Augmented Object and Reduced Effective Inertia in Robot Systems

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Abstract

The paper investigates the dynamic characteristics and control of robot systems involving combinations of parallel and serial mechanical structures, e.g. multiple manipulators and macro/micro-manipulators. A framework for the analysis and control of multiple manipulator systems with respect to the dynamic behavior of the manipulated object is developed. A multi-effector/object system is treated as an augmented object representing the total masses and inertias perceived at some operational point. This system is actuated by the total effector forces acting at that point. The allocation of forces is based on the minimization of the total joint actuator efforts. For serial structures, the effective inertial characteristics of a combined macro/micro-manipulator are shown to be dominated by the inertial characteristics of the micro-manipulator. A new approach for a dextrous dynamic coordination of such mechanisms based on treating the combined system as a single redundant manipulator while minimizing of the deviation from the neutral (mid-range) joint positions of the micro-manipulator is proposed.

Introduction

In quest of higher capabilities and increased performance, robot systems are advancing beyond the traditional single six-degree-of-freedom serial chain mechanism. Recent research and ongoing developments show a clear trend toward robot systems with mechanical stuctures involving a larger number of degrees of freedom distributed between multiple arms and lightweight micro-manipulators.

The investigation of control strategies for the coordination of multiple mechanisms has received increased attention in recent years. Alford and Belyeu (1984) studied the coordination of two arms. Their control system is organized in a master/slave fash-

ion, and a motion coordination procedure is used to minimize the error in the relative position between the two manipulator effectors. Zheng and Luh (1986) have treated the two manipulators as a "leader" and a "follower" system. The joint torques of the follower are obtained directly from the constraint relationships between the two manipulators, thereby producing a coordinated control of the system. Hayati (1986) investigated the problem of motion and force control of multiple manipulators. In his approach, the load is partitioned among the arms. Dynamic decoupling and motion control are then achieved at the level of individual manipulator effectors. In the force control subspace, the magnitude of forces is minimized. In relation with the general problem of object grasping various other criterion have been proposed for an optimal selection of internal forces (Salisbury and Craig 1982, Kerr and Roth 1986, Nakamura, Nagai, and Yoshikawa 1987). Tarn, Bejczy, and Yun (1987) developed the closed chain dynamic model of a two-manipulator system with respect to a selected set of generalized joint coordinates. Nonlinear feedback and output decoupling techniques were then used to linearize and control the system in task coordinates.

Joint space dynamic models only provide a description of the interaction between joint motions. The control of object motion and active forces requires the description of how motions along different axes are interacting, and how the apparent or equivalent inertia or mass of the object varies with configurations and directions. In this paper, the operational space framework which focuses on the dynamics of the manipulated object is extended to the case of a multi-effector robot system. The approach is based on the *augmented object concept* (Khatib 1987b).

The development and use of lightweight stuctures to improve robot's accuracy and dynamic performance is another area of growing interest. The capabil-

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ity of a manipulator to perform fine motions can be significantly enhanced by incorporating a set of small lightweight links –a micro-manipulator– into the manipulator mechanism (Hollis, 1985; Reboulet and Robert 1986; Tilley, Cannon, and Kraft 1986; Cai and Roth, 1987). Clearly, the high accuracy and greater speed of a micro-manipulator is useful for small range motion operations during which the arm is held motionless. During force control operations, a micro-manipulator can also be used to overcome manipulator errors in the directions of active force control by using end-effector force sensing to perform small and fast adjustments (for example, in high speed edge tracking operations).

However, the improvement of the dynamic performance with lightweight links is not limited to small range motion tasks or to force control operations. In this paper we will show that with an adequate control strategy, the dynamic performance of robot systems incorporating lightweight structures can be greatly increased in all manipulation tasks, including large range motion operations.

1 Single Manipulator System

In this section, we summarize the operational space framework for a single manipulator.

