

# Preliminary Results on Two-Foot Balancing on a Seesaw

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**Abstract**—This paper overviews some preliminary results on the control of humanoid robots when balancing on a seesaw. We first model the dynamics of the robot along with the seesaw, and then synthesize simple control strategies for balancing tasks. Preliminary validations of the framework have been conducted both on simulations and real scenarios involving the iCub robot balancing on a semi-cylindrical seesaw.

## I. INTRODUCTION

Forthcoming robotic applications require robots to physically interact with the environment. This requirement is different from those of classical industrial applications where robots are primarily required to perform positioning tasks. Another key element for innovative robotics is the need of increased mobility and adaptation to human-centered environment, e.g. balancing on dynamic surfaces. Within this context, physical interactions between the robot and its surrounding environment influence stability and balance. To allow proper coordination between interaction and posture control, robotics research needs to investigate the principles governing *whole-body control* with *contact dynamics*, as these represent important challenges towards the achievement of robots with augmented physical autonomy and will therefore be the focus of the present paper.

It is worth to recall that industrial robots have been extensively studied since the early seventies. These robots are often fixed-base, and usually confined in a protected and well-known environment. On the contrary, robots with augmented mobility ask for passing from the conventional fixed-base theoretical framework to free-floating approaches, whose control has been addressed only during recent years. Full feedback linearization of free-floating mechanical systems is forbidden because of their underactuated nature [1], which prohibits the application of classical methodologies. The problem becomes even more complex when these systems are constrained, i.e. the dynamics are subject to a set of nonlinear constraints. This is the case for legged robot balancing on a seesaw.

This paper presents the control framework implemented on the humanoid robot iCub for balancing on a seesaw.

## II. BACKGROUND

### A. System modeling

The system dynamics are characterized by the following differential equations (see Figure 1):

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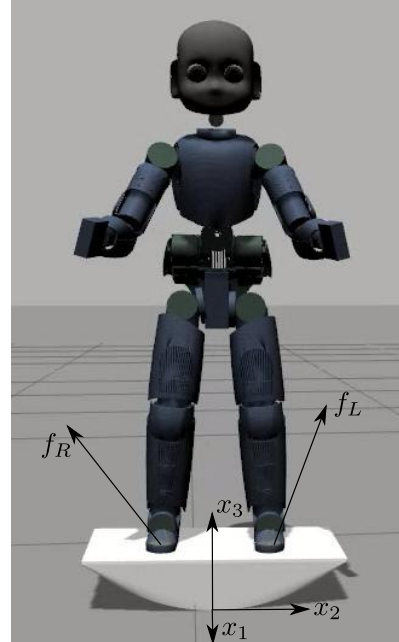


Fig. 1: iCub balancing on a seesaw.

Floating base dynamics

$$M(q)\dot{\nu} + h(q, \nu) - J_R^\top(q)f_R - J_L^\top(q)f_L = S\tau, \quad (1a)$$

Seesaw dynamics

$$\dot{H}_s = {}^s X_c^* f_C - {}^s X_L^* f_L - {}^s X_R^* f_R - m_s g, \quad (1b)$$

Constraint: feet attached to seesaw

$$J(q)\dot{\nu} + \dot{J}(q, \nu)\nu = a_{feet}, \quad (1c)$$

Constraint: rolling of seesaw

$$v_P = 0, \quad (1d)$$

where  $q \in SE(3) \times \mathbb{R}^n$  represents the configuration of the free floating system, which is given by the pose of a base-frame and  $n$  generalized coordinates  $q_j$  characterizing the joint angles. The vector  $\nu \in \mathbb{R}^{n+6}$  represents the robot velocity (it includes both  $\dot{q}_j \in \mathbb{R}^n$  and the linear and angular velocity of the base-frame  $v_b \in \mathbb{R}^6$ ), the system acceleration is denoted as  $\dot{\nu}$ , the derivative of  $\nu$ , the control input  $\tau \in \mathbb{R}^n$  is the vector of joint torques,  $M \in \mathbb{R}^{(n+6) \times (n+6)}$  is the mass matrix,  $h \in \mathbb{R}^{n+6}$  contains both gravitational and Coriolis terms,  $S \in \mathbb{R}^{n \times (n+6)} := (0_{n \times 6}, I_n)^\top$  is the matrix selecting the actuated degrees of freedom, and  $J_L \in \mathbb{R}^{6 \times (n+6)}$ , and  $J_R \in \mathbb{R}^{6 \times (n+6)}$  are proper Jacobians of the frames associated with the robot's feet, and  $J^\top := (J_L^\top, J_R^\top)$ . The six-element vectors  $f_L$  and  $f_R$  represent the contact wrenches acting at the robot's left and right foot, respectively. The six-

