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Real-Time Control of Manipulators in Operational Space

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Abstract

The paper presents a radically new approach to real-time dynamic control and active force control of manipulators. In this approach the manipulator control problem is reformulated in terms of direct control of manipulator motion in operational space, the space in which the task is originally described, rather than controlling the task's corresponding joint space motion obtained after geometric and kinematic transformation. This approach leads to high performance position/force control which is the essential tool for extending robot capabilities in performing more advanced assembly tasks. We also present, a unique real-time obstacle avoidance method for mobile robots and manipulators based on the "artificial potential field approach". These methods have been implemented on a PUMA 560 robot at the Stanford Artificial Intelligence Laboratory.

1. Introduction

Conventional manipulator control, providing linear feedback compensation to control joint positions independently, is unable to meet the high accuracy and performance required in precision manipulator tasks. Addressing this problem, much research has been directed at developing and modelling the dynamic equations of joint motion. Typical models relate joint variables to generalized torques and by necessity force the resulting control scheme to have two levels:

- the first level requires coordinate transformations to convert the description of a desired path from Cartesian to joint space;
- the second level makes use of the arm's dynamic model to calculate generalized force commands.

This first stage of control, the transformation from a Cartesian description of the path into joint trajectories, is very time consuming and prone to problems at kinematic singularities. Considering the dynamic compensation problem, this leads to high computational complexity in real-time control.

On the other hand, these types of joint space control approachs are ill suited to active force control, which is crucial for the extension of robot capabilities in performing more advanced assembly tasks.

In manipulator control, the predominant concern is that the end effector motion and the active forces respond accurately to the desired task. At the level of joint motions, concern is limited to issues of the global stability of the articulated mechanism and the satisfaction of the constraints under which it must operate.

The *operational space control* approach has its roots in the work on end-effector motion control and obstacle avoidance [1,2], that we implemented in 1978 at the "Laboratoire d'Automatique de Montpellier" in France for an MA23 manipulator. This approach has been formalized by constructing its basic tool, the equations of motion in the operational space of the manipulator end-effector [3].

2. Mathematical Models

The end effector configuration is represented by m parameters describing its position and orientation in a frame of reference R_0. These m parameters will be called *End Effector Configuration Parameters* and represented by x. The geometric and kinematic models of a manipulator are:

$$x = G(q) \tag{1}$$
$$\dot{x} = J(q)\,\dot{q} \tag{2}$$

where q is the vector of the n joint coordinates, and $J(q)$ the Jacobian matrix. The dynamic model can be written in the form:

$$A(q)\ddot{q} + b(q,\dot{q}) - g(q) = \Gamma \tag{3}$$

where $A(q)$ is the $n \times n$ kinetic energy matrix, $b(q,\dot{q})$ is the centrifugal and Coriolis forces column matrix and $g(q)$ is the gravity forces column matrix. Γ is the $n \times 1$ column matrix of generalized forces.

3. End Effector Dynamic Model

Definition: An *Operational Coordinate System* is a coordinate system formed by an *independent* set of m_0 end effector configuration parameters.

Let us consider the case of non-redundant manipulators, *i.e.* $n = m_0$, and use a set of independent parameters, *i.e.* operational coordinates, to represent the end effector configuration. Let $\underline{q}_i$ and $\overline{q}_i$ be respectively the minimal and maximal bounds of q_i. The movement of the point q in joint space is confined to the hyperparallelepiped:

$$D_q = \prod [\underline{q}_i, \overline{q}_i] \tag{4}$$

Let $D_q^{\dagger}$ be the domain obtained from D_q by excluding the singular points in the kinematic model (2) and such that the vector function G of (1) is one-to-one. Let $D_x^{\dagger}$ designate the domain:

$$D_x^{\dagger} = G(D_q^{\dagger}) \tag{5}$$

In $D_x^{\dagger}$, the independent parameters $x_1, x_2, \ldots, x_{m_0}$ constitute a set of configuration parameters for the manipulator. Therefore, they constitute a set of generalized coordinates.