1.1 Effector Equations of Motion

The effector position and orientation, with respect to a reference frame $\mathcal{R}_O$ of origin O is described by the relationship between $\mathcal{R}_O$ and a coordinate frame $\mathcal{R}_\odot$ of origin $\odot$ attached to this effector. $\odot$ is called the *operational point*. It is with respect to this point that translational and rotational motions and active forces of the effector are specified. An *operational coordinate system* associated with an m-degree-of-freedom effector and a point $\odot$, is a set x of m *independent* parameters describing the effector position and orientation in a frame of reference $\mathcal{R}_O$. For a non-redundant n-degree-of-freedom manipulator, *i.e.* $n = m$, these parameters form a set of of *generalized operational coordinates*. The effector equations of motion in operational space are given by (Khatib, 1980 and 1987a)

$$\Lambda(x)\ddot{x} + \Pi(x)[\dot{x}\dot{x}] + p(x) = F; \qquad (1)$$

where $\Lambda(x)$ designates the kinetic energy matrix, and $p(x)$ and F are respectively the gravity and the generalized operational force vectors. $\Pi(x)$ represents the $m \times m(m+1)/2$ matrix of centrifugal and Coriolis forces. The elements of the matrix $\Pi(x)$ can be obtained from the Christoffel symbols $\pi_{i,jk}$ given

as a function of the partial derivatives of $\Lambda(x)$ with respect to the generalized coordinates x. With $J(q)$ being the Jacobian matrix associated with the generalized operational velocities $\dot{x}$, the kinetic energy matrix associated with the operational coordinates x is related to the $n \times n$ joint space kinetic energy matrix, $A(q)$ by

$$\Lambda(x) = J^{-T}(q)A(q)J^{-1}(q). \qquad (2)$$

The generalized joint forces Γ required to produce the operational forces F are

$$\Gamma = J^T(q)F; \qquad (3)$$

This relationship is the basis for the actual control of manipulators in operational space.

1.2 Control in Operational Space

The dynamic decoupling and motion control of the manipulator in operational space is achieved by selecting the control structure

$$F = \widehat{\Lambda}(x)F^* + \widehat{\Pi}(x)[\dot{x}\dot{x}] + \widehat{p}(x); \qquad (4)$$

where, $\widehat{\Lambda}(x)$, $\widehat{\Pi}(x)$, and $\widehat{p}(x)$ represent the estimates of $\Lambda(x)$, $\Pi(x)$, and $p(x)$. The system 1 under the command 4 can be represented by

$$I_m\ddot{x} = G(x)F^* + \epsilon(x,\dot{x}) + d(t); \qquad (5)$$

where I_m is the $m \times m$ identity matrix, and

$$\begin{aligned} G(x) &= \Lambda^{-1}(x)\widehat{\Lambda}(x); & (6) \\ \epsilon(x,\dot{x}) &= \Lambda^{-1}(x)[\widetilde{\Pi}(x)[\dot{x}\dot{x}] + \widetilde{p}(x)]. & (7) \end{aligned}$$

with

$$\begin{aligned} \widetilde{\Pi}(x) &= \widehat{\Pi}(x) - \Pi(x); & (8) \\ \widetilde{p}(x) &= \widehat{p}(x) - p(x). & (9) \end{aligned}$$

$d(t)$ includes unmodeled disturbances. With a perfect nonlinear dynamic decoupling, the end-effector becomes equivalent to a *single unit mass*, I_m, moving in the m-dimensional space,

$$I_m\ddot{x} = F^*. \qquad (10)$$

F^* is the input of the decoupled end-effector. This provides a general framework for the selection of various control structures (Slotine, Khatib, and Ruth 1987).

1.3 Redundant Manipulators

A set of operational coordinates, which only describes the end-effector position and orientation, is obviously not sufficient to completely specify the configuration of a redundant manipulator. Therefore, the dynamic behavior of the entire system cannot be described by a dynamic model in operational coordinates. The dynamic behavior of the end-effector itself, nevertheless, can still be described, and its equations of motion in operational space can still be established. In fact, the structure of the effector dynamic model is identical to that obtained in the case of non-redundant manipulators (equation 1). In the redundant case, however, the matrix Λ should be interpreted as a *"pseudo kinetic energy matrix"*. This matrix is related to the joint space kinetic energy matrix by

$$\Lambda(q) = [J(q)A^{-1}(q)J^T(q)]^{-1}; \qquad (11)$$

Another important characteristic of redundant manipulator is concerned with the relationship between operational forces and joint forces. In the case where $n = m$, an operational force vector F is produced by the unique joint force vector $J^T F$. The additional freedom of redundant mechanism results in an infinity of possible joint force vectors. This due to the fact that some joint force vectors will only act internally. Those are the joint forces acting in the null space associated with J^T and defined by some generalized inverse P^T. A straightforward static analysis would lead to an infinity of generalized inverse matrices. However, a generalized inverse that is consistent with the system's dynamics is unique (Khatib 1987a) and given by

