the precomputed dynamic model reference point and unsubscripted values refer to values at $\mathcal{O}$. The following identity between kinetic energy matrices follow from the above relationship.

$$\Lambda(\mathrm{x}) = M^{-T}(\mathrm{q})\Lambda_w(\mathrm{q})M^{-1}(\mathrm{q}). \qquad (18)$$

The relationship for coriolis and centrifugal terms can be established similarly.

3 Motion Under Force Control

The specification of motion under hybrid scheme involves the following:

1. Reference point $\mathcal{O}$ (constant, function of moving object's configuration, or function of time; either on or off the moving body).
2. Position/force frame of reference S_f (constant, function of moving object's configuration, or function of time).
3. Orientation/moment frame of reference S_τ (constant, function of moving object's configuration, or function of time).
4. Position/force compliance selection matrix Σ_f.
5. Orientation/moment compliance selection matrix Σ_τ.
6. Vector of desired position/force $\mathbf{p}_D/\mathbf{f}^f_D$.
7. Vector of desired orientation/moment $\theta_D/\mathbf{m}_D$.

The position trajectory $\{\mathbf{p}_D|\theta_D\}$ governs the motion in position controlled direction. The motion trajectory in force control direction is dependent on the desired force $\{\mathbf{f}^f_D|\mathbf{m}_D\}$ trajectory and the reaction forces $\mathbf{f}_R$ from the environment. Thus, if $\mathbf{f}_{net}$ represents the net forces acting on the manipulator

$$\mathbf{f}_{net} = \mathbf{f}_D + \mathbf{f}_R; \qquad (19)$$

The component of $\mathbf{f}_{net}$ along force control direction generates motion along them. For instance, a manipulator is force controlled along vertical axis with zero desired force. A push on the manipulator along vertically up direction will make the manipulator move up. By complying, the manipulator maintains the desired zero force. Similarly, consider a manipulator moment controlled about an axis with zero desired moment. A force applied from the environment that has a net moment about the axis of moment control will cause the manipulator to rotate. The direction of rotation is determined by the component of applied moment along the axis of moment control. By complying, the manipulator maintains the desired zero moment.

3.1 Motion under Moment Control

The location of reference point $\mathcal{O}$ determines the magnitude and direction of the net moment.

$$\begin{bmatrix} \mathbf{f}^f_{net} \\ \mathbf{m}_{net} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^f_D \\ \mathbf{m}_D \end{bmatrix} + \begin{bmatrix} \mathbf{f}^f_R \\ -\mathbf{r}_{OR} \times \mathbf{f}^f_R + \mathbf{m}_R \end{bmatrix}; \qquad (20)$$

where $\mathbf{r}_{OR}$ is a vector from the reference point $\mathcal{O}$ to the *reaction point*, R. The system of reaction forces from the environment $\{\mathbf{f}_R^f | \mathbf{m}_R\}$ is represented at R. For a given set of reaction forces from the environment, the net moment changes as a function of the reference point $\mathcal{O}$. A plane that is defined by two lines - the line of action of $\mathbf{f}_R^f$ and the axis of moment control, *partitions* the 3D space. This is the case of $\mathbf{m}_D = 0$. The component of net moment along the axis of moment control is zero for $\mathcal{O}$ lying on this plane and the two sides of the plane have opposite signs. Thus, an object can virtually be rotated in either clockwise or counterclockwise direction about any given axis by selecting the reference point in the appropriate half of the space (fig. 1). There are two degenerate cases though. When the reaction force $\mathbf{f}_R^f$ is zero or when the $\mathbf{f}_R^f$ and the axis of moment control are parallel. In either case, the *partioning plane* is not defined. The case of $\mathbf{m}_D \neq 0$ differentiates the scheme from a remote compliance center (*RCC*)[Drake 75; Whitney 1982] where $\mathbf{m}_D$ is by default zero.

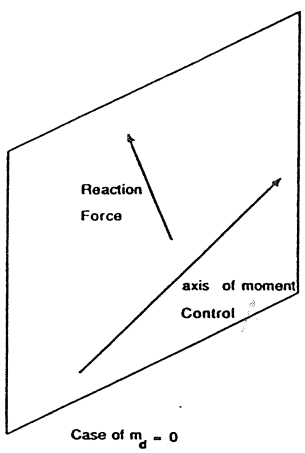

Figure 1: Partitioning Plane

3.2 Rotation motion in Hybrid Scheme

A large set of motions are possible if all the hybrid scheme parameters are usefully exploited. The following three varieties of rotation present one such set of motions.

1. Rotation such that a point of the body is fixed in space.
2. Rotation such that a point of the body remains on a straight line.
3. Rotation such that a point of the body remains on a plane.

These three kinds of rotation are achieved by enforcing position control in all three directions, along two directions and along one direction respectively with appropriate moment control.
Another set of motions come from specifying the force frame $\mathcal{R}_f$ and the moment frame $\mathcal{R}_\tau$ as a function of the configuration of the moving object. Or simply, the force frame and the moment frame can be fixed with the object. It is possible to rotate a crank in this scheme while the plane of rotation of the crank is changing.

4 Illustrations

4.1 Stacking

Stacking of a bounded planar surface on another planar surface is a simple problem. The solution is simple as well. The motion consists of a *guarded move* until contact. Then a compliant move with moment control about two axes lying in the nominal orientation of the plane of contact and force control along perpendicular to the nominal orientation of plane of contact. The refernce point should ideally be the first point of contact. This will reduce any sliding of the moving plane on the fixed plane to zero. Due to uncertainty in the orientation of the moving plane the first point of contact is not known. Hence a point that keeps the travel for all possible points of contact is chosen as the reference point $\mathcal{O}$. For a rectangular and circular surfaces, it is the centroid of the face of contact.

