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DYNAMIC CONTROL OF MULTI-STRUCTURE ROBOT SYSTEMS AT THE MANIPULATION OBJECT LEVEL

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ABSTRACT

The paper discusses basic methodologies developed within the operational space framework for the analysis and control of robot systems involving combinations of serial and in-parallel mechanical structures. First, we present the fundamentals of the operational space framework and describe the unified approach for motion and active force control of manipulators. For serial structures such as a macro/-manipulator, the effective inertial characteristics of the combined system are shown to be dominated by the inertial properties of the micro-manipulator. This result is the basis for the development of a new approach for dextrous dynamic coordination. In this approach, the combined system is treated as a single redundant manipulator. The dexterity is achieved by minimizing the deviation from the neutral (mid-range) joint positions of the micro-manipulator. In the case where several arms, *i.e.* in-parallel structures, are involved in the manipulation of object, the multi-effector/object system is treated as an augmented object representing the total masses and inertias perceived at some operational point actuated by the total effector forces acting at that point. This model is used for the dynamic decoupling, motion, and active force control of the system using a criterion based on the minimization of the total actuator joint force activities.

KEYWORDS

Augmented object, dextrous dynamic coordination, dynamic decoupling, reduced effective inertia, active force control, macro/micro-manipulator, multiple manipulators, operational space, redundancy.

INTRODUCTION

In quest of higher capabilities and increased performance, robot systems are advancing beyond the traditional single six-degree-of-freedom serial chain mechanism. Recent research and ongoing developments show a clear trend toward robot systems with mechanical structures involving a larger number of degrees of freedom distributed between multiple arms and lightweight micro-manipulators [11,14].

The investigation of control strategies for the coordination of systems with combined mechanical structures has been generally based on the *joint space framework*, where the dynamics is viewed from the perspective of manipulators' joint motions. Based on these models, various control structures for the dynamic control of joint motions have been proposed. The involvement of task specifications in these control systems has required transformations whereby joint motion descriptions are obtained from the task specifications at the manipulated object level.

Task specification for motion and contact forces, dynamics, and force sensing feedback, are most closely linked to the end-effector's motion, or more generally to the manipulated object's motion. The issue of dynamic modeling and control at the manipulated object level is yet more acute for tasks that require simultaneous motion and contact force control of the object. The unability of joint space models to deal with effector or object dynamic control has resulted in force control methodologies that has been essentially based on *kinematic and static* considerations. Force sensing has been used to correct manipulator joint motions. Desired stiffness at the end-effector has been achieved by controlling the corresponding joint stiffnesses using kinematic and static relationships. The performance of the resulting implementations is obviously limited when dynamic effects need to be considered. In free motion, the effects of dynamics increase with the range of motion, speed, and acceleration at which the robot is operating. In assembly operations, the effects of dynamics increase in addition with the rigidity of the mating object. There is clearly a need for the description of the dynamic of the end-effector or manipulated object and its interaction with the environment.

This has been precisely the basic motivation in the development of the *operational space formulation* [4,7]. In this framework both motion and active forces are addressed at the same level of end-effector or manipulated object. The formulation provides a unified approach for the dynamic control of end-effector motions and forces.

Combined Mechanical Structures. The limitations of the joint space framework, mentioned above, become yet more difficult when a larger number of degrees of freedom is involved in the robot system. Motion redundancy is important for extending robot applications to complex tasks and workspaces. But beyond enhancing the system's kinematic and workspace characteristics, motion redundancy can provide effective means to improve the system's dynamic response. The capability of a manipulator to perform fine motions can be significantly improved by incorporating a set of small lightweight links –a micro-manipulator– into the manipulator mechanism [10.11]. Clearly, the high accuracy and greater speed of a micro-manipulator is useful for small range motion operations during which the arm is held motionless. During force control operations, a micro-manipulator can also be used to overcome manipulator errors in the directions of active force control by using end-effector force sensing to perform small and fast adjustments (for example, in high speed edge tracking operations).