$$\bar{J}(q) = A^{-1}(q)J^T(q)\Lambda(q). \qquad (12)$$

$\bar{J}(q)$ in equation 12 is actually a generalized inverse of the Jacobian matrix corresponding to the solution that minimizes the manipulator's instantaneous kinetic energy. The relationship between forces is

$$\Gamma = J^T(q)F + [I_n - J^T(q)\bar{J}^T(q)]\Gamma_o; \qquad (13)$$

where I_n is the $n \times n$ identity matrix and Γ_o is an arbitrary joint force vector. Joint forces of the form $[I_n - J^T(q)\bar{J}^T(q)]\Gamma_o$ correspond to a zero operational vector.

1.4 Control of Redundant Manipulators

Similar to the case of non-redundant manipulators, the dynamic decoupling and control of the end-effector can be achieved by selecting an operational command vector of the form 4. The manipulator joint motions produced by this command vector are those that minimize the instantaneous kinetic energy of the mechanism. Analysis shows the system to be stable; however, while the end-effector is asymptotically stable, the manipulator joints can still describe internal motions in the nullspace. Asymptotic stabilization of the entire system can be achieved by the addition of dissipative joint forces. In order to preclude any effect of the additional forces on the end-effector and maintains its dynamic decoupling, these forces must be selected to only act in the nullspace of the Jacobian matrix. These additional stabilizing joint forces are of the form

$$\Gamma_{ns} = [I_n - J^T(q)\bar{J}^T(q)]\Gamma_s. \qquad (14)$$

In the actual implementation, the global control vector will be developed in a form that avoids the explicit evaluation of the expression of the generalized inverse of the Jacobian matrix.

1.5 Basic Jacobian

Different kinematic models and different Jacobian matrices are associated with different selections of systems of operational coordinates. The kinematic characteristics of a manipulator, which are independent of the selected system of operational coordinates, are described by the model

$$\begin{bmatrix} \mathbf{v} \\ \omega \end{bmatrix} = J_O(q)\dot{q}. \qquad (15)$$

This model establishes the relationship between generalized joint velocities $\dot{q}$ and the end-effector linear and angular velocities $\mathbf{v}$ and ω. The matrix $J_O(q)$, termed *the basic Jacobian*, is defined independently of the particular set of parameters used to describe the end-effector configuration. The Jacobian matrix $J(q)$ associated with a given selection, $\mathbf{x}$, of operational coordinates can then be expressed in the form (Khatib 1980)

$$J(q) = E(\mathbf{x})J_O(q). \qquad (16)$$

The matrix $E(\mathbf{x})$ is dependent on the type of coordinates selected to represent the position and orientation of the effector.

1.6 Effect of a Load

The kinetic energy matrix $\Lambda(\mathbf{x})$ associated with the operational coordinates $\mathbf{x}$ describes the inertial characteristics of the effector as perceived at the point $\odot$. The addition of a load will result in an increase in the total kinetic energy. Let m_L and $\mathcal{I}_L$ be the mass

and inertia matrix of the load with respect to $\mathcal{R}_\odot$. The additional kinetic energy due to the load is

$$T_\mathcal{L} = \frac{1}{2}[m_\mathcal{L}\mathbf{v}^T\mathbf{v} + \omega^T\mathcal{I}_\mathcal{L}\omega]; \qquad (17)$$

where $\mathbf{v}$ and ω are the vectors of linear and angular velocities. The generalized operational velocities $\dot{x}$ are related to the linear and angular velocities by the matrix $E(\mathbf{x})$. The kinetic energy due to the load can be written in the form

$$T_\mathcal{L} = \frac{1}{2}\dot{x}^T\Lambda_\mathcal{L}(\mathbf{x})\dot{x}; \qquad (18)$$

where the matrix of kinetic energy with respect to $\mathbf{x}$ is

$$\Lambda_\mathcal{L}(\mathbf{x}) = E^{-T}(\mathbf{x})M_\mathcal{L}E^{-1}(\mathbf{x}); \qquad (19)$$

with

$$M_\mathcal{L} = \begin{bmatrix} m_\mathcal{L}I & 0 \\ 0 & \mathcal{I}_\mathcal{L} \end{bmatrix}; \qquad (20)$$

where I and 0 are the unit and zero matrices of appropriate dimension.