4.2 Peg-in-hole

The commonly known compliant motion strategy for peg-in-hole problem refers to the assembly motion after peg has entered the hole. The same hybrid control parameters may or may not produce a successful entry of a peg into a hole that has a chamfer. The success depends on the relative stiffness/damping in the translation and rotation directions [Whitney 1982]. The strategies for entry of a 3D peg into a hole without chamfer has so far been limited to biased spiral search and tilted peg strategy. Assuming that the hole has been found, a compliant motion is still required to successfully make the peg enter the hole. Based on different categories of rotation a hybrid scheme can generate, we present strategy for 2D peg insertion in a chamferless hole for a planar peg in hole case. We extend these to 3D peg in hole.
One measure of difficulty in getting a peg into hole is the clearance between the hole and peg. Lets define an entry angle $\theta_e = cos^{-1}(r/R)$ where r is peg's radius and R is hole's radius. $\theta_e = 0$ is peg and hole with no clearance. If the peg's angle is within $\pm\,\theta_e$ of the hole's axis, the peg will enter the hole. Let θ_{peg} and θ^*_{peg} be the actual orientation and the measured orientation of

$$\mu \quad = \quad coefficient\ of\ friction.$$

With angular uncertainty, a higher clearance is required to succeed under same position uncertainty and friction coefficient. With angular uncertainty $\pm\theta_{unc}$, μ in equation (21) is replaced by

$$\mu' = \mu\ cos(\theta_{unc}) - sin(\theta_{unc}). \qquad (22)$$

The graph of figure 3 shows the zone of *one point contact* entry as a function of given clearance and position uncertainty. The zone shrinks as θ_{unc} is increased.

Two point contact case

Two point contact situtations occur when reference point can be positioned within $\pm$ radius of the hole but the inequality in (21) is not satisfied, or when the hole has been searched through tilting the peg strategy. Several distinct cases can be isolated based on relative values of θ_{unc}, θ_e and the way hole has been found. Some of them ensure one point entry. The rest may or may not get in two point contact. We assume that two point contact condition is recognizable *i.e.*this situation is distinguishable from the *one point contact* case.

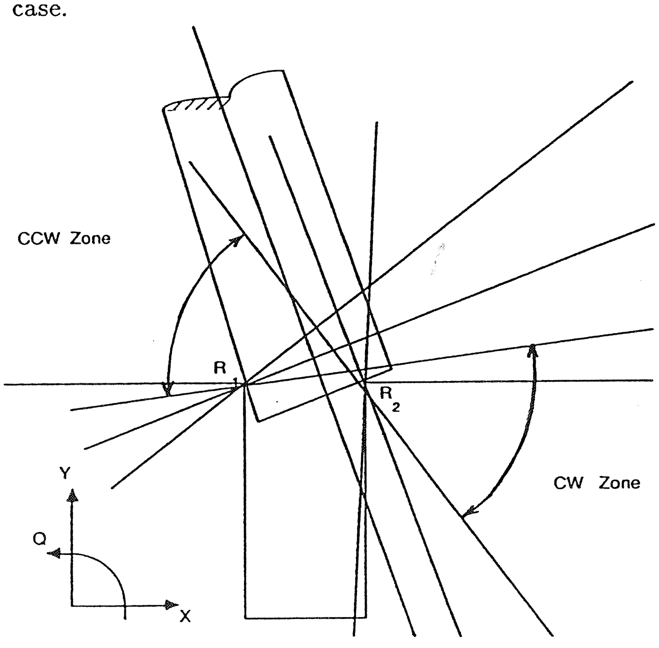

Figure 4: Two Point Contact Case

The reaction forces at the two points of contact lie in the friction cone. They define two zones - *ccw zone* where moment due to the two reaction forces are always positive and *cw zone* where it is negative (fig. 4). The strategy to insert the peg is to maintain the two contact points while rotating in the appropriate direction. In order to maintain the contact a vertically downward force in -ve y direction is applied and a zero horizontal force along x direction is maintained. In order that the peg rotates in the direction that gets it into the hole, the sign of θ_{peg} should be known. That is either the peg is clockwise to the hole axis or counterclockwise to the axis of the hole. Based on this, an appropriate reference point $\mathcal{O}$ can be selected either in *cw zone* or *ccw zone*. Thus, the orientation axis is moment controlled.

4.2.2 3D Peg-in-hole

In principle the strategy for 3D insertion remains same as the 2D case. The *one point contact* and *two point contact* cases become *two point contact* case and *three point contact* case respectively. The possiblity of one point contact occurs at first contact stage. This is reduced to either planar contact or two point contact with moment control about two axes in the plane of contact and reference point $\mathcal{O}$ at the center of the peg. The 3D constraint for *two point contact* entry equivalent to (21) is much tighter. For *three point contact* case, the *cw/ccw zones* of 2D case are volumes in 3D.

5 Conclusion

Study of constrained motion under force control so far has been limited to motions of object modeled by *damper model*. We present motions that the object can make under hybrid force/position control scheme. The concept of natural and artificial constraints extended earlier for deriving hybrid control parameters is dependent on the selection of reference point in addition to the shape of the objects in contact. By selecting the reference point appropriately, rotation motion under moment compliance can be made in either clockwise or counterclockwise direction. Thus, assembly of parts can be made robust under larger uncertainty.

We have successfully implemented the force strategies presented in this paper. An earlier implementation of operational space control system, COSMOS [Khatib and Burdick 1986], in the NYMPH [Chen 1986] multiprocessor system has been extended for moment control and dynamic selection of decoupling point/reference point.

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