However, the improvement of the dynamic performance with lightweight links is not limited to small range motion tasks or to force control operations. In this paper, a fundamental characteristic associated with the effective inertia of macro/micro-manipulator systems is identified. It is shown that with an adequate control strategy, the dynamic performance of robot systems incorporating lightweight structures can be greatly increased in all manipulation tasks, including large range free motion operations.

The development and use of multiple manipulators is another area of growing interest. In recent years, the control problems associated with multi-arm robot systems has received increased attention. Alford and Belyeu [1] studied the coordination of two arms. Their control system is organized in a master/slave fashion, and a motion coordination procedure is used to minimize the error in the relative position between the two manipulator effectors. Zheng and Luh [15] have treated the two manipulators as a "leader" and a "follower" system. The joint torques of the follower are obtained directly from the constraint relationships between the two manipulators. Hayati [3] investigated the problem of motion and force control of multiple manipulators. In his approach, the load is partitioned among the arms. Tarn, Bejczy, and Yun [13] developed the closed chain dynamic model of a two-manipulator system with respect to a selected set of generalized joint coordinates.

In this paper we describe the extension of the operational space framework to robot systems involving multiple arms. A multi-effector/object system will be treated as an *augmented object* representing the total masses and inertias perceived at some operational point.

SINGLE MANIPULATOR SYSTEM

In this section, the operational space framework for a single manipulator is summarized.

Effector Equations of Motion

The effector position and orientation, with respect to a reference frame $\mathcal{R}_O$ of origin O is described by the relationship between $\mathcal{R}_O$ and a coordinate frame $\mathcal{R}_\odot$ of origin $\odot$ attached to this effector. $\odot$ is called the *operational point.* It is with respect to this point that translational and rotational motions and active forces of the effector are specified. An *operational coordinate system* associated with an m-degree-of-freedom effector and a point $\odot$, is a set $\mathbf{x}$ of m *independent* parameters describing the effector position and orientation in a frame of reference $\mathcal{R}_O$. For a non-redundant n-degree-of-freedom manipulator, *i.e.* $n = m$, these parameters form a set of of *generalized operational coordinates*. The effector equations of motion in operational space [4,7] are given by

$$\Lambda(\mathbf{x})\ddot{\mathbf{x}} + \Pi(\mathbf{x})[\dot{\mathbf{x}}\dot{\mathbf{x}}] + \mathbf{p}(\mathbf{x}) = \mathbf{F}; \tag{1}$$

where $\Lambda(\mathbf{x})$ designates the kinetic energy matrix, and $\mathbf{p}(\mathbf{x})$ and $\mathbf{F}$ are respectively the gravity and the generalized operational force vectors. $\Pi(\mathbf{x})$ represents the $m \times m(m+1)/2$ matrix of centrifugal and Coriolis forces. With $\mathrm{J}(\mathbf{q})$ being the Jacobian matrix associated with the generalized operational velocities $\dot{\mathbf{x}}$, the kinetic energy matrix $\Lambda(\mathbf{x})$ is related to the $n \times n$ joint space kinetic energy matrix, $A(\mathbf{q})$ by

$$\Lambda(\mathbf{x}) = J^{-T}(\mathbf{q})A(\mathbf{q})J^{-1}(\mathbf{q}). \tag{2}$$

The generalized joint forces Γ required to produce the operational forces $\mathbf{F}$ are

$$\mathbf{\Gamma} = J^T(\mathbf{q})\mathbf{F}; \tag{3}$$

This relationship is the basis for the control of manipulators in operational space. The dynamic decoupling and motion control of the manipulator in operational space is achieved by selecting the control structure

$$\mathbf{F} = \widehat{\Lambda}(\mathbf{x})\mathbf{F}^\star + \widehat{\Pi}(\mathbf{x})[\dot{\mathbf{x}}\dot{\mathbf{x}}] + \widehat{\mathbf{p}}(\mathbf{x}); \tag{4}$$

where, $\widehat{\Lambda}(\mathbf{x})$, $\widehat{\Pi}(\mathbf{x})$, and $\widehat{\mathbf{p}}(\mathbf{x})$ represent the estimates of $\Lambda(\mathbf{x})$, $\Pi(\mathbf{x})$, and $\mathbf{p}(\mathbf{x})$. With a perfect nonlinear dynamic decoupling, the end-effector becomes equivalent to a *single unit mass*, I_m, moving in the m-dimensional space,

$$I_m\ddot{\mathbf{x}} = \mathbf{F}^\star. \tag{5}$$

$\mathbf{F}^\star$ is the input of the decoupled end-effector. This provides a general framework for the selection of various control structures [11].