<u>Lemma 1</u>   *The kinetic energy matrix of the effector and load system is the matrix*

$$\Lambda_{\text{effector+load}}(\mathbf{x}) = \Lambda_{\text{effector}}(\mathbf{x}) + \Lambda_\mathcal{L}(\mathbf{x}).$$

This is a straightforward implication of the evaluation, with respect to the operational coordinates, of the total kinetic energy of the system.

2   In-Parallel Structures

Let us consider the problem of manipulating an object with a system of N robot manipulators, as illustrated in Figure 1. The effectors of each of these manipulators are assumed to have the same number of degrees of freedom, m, and to be rigidly connected to the manipulated object. Let $\odot$ be the selected operational point attached to this object. This point is fixed with respect to each of the effectors. Let $\Lambda_\mathcal{L}(\mathbf{x})$ be the kinetic energy matrix associated with the object's load alone, expressed with respect to $\odot$ and the operational coordinates $\mathbf{x}$. Being held by N effectors, the inertial characteristics of the object as perceived at the operational point are modified. The N-effector/object system can be viewed as an *augmented object* representing the total inertias perceived at $\odot$. Let $\Lambda_i(\mathbf{x})$ be the kinetic energy matrix associated with the ith effector.

Theorem 1   *The kinetic energy matrix of the augmented object is*

$$\Lambda_\oplus(\mathbf{x}) = \Lambda_\mathcal{L}(\mathbf{x}) + \sum_{i=1}^N \Lambda_i(\mathbf{x}).$$

This results from the evaluation of the total kinetic energy of the N effectors and object system expressed with respect to the operational velocities,

$$T = \frac{1}{2}\dot{x}^T\Lambda_\mathcal{L}(\mathbf{x})\dot{x} + \sum_{i=1}^N \frac{1}{2}\dot{x}^T\Lambda_i(\mathbf{x})\dot{x}$$
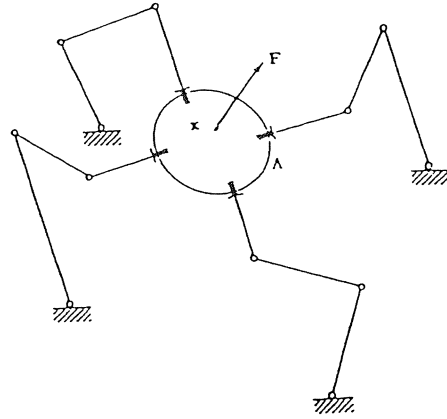

Figure 1: A Multi-Effector/Object System

2.1   Augmented Object Model

The system considered here is the system resulting from rigidly connecting an object, to the effectors of N n-degree-of-freedom manipulators. In the case of nonredundant manipulators, the number of degrees of freedom, $n_\oplus$ associated with the combined system can be shown (Khatib 1987b) to be equal to 3 in the planar case and to 6 in the spatial case. Thus, the number of operational coordinates, m, is therefore equal to the number of degrees of freedom, $n_\oplus$, of the mechanism. These coordinates form, therefore, a set of generalized coordinates for the system. The kinetic energy matrix of the system expressed with respect to the generalized operational coordinates $\mathbf{x}$ is given in Theorem 1. The augmented object equations of motion are

$$\Lambda_\Theta(\mathbf{x})\ddot{x} + \Pi_\oplus(\mathbf{x})[\dot{x}\dot{x}] + p_\oplus(\mathbf{x}) = \mathbf{F}_\oplus; \qquad (21)$$

where the matrix, $\Pi_\Theta(\mathbf{x})$, of centrifugal and Coriolis forces also possesses the additive property

$$\Pi_\Theta(\mathbf{x}) = \Pi_\mathcal{L}(\mathbf{x}) + \sum_{i=1}^N \Pi_i(\mathbf{x}); \qquad (22)$$