element vector  $f_C$  represents the pure contact force between the seesaw and the floor. The matrices  ${}^a X_b^*$  are the classical transformation matrices in the space of wrenches [2]. The twelve dimensional vector  $a_{feet}$  is obtained by stacking the derivative of the robot's feet twists. The three dimensional vector  $v_P$  is the velocity of the contact point between floor and seesaw. The six-dimensional vector  $H_s$  is the seesaw's momentum,  $m_s$  the seesaw's mass, and  $g \in \mathbb{R}^6$  is the gravitational acceleration.

### B. Problem statement

The control objective is the asymptotic stabilization of a desired robot's centroidal dynamics [3], and a desired seesaw orientation. In this short paper, however, we focus on the stabilization of the robot and seesaw pose shown in Figure 1, i.e. seesaw's rectangular surface parallel to the floor, robot's center of mass above the contact point between the seesaw and the floor, and zero robot's angular momentum.

The stabilization of the robot's center of mass and angular momentum is achieved by means of the robot's centroidal dynamics. Let  $H_G$  denote the centroidal momentum of the robot. Then, the time derivative of  $H_G$  equals the summation of the external wrenches acting on the multi-body system.

$$\dot{H}_G = {}^G X_L^* f_L + {}^G X_R^* f_R - m_r g, \quad (2)$$

where  $m_r$  is the mass of the robot,  $G$  the robot's center of mass, and  $\ddot{x} \in \mathbb{R}^3$  is the acceleration of the center of mass.

The control objective is to find a control law for the inputs  $\tau$  such that  $x \rightarrow x_d$ , with

$$x_d := (x_1(0), 0, x_3(0)),$$

$H_\omega \rightarrow 0$ , and  $\theta \rightarrow 0$ , where  $\theta$  is the angle between the horizon and the seesaw's rectangular surface, and  $H_\omega$  the *angular momentum* of the robot. This choice is sufficient for balancing purposes. Also, while achieving this control objective, the system shall have a degree of compliance.

## III. A PRELIMINARY CONTROL STRATEGY

The control strategy is composed of three steps. We first choose the external wrenches  $f_L$  and  $f_R$  such that  $x \rightarrow x_d$ , and  $H_\omega \rightarrow 0$ . Secondly, we use the six-dimensional null space of these wrenches to stabilize  $\theta \rightarrow 0$ , which still leave a five-dimensional space null space of the contact wrenches free. Finally, we generate these contact wrenches through the internal torques. Since iCub possesses more than twelve degrees-of-freedom, we choose the remaining control inputs so that to have compliance at the joint level, and we use the left redundancy at the contact wrenches level to minimize the joint torques.

### A. The choice of the feet contact wrenches

The contact wrenches  $f_L$  and  $f_R$  such that  $x \rightarrow x_d$  and  $H_\omega \rightarrow 0$  follow directly from the equation (2) [4] [5], i.e.

$$\begin{pmatrix} f_L \\ f_R \end{pmatrix} = A^+ (\dot{H}^{des} + m_r g) + N_A f_0 \quad (3)$$

with  $A := ({}^G X_L^*, {}^G X_R^*)$ ,  $A^+$  the pseudo inverse of  $A$ , and  $N_A$  the null space of  $A$ . Then, the control task  $x \rightarrow x_d$  and  $H_\omega \rightarrow 0$  is achieved by choosing

$$\dot{H}^{des} = -k_d H - k_p \begin{pmatrix} x - x_d \\ 0_{3 \times 1} \end{pmatrix}, \quad (4)$$

with  $k_d$  and  $k_p$  two positive constants. The vector  $f_0$  in Eq. (3) is free, and can be used for the stabilization of  $\theta = 0$ .