Active Force Control. The operational space formulation provides a natural framework for integrating motion control and active force control in a unified manner. In part mating operations, both motions and active forces need to be simultaneously controlled. Such operations typically involve motion control in some directions and active force control in the orthogonal directions, as illustrated in Fig.1. To that purpose, we have introduced the concept of generalized specification matrices, Ω and its complement $\overline{\Omega}$ [7]. Using these matrices, the unified control vector for end-effector motion and active force control is:

$$\mathbf{F} = \Omega\ \mathbf{F}_{\text{Motion}} + \overline{\Omega}\ \mathbf{F}_{\text{Active-Force}}; \tag{6}$$

where $\mathbf{F}_{\text{Motion}}$ is given as in equation 4 and $\mathbf{F}_{\text{Active-Force}}$ is the active force control vector [7]. The control system is developed following a two-level architecture: a low rate dynamic parameter evaluation level updating the dynamic parameters; and a high rate servo control level that computes the command vector using the updated dynamic coefficients.

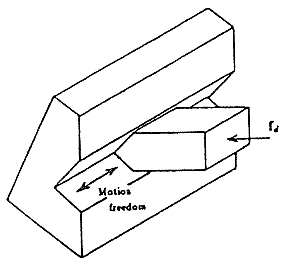

Fig.1. A Constrained Motion Operation

This architecture is illustrated in Fig. 2, where k_v and k_p are the velocity and position gains; k_f, and k_{vf} are the force error gain and the velocity damping in the force control directions; $\tilde{B}$, and $\tilde{C}$ are associated with the Coriolis and centrifugal forces; and g is associated with the gravity.

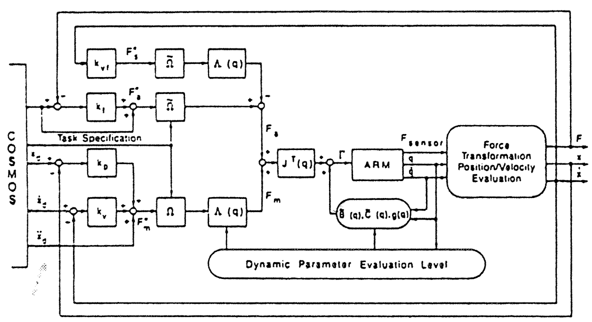

Fig.2. Operational Space Control System Architecture

Redundant Manipulators

A set of operational coordinates, which only describes the end-effector position and orientation, is obviously not sufficient to completely specify the configuration of a redundant manipulator. Therefore, the dynamic behavior of the entire system cannot be described by a dynamic model in operational coordinates. The dynamic behavior of the end-effector itself, nevertheless, can still be described, and its equations of motion in operational space can still be established. In fact, the structure of the effector dynamic model is identical to that obtained in the case of non-redundant manipulators (equation 1). In the redundant case, however, the matrix Λ should be interpreted as a *"pseudo kinetic energy matrix"*. $\Lambda(\mathbf{q})$ is related to the joint space kinetic energy matrix by

$$\Lambda(\mathbf{q}) = [J(\mathbf{q})A^{-1}(\mathbf{q})J^T(\mathbf{q})]^{-1}; \tag{7}$$