where $\Pi_{\mathcal{L}}(x)$ and $\Pi_i(x)$ are the $m \times m(m+1)/2$ matrix of centrifugal and Coriolis forces associated with $\Lambda_{\mathcal{L}}(x)$ and $\Lambda_i(x)$ respectively. The gravity vector is

$$\mathbf{p}_{\oplus}(x) = \mathbf{p}_{\mathcal{L}}(x) + \sum_{i=1}^{N} \mathbf{p}_i(x), \qquad (23)$$

where $\mathbf{p}_{\mathcal{L}}(x)$ and $\mathbf{p}_i(x)$ are the gravity vectors associated with the object and the ith effector. The generalized operational forces $\mathbf{F}_{\oplus}$ are the resultants of the forces produced by each of the N effectors at the operational point $\odot$.

$$\mathbf{F}_{\oplus} = \sum_{i=1}^{N} \mathbf{F}_i. \qquad (24)$$

The effector's operational forces $\mathbf{F}_i$ are generated by the corresponding manipulator actuators. The generalized joint force vector Γ_i corresponding to $\mathbf{F}_i$ is given by

$$\Gamma_i = J_i^T(\mathbf{q}_i) \, \mathbf{F}_i;$$

where $\mathbf{q}_i$ and $J_i^T(\mathbf{q}_i)$ are, respectively, the vector of joint coordinates and the Jacobian matrix computed with respect to x_o and associated with the ith manipulator. The dynamic decoupling and motion control of the augmented object in operational space is achieved by selecting a control structure similar to 4, i.e.

$$\mathbf{F}_{\ominus} = \hat{\Lambda}_{\oplus}(x)\mathbf{F}_o^* + \hat{\Pi}_{\oplus}(x)[\dot{x}\dot{x}] + \hat{\mathbf{p}}_{\oplus}(x). \qquad (25)$$

2.2 Allocation of Effector Forces

The control structure 25 provides the net force $\mathbf{F}_o$ to be applied to the augmented object at $\odot$. The criterion for distributing this force between effectors will be based on the minimization of total actuator activities. The force vector, $\mathbf{F}_i$, to be produced by the ith effector will be selected to be aligned with $\mathbf{F}_{\ominus}$ and acting in the same direction,

$$\mathbf{F}_i = \alpha_i \mathbf{F}_{\oplus}; \quad \text{with} \quad \alpha_i > 0. \qquad (26)$$

In addition, the set of N positive numbers α_i must satisfy

$$\sum_{i=1}^{N} \alpha_i = 1. \qquad (27)$$

The actuator joint forces required by the ith manipulator is

$$\Gamma_i = \alpha_i J_i^T(\mathbf{q}_i) \, \mathbf{F}_{\oplus}.$$

The problem now is to find the set of N positive numbers, $\alpha_1, \alpha_2, ..., \alpha_N$ such that the overall effort of the actuators is minimized. Let us consider the

vector of joint forces τ_i corresponding to the total operational forces $\mathbf{F}_{\oplus}$

$$\tau_i = J_i^T(\mathbf{q}_i) \, \mathbf{F}_{\oplus};$$

τ_i represents the actuator joint forces that would be assigned to the ith manipulator, if this manipulator alone were to produce the total operational force $\mathbf{F}_{\oplus}$. Let τ_{ij} be the jth component of τ_i. Actuator joint forces are limited. Let $\overline{\gamma}_{ij}$ be the magnitude of the maximal bounds on the jth actuator force of the ith manipulator. The number $|\tau_{ij}|/\overline{\gamma}_{ij}$ represents a measure of the effort that will be required by the jth actuator if the ith manipulator alone produced the total operational forces $\mathbf{F}_{\oplus}$. The effort of the ith manipulator can be characterized by

$$r_i = \max_j \{|\tau_{ij}|/\overline{\gamma}_{ij}\};$$

which corresponds to the greatest effort. r_i is a positive number, which would be greater than one if the requested joint forces cannot be achieved by the ith manipulator alone. In order to minimize the overall effort, the weighting numbers α_1, α_2, ..., and α_N will be selected so that the effort is equally distributed, that is