To stabilize the seesaw orientation  $\theta$ , we need to use the seesaw dynamics (1b). This dynamics is influenced by the contact force  $f_C$  between the seesaw and the floor. The contact force can be obtained by imposing the constraint (1d), which leads to  $f_C = f_C(f_L, f_R)$ , i.e. the contact force does depend upon the feet wrenches. These dependencies, however, are linear, which allows us to find a null space  $f_0$  such that  $\theta \rightarrow 0$ , i.e. we can find  $f_0$  that feedback linearizes the dynamics  $\dot{\theta}$ . Yet, this still leave a five-dimensional null space for the contact wrenches, which we denote as  $f_1$ .

### B. The choice of the joint torques

The control input  $\tau$  must generate the wrenches  $f_L$  and  $f_R$ . The relationship between the contact wrenches and the joint torques can be obtained by using the constraint equation (1c). More precisely, the feet acceleration is related to the angular acceleration of the seesaw as long as the feet are attached to the seesaw's surface. Then, by using also the free-floating dynamics (1), one can show that the torques generating  $f_L$  and  $f_R$  are given by the summation of two terms, i.e.,

$$\tau = \tau_f(f_1) + N\tau_0(f_1), \quad (5)$$

where  $\tau_f$  ensures the stabilization of the desired contact wrenches, the matrix  $N \in \mathbb{R}^{n \times n}$  is a proper null space projector, and  $\tau_0$  is a vector that can be chosen at will. To obtain compliance at joint level, we choose  $\tau_0$  similar to a gravity and external force compensation, plus a term of the form

$$-k(q_j - q_d),$$

which ensures compliance at joint level. Then, we use the aforementioned five-dimensional null space of the contact wrenches, represented by the vector  $f_1$ , to minimize the joint torques  $\tau$ .

### Remark

Instead of using pseudo-inverses and null-space projectors, the above minimization problems can be formulated as a cascade of quadratic programming problems. By doing so, we also ensure the satisfaction of inequality constraints, which in turn ensure that the feet remain attached to the seesaw. The implementation in the simulations/experiments presented next is carried on in the context of the quadratic programming problems subject to inequality constraints.

#### IV. SIMULATIONS AND EXPERIMENTS

We implemented the proposed control strategy on the iCub platform, and we verified the effectiveness of the control laws first in simulations carried on Gazebo, and then in a real scenario.

The simulation consists in two phases. In the first phase, no external perturbation is applied, and in the second phase an external perturbation is applied to the system. The simulation can be found at<sup>1</sup>. Observe that when the system is not perturbed, the controller stabilizes the robot's and seesaw's pose. Let us highlight that the system's configuration is unstable, i.e., if one blocks all joints the robot would fall down. After applying the perturbation, the robot reacts so as to stabilize the desired equilibrium, and slowly converges to the desired configurations. The settling time is kept high for avoiding instabilities in the real scenario.

Then, we carried on some experiments on the real platform. The control architecture is composed of two loops [6]. The inner loop is in charge of stabilizing *desired* joint torques, while the outer loop is governed by Eq. (5). Both loops runs at the same frequency of 100 Hz.

A video of the experiment can be found at <sup>2</sup>. The experiments do not perform as well as simulations do, however. This may be due to several reasons. First, the loop for stabilizing desired joint torques runs at 100 Hz, which impairs the performances of the entire control architecture. Then, the estimation of the floating base is a far cry from being robust, since poor filtering techniques forbade us to increase the low pass cut-off frequencies, which was set at 2 Hz. The effects of these two problems render the system slow, which results to a very poor control performance.

#### V. CONCLUSIONS

This paper has presented preliminary results on the control of humanoid robots when standing on semi-cylindrical seesaws. The control strategy has been verified both in simulations and in a real scenario involving the iCub humanoid robot. Future work consists in thorough analyses of the associated nonlinear systems, which shall give us insights into the system equilibrium configurations, and into their associated control laws. We believe that the poor performance of the control are also due to the very simple control strategy presented here, which may neglect fundamental points when synthesizing the control laws.

From the practical perspective, we are currently implementing the controller for stabilizing joint torques at the firmware level, which runs at 1 KHz, and this should cut time delays in the low-level control loop. Also, better estimations of the floating base may allow us increase the cut off frequencies of the associated low pass filters. We believe that improvements at the low level joint torque control and

at the estimation of the floating base, along with a deep analysis on the nonlinear system, can improve the entire control architecture, so as to provide the user with acceptable performances.

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<sup>1</sup><https://youtu.be/3dvx8BhDRMk>

<sup>2</sup><https://youtu.be/LAnCXAlfNAE>