Another important characteristic of redundant manipulator is concerned with the relationship between operational forces and joint forces. In the case where $n = m$, an operational force vector F is produced by the unique joint force vector $J^T F$. The additional freedom of redundant mechanism results in an infinity of possible joint force vectors. This is due to the fact that some joint force vectors will only act internally. Those are the joint forces acting in the null space associated with J^T and defined by some generalized inverse P^T. A straightforward static analysis would lead to an infinity of generalized inverse matrices. However, a generalized inverse that is consistent with the system's dynamics is unique [7] and given by

$$\overline{J}(\mathbf{q}) = A^{-1}(\mathbf{q})J^T(\mathbf{q})\Lambda(\mathbf{q}). \tag{8}$$

$\overline{J}(\mathbf{q})$ in equation 8 is actually a generalized inverse of the Jacobian matrix corresponding to the solution that minimizes the manipulator's instantaneous kinetic energy. The relationship between forces is

$$\mathbf{\Gamma} = J^T(\mathbf{q})\mathbf{F} + [I_n - J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]\mathbf{\Gamma}_o; \tag{9}$$

where I_n is the $n \times n$ identity matrix and $\mathbf{\Gamma}_o$ is an arbitrary joint force vector. A Joint force vector of the form $[I_n - J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]\mathbf{\Gamma}_o$ does not only correspond to a zero-vector of operational forces at static equilibrium, but it does also during motion. When the redundant manipulator is submitted to $\mathbf{\Gamma}$ of equation 9, the dynamic behavior in operational space of the end-effector is still governed by an equation identical to 1.

Control of Redundant Manipulators. Similar to the case of non-redundant manipulators, the dynamic decoupling and control of the end-effector can be achieved by selecting an operational command vector of the form 4. The manipulator joint motions produced by this command vector are those that minimize the instantaneous kinetic energy of the mechanism. Analysis shows the system to be stable; however, while the end-effector is asymptotically stable, the manipulator joints can still describe internal motions in the nullspace. Asymptotic stabilization of the entire system can be achieved by the addition of dissipative joint forces. In order to preclude any effect of the additional forces on the end-effector and maintains its dynamic decoupling, these forces must be selected to only act in the dynamically consistent nullspace associated with $\overline{J}(\mathbf{q})$. These additional stabilizing joint forces are of the form

$$\mathbf{\Gamma}_{ns} = [I_n - J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]\mathbf{\Gamma}_s. \tag{10}$$

In the actual implementation, the global control vector will be developed in a form [7] that avoids the explicit evaluation of the expression of the generalized inverse of the Jacobian matrix.

MACRO/MICRO-MANIPULATOR SYSTEMS

Let us consider the case of systems resulting from a serial combination of two manipulators. The manipulator connected to the ground will be referred to as the "macro-manipulator", it has n_M degrees of freedom. The second manipulator, referred to as the "micro-manipulator", has n_μ degrees of freedom. The resulting structure is an n-degree-of-freedom macro/micro-manipulator with $n = n_M + n_\mu$. If m represents the number of effector degrees of freedom of the combined structure, n_M and n_μ are assumed to obey: $n_M > 1$ and $n_\mu \geq m$. This assumption states that what is considered to be the micro-manipulator must possess the full freedom to move in the operational space and can possibly have a redundant structure. The macro-manipulator must have at least one-degree-of-freedom, and can also be redundant. Let Λ_μ be the kinetic energy matrix associated with the micro-manipulator considered alone, and Λ the pseudo kinetic energy matrix associated with the combined mechanism, *i.e.* macro-micro-manipulator.

Theorem 1: *(Reduced Effective Inertia)* The operational space *pseudo kinetic energy matrix* Λ satisfy [9]

$$\frac{1}{1+\eta\cdot\lambda_k(\Lambda_\mu)} \leq \frac{\lambda_k(\Lambda)}{\lambda_k(\Lambda_\mu)} \leq 1; \quad k = 1, 2, \ldots, m$$

where $\eta \geq 0$, and $\lambda_k(.)$ denote the k^{th} largest eigenvalue of (.), *i.e.* $\lambda_m(.) \leq \ldots \leq \lambda_1(.)$. Fig.3 illustrates the inertial characteristics stated in this theorem: the effective inertia, in any direction, of the combined mechanism is smaller than or equal to the inertias associated with the micro-manipulator.