$$\alpha_1 r_1 = \alpha_2 r_2 = ... = \alpha_N r_N.$$

Using equation 27, this corresponds to the solution

$$\alpha_i = \frac{\beta_i}{\beta_1 + \beta_2 + ... + \beta_N}; \qquad (28)$$

where

$$\beta_i = \frac{r_1 . r_2 ... r_N}{r_i}. \qquad (29)$$

3 In-Serial Structure

In this section, we analyze the dynamic characteristics of the system resulting from the combination in-serial of two manipulators. The manipulator connected to the ground will be refered to as the "heavy-weight" manipulator. It has n_H degrees of freedom and its configuration is described by the system of n_H generalized joint coordinates $\mathbf{q}_H$. The second manipulator, refered to as the "light-weight" manipulator, has n_L degrees of freedom and its configuration is described by the generalized coordinates $\mathbf{q}_L$. The resulting structure is an n-degree-of-freedom manipulator with $n = n_H + n_L$. Its configuration is described by the system of generalized joint coordinates $\mathbf{q} = [\mathbf{q}_H^T \ \mathbf{q}_L^T]^T$. If m represents the number of effector degrees of freedom of the combined structure, n_H and n_L are assumed to obey

$$n_H > 1 \text{ and } n_L \geq m. \qquad (30)$$

This assumption states that what is considered to be the "light-weight" manipulator must possess the full freedom to move in the operational space, and can possibly have a redundant structure. The "heavy weight" manipulator must have at least one-degree-of-freedom, and can also be redundant.

3.1 Kinematics

The kinematics of the two manipulators, considered separately, are described with respect to the reference frames $\mathcal{R}_{OH}$ and $\mathcal{R}_{OL}$. The coordinate frames associated with their operational points, Θ_H and Θ_L, are denoted $\mathcal{R}_{\Theta H}$ and $\mathcal{R}_{\Theta L}$ respectively. The transformation matrix describing the rotation between the frames $\mathcal{R}_{OH}$ and $\mathcal{R}_{\Theta H}$ is $S_H(q_H)$. $S_L(q_L)$ is the transformation matrix associated with $\mathcal{R}_{OL}$ and $\mathcal{R}_{\Theta L}$. The operational coordinates are x_H and x_L, and $J_H(q_H)$ and $J_L(q_L)$ are the respective Jacobian matrices. If $J_{OH}(q_H)$ and $J_{OL}(q_L)$ are the basic Jacobian matrices associated with two individual manipulators, the basic Jacobian matrix associated with the in-serial combination can be expressed as

$$J_O(q) = [J_{Oh}(q) \; J_{Ol}(q)]; \qquad (31)$$

where

$$J_{Oh}(q) = \begin{bmatrix} I & -\hat{v} \\ 0 & I \end{bmatrix} J_{OH}(q_H); \qquad (32)$$

$$J_{Ol}(q) = \Omega(q_H) J_{OL}(q_L). \qquad (33)$$

$\hat{v}$ is the cross product operator on the position vector associated with the "lightweight" manipulator and expressed in $\mathcal{R}_{OH}$, and

$$\Omega(q_H) = \begin{bmatrix} S_H & 0 \\ 0 & S_H \end{bmatrix}. \qquad (34)$$

3.2 Dynamics

The kinetic energy matrix, $A(q)$, of the combined system can be decomposed in diagonal blocks corresponding to the dimensions of the two manipulators' individual kinetic energy matrices

$$A(q) = \begin{bmatrix} A_h & A_{hl}^T \\ A_{hl} & A_l \end{bmatrix}. \qquad (35)$$

It can be easily shown that the matrix A_l of dimensions $n_L \times n_L$ in equation 35 is identical to the kinetic energy matrix A_L associated with "light-weight" manipulator, i.e. $A_l = A_L$. The inverse of the kinetic energy matrix $A(q)$ is

$$A^{-1}(q) = \begin{bmatrix} \overline{A}_h & \overline{A}_{hl} \\ \overline{A}_{hl}^T & \overline{A}_l \end{bmatrix}. \qquad (36)$$