The end effector dynamic model of a non-redundant manipulator in the domain $D_x^\dagger$ is given by [4]:

$$\Lambda(x)\ddot{x} + \mu(x, \dot{x}) - p(x) = F \tag{6}$$

$\Lambda(x), \mu(x, \dot{x})$ and $p(x)$ represent respectively the kinetic energy matrix, the centrifugal and Coriolis forces column matrix and the gravity forces column matrix. F is the $m_0 \times 1$ operational forces column matrix. The operational space dynamic parameters are related to the joint space dynamic parameters by [4]:

$$\begin{aligned} \Lambda(x) &= J^{-T}(q)A(q)J^{-1}(q) \\ \mu(x, \dot{x}) &= J^{-T}(q)b(q, \dot{q}) - \Lambda(q)h(q, \dot{q}) \\ p(x) &= J^{-T}(q)g(q) \end{aligned} \tag{7}$$

where

$$h(q, \dot{q}) = \dot{J}(q)\dot{q}\,; \tag{8}$$

Since the evaluation of the matrices Λ, μ, p in the foregoing expressions has been obtained in terms of the joint coordinates, the domain $D_x^\dagger$ of applicability of the dynamic model (6) may be extended to the domain D_x^* defined by:

$$D_x^* = G(D_q^*) \tag{9}$$

where D_q^* is the domain resulting from D_q of (4) when the singular points in the kinematic model are excluded. Indeed, the restriction to a domain where G is one-to-one then becomes unnecessary.

4. Decoupling of End Effector Motions

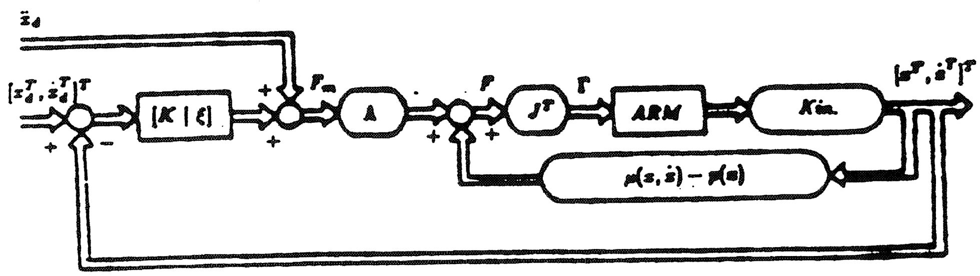
The dynamic model (6) provides a description of the dynamic behavior of the end effector motions in operational space. The control of the manipulator for a desired motion in this space becomes feasible by selecting F as control vector. In order to produce this control vector of operational forces, specific forces Γ must be applied with joint-based actuators. The relationship between F and the joint forces Γ may be obtained by exploiting the identity between the virtual work of F in an elementary displacement δx and the virtual work of Γ in the corresponding displacement δq, according to the *virtual work principle*. Using equation (2) this leads to:

$$\Gamma = J^T(q)\,F \tag{10}$$

The decoupling of the end effector motion in the domain D_x^* of the operational space is achieved by employing the following structure of control:

$$F = \Lambda(x)F_m + \mu(x, \dot{x}) - p(x) \tag{11}$$

where F_m represents the command vector of the decoupled end-effector which becomes equivalent to a ***single mass***. The control system is shown in Figure 1 where K and ξ represent the $m_0 \times m_0$ constant gain matrices and the subscript $_d$ denotes the desired motion.



End effector motion decoupling.
Figure 1

7. Redundant Manipulators

In the case of redundancy, the operational coordinates can't constitute a generalized coordinate system since their number is less than the manipulator *dof*, *i.e.*, $m_0 < n$. Therefore, the manipulator's dynamic behaviour cannot be described by a dynamic model worked out in the operational space. However, equations of motion in this space may be obtained for the end effector. These equations are [4]:

$$\Lambda_r(x)\ddot{x} + \mu_r(x, \dot{x}) - p_r(x) = F \tag{12}$$

with

$$\begin{aligned} \Lambda_r(q) &= [J(q)A^{-1}(q)J^T(q)]^{-1} \\ \mu_r(q, \dot{q}) &= \bar{J}^T(q)b(q, \dot{q}) - \Lambda_r(q)h(q, \dot{q}) \\ p_r(q) &= \bar{J}^T(q)g(q) \end{aligned} \tag{13}$$

where

$$\bar{J}(q) = A^{-1}(q)J^T(q)\Lambda_r(q) \tag{14}$$

$\bar{J}(q)$ is actually a right pseudo-inverse of the Jacobian matrix corresponding to the solution that minimizes the manipulator's kinetic energy. The system's asymptotic stabilization may be achieved by the addition of dissipative forces proportional to $\dot{q}$ in the control law (11).