Dextrous Dynamic Coordination

The previous result shows that the dynamic characteristics of the combined system can be made to be comparable to (and, in some cases, better than) those of the micro-manipulator. The basic idea in approaching the control problem associated with coordinating a manipulator and a micro-manipulator system is to treat the manipulator and micro-manipulator as a single redundant system. However, this type of control cannot be directly applied to the macro/micro motion coordination problem. In effect, given the mechanical limits on the range of joint motions of the micro-manipulator, such a controller would rapidly lead to joint saturation of the micro-manipulator degrees of freedom.

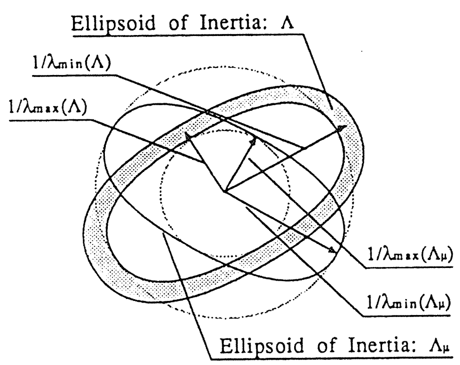

Fig.3.: Reduced Effective Inertia

The *dextrous dynamic coordination* we propose is developed within the framework of redundant manipulator control in operational space. It is based on the minimization of the deviation from the neutral (mid-range) joint positions of the micro-manipulator. This minimization is achieved by the use of joint forces selected from the dynamically consistent null space associated with $\bar{J}(\mathbf{q})$. This will preclude any effects of the additional forces on the primary task. Let $\bar{q}_i$ and $\underline{q}_i$ be the upper and lower bounds on the i^{th} joint position q_i. We construct the potential function

$$V_{\text{Dextrous}}(\mathbf{q}) = k_d \sum_{i=n_M+1}^{n} (q_i - \frac{\bar{q}_i + \underline{q}_i}{2})^2; \qquad (11)$$

where k_d is a constant gain. The gradient of this function

$$\Gamma_{\text{Dextrous}} = -\nabla V_{\text{Dextrous}}; \qquad (12)$$

provides the required attraction [5] toward the mid-range joint positions of the micro-manipulator. The interference of these additional torques with the end-effector dynamics is avoided by selecting them from the null space. This is

$$\Gamma_{nd} = [I_n - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\Gamma_{\text{Dextrous}}. \qquad (13)$$

The avoidance of joint limits is achieved using an "artificial potential field" function [5]. It is essential that the range of motion of the joints associated with the micro-manipulator accommodate the relatively slower dynamic response of the arm. A sufficient motion margin is required for achieving dextrous dynamic coordination.

MULTI-EFFECTOR ROBOT SYSTEM

Let us consider the problem of manipulating an object with a system of N robot manipulators, as illustrated in Fig.4. The effectors of each of these manipulators are assumed to have the same number of degrees of freedom, m, and to be rigidly connected to the manipulated object. Let $\odot$ be the selected operational point attached to this object. This point is fixed with respect to each of the effectors. Let $\Lambda_{\mathcal{L}}(\mathbf{x})$ be the kinetic energy matrix associated with the object's load alone, expressed with respect to $\odot$ and the operational coordinates $\mathbf{x}$. Being held by N effectors, the inertial characteristics of the object as perceived at the operational point are modified. The N-effector/object system can be viewed as an *augmented object* [8] representing the total inertias perceived at $\odot$. Let $\Lambda_i(\mathbf{x})$ be the kinetic energy matrix associated with the i^{th} effector.