The operational space *pseudo kinetic energy matrix* Λ_O associated with the linear and angular velocities is defined by $(J_O A^{-1} J_O^T)^{-1}$. Using equations 31, 32, and 33 the inverse of this matrix can be written as

$$\Lambda_O^{-1} = \Lambda_{Ol}^{-1} + \overline{\Lambda}_C; \qquad (37)$$

where

$$\overline{\Lambda}_C = J_O \overline{A}_C J_O^T; \qquad (38)$$

$$\Lambda_{Ol}^{-1} = \Omega \Lambda_{OL}^{-1} \Omega^T; \qquad (39)$$

and

$$\overline{A}_C = \begin{bmatrix} \overline{A}_h & \overline{A}_{hl} \\ \overline{A}_{hl}^T & \overline{A}_l - A_L^{-1} \end{bmatrix}. \qquad (40)$$
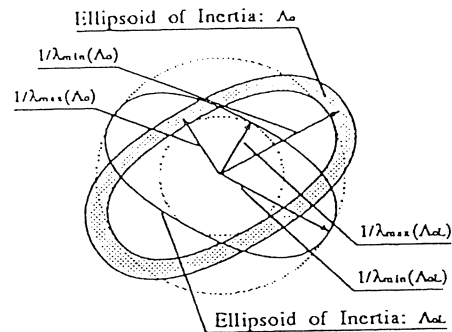

Figure 2: Reduced Effective Inertia

Theorem 2: *(Reduced Effective Inertia).* The operational space *pseudo kinetic energy matrices* Λ_O (combined mechanism), and Λ_{OL} ("light-weight" mechanism) verify

$$\frac{1}{1 + \|\overline{\Lambda}_C\| \cdot \lambda_k(\Lambda_{OL})} \leq \frac{\lambda_k(\Lambda_O)}{\lambda_k(\Lambda_{OL})} \leq 1; \quad k = 1, 2, \ldots, m$$

Figure 2 illustrates the inertial characteristic stated in this theorem: the effective inetrias, in all directions, of the entire system are smaller than or equal to those of the "light-weight" mechanism. The proof involves the two steps:

Step 1: *(Eigenvalue Characteristic)* This first step is based on an important characteristic of symmetric matrices. It is possible to show that (Wilkinson, 1965): *If M and $M+E$ are $n \times n$ symmetric matrices, then for $k = 1, 2, \ldots, n$*

$$\lambda_k(M) + \lambda_n(E) \leq \lambda_k(M + E) \leq \lambda_k(M) + \lambda_1(E);$$

where $\lambda_k(.)$ denote the kth largest eigenvalue of $(.)$, i.e. $\lambda_n(.) \leq \ldots \leq \lambda_1(.)$.

Applying this relation to equation 37 for $k = 1, 2, \ldots, m$, and noting that Λ_{Ol} and Λ_{OL} are similar (equations 33 and 39) positive definite matrices

with the identical eigenvalues $1/\lambda_k(\Lambda_{0L}^{-1})$, yields

$$\frac{1}{1 + \lambda_1(\overline{\Lambda}_C) \cdot \lambda_k(\Lambda_O L)} \leq \frac{\lambda_k(\Lambda_O)}{\lambda_k(\Lambda_{0L})} \leq$$

$$\frac{1}{1 + \lambda_n(\overline{\Lambda}_C) \cdot \lambda_k(\Lambda_O L)}. \qquad (41)$$

Step 2: *(Positive Semidefinition of $\overline{\Lambda}_C$).* Let us consider the symmetric matrix

$$M = \begin{bmatrix} E & C^T \\ C & D \end{bmatrix}.$$

If E is a nonsingular matrix, M can be decomposed (Bruch and Prelett, 1971) following

$$M = \begin{bmatrix} I & 0 \\ CE^{-1} & I \end{bmatrix} \begin{bmatrix} E & 0 \\ 0 & D - CE^{-1}C^T \end{bmatrix} \begin{bmatrix} I & E^{-1}C^T \\ 0 & I \end{bmatrix}. \qquad (42)$$

Applying this decomposition to the matrix $\overline{A}_C$ ($\overline{A}_h$ is nonsingular, since A^{-1} is nonsingular), and using the relationships between the block matrices resulting from $AA^{-1} = I$, yields

$$\overline{A}_C = \begin{bmatrix} I & 0 \\ \overline{A}_{hl}^T \overline{A}_h^{-1} & I \end{bmatrix} \begin{bmatrix} \overline{A}_h & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I & \overline{A}_h^{-1}\overline{A}_{hl}^T \\ 0 & I \end{bmatrix}. \qquad (43)$$