8. Singular Configurations

A *singular configuration* is a configuration q from which the end effector cannot move along or rotate around a given direction of the Cartesian space. In such a configuration, the manipulator's mobility locally decreases. To a singular configuration corresponds a singular direction attached to the end effector. It is for that direction in fact that the effector presents an infinite inertial mass for a displacement or an infinite inertia for a rotation. Its movements remain free in the sub-space orthogonal to this direction. At a singular configuration, the manipulator can be treated as a redundant system with respect to the end effector motion in the sub-space orthogonal to its singular direction.

9. Active Force Control

In previous approachs [5,6], the active force control problem has been treated within the frame of a joint space control system. However, the wrist or finger sensing, end-effector desired contact forces, and end-effector stiffness and dynamics involved in this problem are closely linked to the operational space. Active force control can be naturally integrated in the operational space control system using the same operational force command vector. By decoupling the end-effector motion, compliance in a direction of the operational space is directly controlled by the position gain matrix K_p (see figure 2). Active force control in a direction is then simply achieved by setting the end-effector stiffness in that direction to zero and selecting the corresponding force servo using the matrix S. The system stabilization is obtained by maintaining the damping ξ in both position and force control.

10. Real-Time Implementation

The control law (11) may be written in the form [4]:

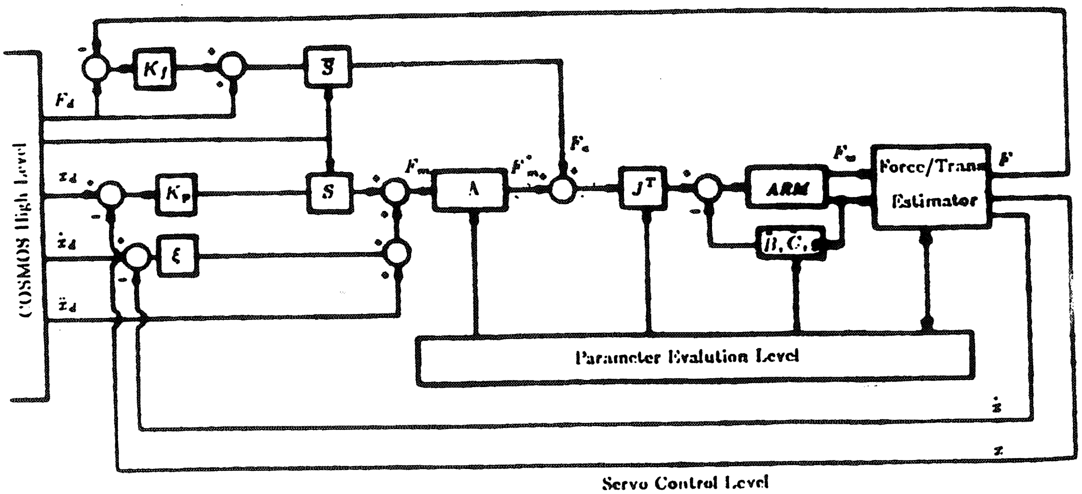
$$\Gamma = J^T(q)\Lambda(q)F_m + \tilde{B}(q)[\dot{q}\dot{q}] + \tilde{C}(q)[\dot{q}^2] - g(q) \tag{15}$$

Where the matrices $\tilde{B}(q)$ and $\tilde{C}(q)$ have respectively the dimensions $n \times n(n-1)/2$ and $n \times n$. $[\dot{q}\dot{q}]$ and $[\dot{q}^2]$ are defined by:

$$\begin{aligned} [\dot{q}\dot{q}] &= [\dot{q}_1\dot{q}_2 \ \dot{q}_1\dot{q}_3 \cdots \dot{q}_{n-1} \ \dot{q}_n]^T \\ [\dot{q}^2] &= [\dot{q}_1^2 \ \dot{q}_2^2 \cdots \dot{q}_n^2]^T \end{aligned} \tag{16}$$