Theorem 2: *(Augmented Object)* The kinetic energy matrix of the augmented object is [8]

$$\Lambda_{\oplus}(\mathbf{x}) = \Lambda_{\mathcal{L}}(\mathbf{x}) + \sum_{i=1}^{N} \Lambda_i(\mathbf{x}).$$

The augmented object equations of motion are

$$\Lambda_{\oplus}(\mathbf{x})\ddot{\mathbf{x}} + \Pi_{\oplus}(\mathbf{x})[\dot{\mathbf{x}}\dot{\mathbf{x}}] + \mathbf{p}_{\oplus}(\mathbf{x}) = \mathbf{F}_{\oplus}; \qquad (14)$$

where the matrix $\Pi_{\oplus}(\mathbf{x})$, of centrifugal and Coriolis forces, the vector $\mathbf{p}_{\oplus}(\mathbf{x})$, of gravity forces, and the generalized operational forces $\mathbf{F}_{\oplus}$ also possess the additive property.

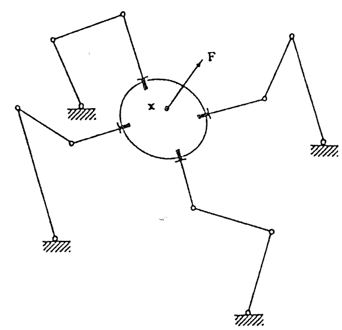

Fig.4. A Multi-Effector/Object System

The augmented object represents the total masses and inertias perceived at the operational point and actuated by the total effector forces acting at that point. Based on this model, a control structure similar to 4 has been used to achieve the dynamic decoupling and control of the combined system. The criterion in the allocation of forces has been based on the minimization of the total joint actuator efforts [8].

SUMMARY AND DISCUSSION

The unified approach for motion and active force control in the operational space framework has been implemented in the COSMOS robot programming system [6]. Results of the experimentation on a PUMA 560 manipulator equipped with wrist and finger force sensors have shown the effectiveness of this approach in achieving high dynamic performance in real-time assembly operations.

The inertial analysis of macro/micro-manipulator system has shown that the dynamic performance of the combined system can be made to be superior to the performance of the micro-manipulator considered alone. This is achieved using a *dextrous dynamic coordination* based on minimizing the deviation from the neutral (mid-range) joint positions of the micro-manipulator. In order to preclude any effects of the additional forces on the primary task, this minimization is realized using joint forces selected from the null space associated with the mapping between operational and joint forces and consistent with system's dynamics.

The augmented object model proposed in this paper constitutes a natural approach for the dynamic modelling and control of multi-effector/object systems. In this approach, the control structure only uses the necessary forces, *i.e.* net force, required to achieve the dynamic decoupling and control of the system. This methodology constitutes a powerful tool for dealing with the problem of object manipulation in a multi-fingered hand system.

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TOPOLOGICAL AND KINEMATICAL STUDY OF TREE STRUCTURE ROBOT MANIPULATORS

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ABSTRACT

Tree structure robot manipulators have been very little studied because of the complexity of their topological structures compared to simple chain type. This paper presents the topological and kinematical analysis of such systems. The kinematic analysis is based on the idea of decomposing the whole model to a number of sub-models, one for each arm of the tree. A complete symbolic kinematical analysis of a five link tree structure with two end effectors is presented. The necessary number of arithmetic operations for computing each model is also computed. This example shows the effectiveness of the used approach for kinematic analysis of this type of robot manipulators.

KEYWORDS

Robotics, Kinematics, Jacobian matrix, Geometric constraints, Articulated hands.

INTRODUCTION

Human has been created with two arms, two hands, and two legs with five fingers each; i.e. tree structure. For replacing a human, in a factory, by an industrial robot manipulator, it is natural to think in a similar structure. In practice, the industrial robots have simple chain structures. This is because of the complexity of the analysis and control of tree structure robot arms. One of the disadvantages of the simple chain type is the singularity of its structure which means that it has more than one conifuguration for the same position and orientation of its end effector [1,2,3]. This problem has been solved for human arms by decreasing the range of motion of its joints. This solution has the side effect of decreasing the working volume of each arm. The total working volume of two arms (left and right) is more than that of one industrial robot arm for same geometrical dimensions. Despite these advantages of tree structure robot arms, they are very little studied. The few available studies are those concerning the articulated hands [4,5,6,7,8].

This paper presents a complete description of the topological and