Like A, the diagonal block $\overline{A}_C$ is positive definite. The decomposition 43 shows $\overline{A}_C$ to be positive semidefinite, $\lambda_n(\overline{A}_C) = 0$ and $\lambda_1(\overline{A}_C) \geq 0$. The matrix $\overline{\Lambda}_C$ which results from a congruence transformation (equation 38) of $\overline{A}_C$ is similarly defined, *i.e.* positive semidefinite with $\lambda_n(\overline{\Lambda}_C) = 0$ and $\lambda_1(\overline{\Lambda}_C) \geq 0$. Substituting this result in equation 41 completes the proof of the Theorem.

3.3 Dextrous Dynamic Coordination

The previous result on the reduction of the effective inertial characteristics at the effector of a redundant manipulator is very useful in approaching the control problem associated with coordinating a manipulator and a micro-manipulator system. The basic idea in our control strategy is to treat the manipulator and micro-manipulator as a single redundant system. This approach for the control of the manipulator and micro-manipulator system will clearly result in a substantial increase of the dynamic performance of the system. Theorem 2 shows that the dynamic characteristics of the combined system can be made to be comparable to (and, in some cases, better than) those of the micro-manipulator. The problem, however, is that this type of control cannot be directly applied to the macro/micro motion

coordination problem. In effect, given the mechanical limits on the range of joint motions of the micro-manipulator, such a controller would rapidly lead to joint saturation of the micro-manipulator degrees of freedom.

The *dextrous dynamic coordination* we proposed, which is essentially based on the framework of redundant manipulator control in operational space, involves the minimization of the deviation from the neutral (mid-range) joint positions of the micro-manipulator. This minimization will be achieved using joint forces selected from the null space associated with the mapping between operational and joint forces. This will preclude any effects of the additional forces on the primary task. Let $\overline{q}_i$ and q_i be the upper and lower bounds on the ith joint position q_i. We construct the potential function

$$V_{\text{Dextrous}}(\mathbf{q}) = k_d \sum_{i=n_H+1}^{n} (q_i - \frac{\overline{q}_i + q_i}{2})^2; \qquad (44)$$

where k_d is a constant coefficient. The gradient of this function

$$\Gamma_{\text{Dextrous}} = -\nabla V_{\text{Dextrous}}; \qquad (45)$$

provides the required attraction (Khatib 1986) to the mid-range joint positions of the micro-manipulator. The interference of these additional torques with the end-effector dynamics is avoided by selecting them from the null space. This is

$$\Gamma_{nd} = [I_n - J^T(\mathbf{q})\overline{J}^T(\mathbf{q})]\Gamma_{\text{Dextrous}}. \qquad (46)$$

The avoidance of joint limits is achieved using an "artificial potential field" function. It is essential that the range of motion of the joints associated with the micro-manipulator accommodate the relatively slower dynamic response of the arm. A sufficient motion margin is required for achieving dextrous dynamic coordination.

4 Conclusion

The augmented object model proposed in this paper constitutes a natural framework for the dynamic modelling and control of multi-effector/object systems. In this approach, the control structure only uses the necessary forces, *i.e.* net force, required to achieve the dynamic decoupling and control of the system. Compared to control structures where joint motions or effector motions are individually decoupled and controlled, the proposed control system presents a significant reduction in actuator activities.

Indeed, in this approach, the inertial coupling, centrifugal, and Coriolis forces acting on one effector are used to compensate for parts of the coupling forces acting on the others. The actuator joint force activity is further minimized by the criterion used for the allocation of effector forces. The methodology developed in this framework constitutes a powerful tool for dealing with the problem of object manipulation in a multi-fingered hand system. The extension of the augmented object concept to systems involving combinations of redundant manipulators is presented in (Khatib 1987a).

The inertial analysis of combined manipulator and micro-manipulator system has shown that the dynamic performance of the combined system can be made to be superior to that of the micro-manipulator considered alone. Treating the manipulator and micro-manipulator as a single redundant system, the proposed *dextrous dynamic coordination* is based on minimizing the deviation from the neutral (midrange) joint positions of the micro-manipulator. In order to preclude any effects of the additional forces on the primary task, this minimization is achieved using joint forces selected from the null space associated with the mapping between operational and joint forces.

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