In this new control structure the end-effector dynamic decoupling is obtained using the end-effector dynamic parameters (EEDP) $\Lambda(q)$, $\tilde{B}(q)$, $\tilde{C}(q)$ and $g(q)$, which are configuration-dependant. With respect to the servo control rate, the EEDP parameters can be computed at a relatively low rate. This, with the integration of an operational position/velocity estimator, leads to a two-level control system architecture (see figure 2):

- low rate *parameter evaluation level*: updating the EEDP, the Jacobian matrix and the geometric model;
- high rate *servo control level*: computing the command vector using the estimator and the updated parameters.



Operational Space Control System Architecture.
Figure 2

10. Real-Time Obstacle Avoidance

The operational space control approach enabled the development of a unique obstacle avoidance scheme based on the use of potential functions around obstacles, rather than actually planning paths. The philosophy of this approach can be schematically described as follows: *The manipulator moves in a field of forces. The position to be reached is an attractive pole for the end effector, and the obstacles are repulsive surfaces for the manipulator parts* [2]. Obstacles are described by composition of *primitives*. Analytic equations representing envelopes best approximating the primitives' shapes have been developed (parallelepiped, cone, cylinder, *etc.*). The control of a given point of the manipulator vis-à-vis an obstacle is achieved by submitting it to a *Force Inducing an Artificial Repulsion from the Surface* (FIRAS, from the french). These forces are created by an artificial potential field V obtained as a function of the normal distance to the obstacle's approximating surface ρ [3]:

$$V(\rho) = \begin{cases} \left(\frac{1}{\rho^2} - \frac{1}{\rho_0^2}\right)^2, & \text{if } |\rho| < |\rho_0|; \\ 0, & \text{if } |\rho| > |\rho_0|; \end{cases} \qquad (47)$$

where ρ_0 represents the limit distance of the potential field influence. ρ is easily obtained using a variational procedure. Considering the small amount of calculation needed, this method allows obstacle avoidance to occur in real time as an integral part of the servo-control. Collision avoidance, traditionally considered as a high-level planning problem, can be effectively distributed between different levels of control. This allows real-time robot operations in complex environments.

11. Applications

An experimental manipulator programming system "COSMOS" (Control in Operational Space of a Manipulator-with-Obstacles System) has been designed at the Stanford Artificial

Intelligence Laboratory for implementation of the operational space control approach for the Unimation PUMA 560 arms. In the absence of an effective force control, a simplified end effector dynamic model of the PUMA arm is used. The COSMOS system is currently implemented on a PDP 11/45 computer that is interfaced to a PUMA 560. The PDP 11/23 and VAL are disconnected, and only the joint microprocessors in the PUMA controller are used for motor current control. The PUMA is equipped with a six degree of freedom force wrist that is interfaced to the PDP 11/45 via an A/D convertor. The PUMA is also interfaced to a Machine Intelligence Corporation vision module.

In the current COSMOS implementation, the servo control level rate is 125 Hz while the rate of the parameter evaluation level is 40 Hz. With the new multiprocessor implementation (PDP 11/45 and PDP 11/60), COSMOS is expected to achieve a dynamic and kinematic update rate of 100 Hz and a position/force servo rate of 300 Hz.

Demonstrations of real-time end effector free and constrained motion including contact, slide and compliance operations as well as real-time collision avoidance with links and moving obstacle have been performed with the COSMOS system.

12. Conclusion

The operational space formulation has been shown to be an effective means of achieving high dynamic performance in real-time motion control and active force control of robot manipulators for complex assembly tasks. In addition, the complex transformation of the task into joint coordinates, required in conventional joint space control approaches, is eliminated. This leads to a reduction in the amount of computation and avoids singularity problems. In a complex environment, the artificial potential field approach constitutes a unique tool for real-time obstacle avoidance. Collision avoidance, generally treated at the highest level of control, has been demonstrated here to be an effective component of low-level real-time control.

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