New approach for testing dynamic balance and motion feasibility of humanoids in presence of multiple spatial contacts*

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***Abstract*— Besides the contacts between the feet and the ground, in some cases motion of humanoids may require additional contacts with environment. When spatial distribution of contacts is needed in order to perform planned motion, decision where the next contact will be established may be of fundamental importance. To ensure that planned motion will be feasible and dynamically balanced, all contacts with environment should remain sustained. However, the main indicator of dynamical balance, the ZMP, is only applicable to the robots walking on flat horizontal surfaces. For that reason, method for testing dynamical balance when contacts are distributed spatially will be presented. To do so, constraints on all contact wrenches are written in the form of *Composite Dynamic Balance Matrix*. Based on that, condition which can tell if desired motion is feasible or not, is derived. Those conditions are in form of inequalities and maintaining them satisfied is of highest importance. For synthesizing whole body motion where robot has to fulfill several tasks (in form of equalities and inequalities), generalized task prioritization framework could be used.**

I. Introduction

When all contacts are sustained, humanoid's motion will be feasible and dynamically balanced. The main indicator of dynamical balance is ZMP [1], [2], but it is only applicable to the robots walking on flat horizontal surfaces. Several authors tried to generalize this condition. In [3] authors proposed generalized ZMP which can be used when multiple contacts exist. Feasible solution of wrench, introduced in [4], is used to check if motion is feasible when robot walks on uneven terrain. Recently, Caron et al. [5] introduced a closed-form formulae for the contact wrench cone for rectangular support areas. These generalizations are often very hard to apply and cover only a subset of all possible contact configurations. In [6] dynamically balanced motion was created using optimization scheme together with simplified dynamics and full kinematics. That method is unique in the sense that there is no need to schedule contact sequence in advance, but it's biggest problem is the use global optimization, so it can not be applied on-line.

Humanoid robots often have to perform multiple tasks simultaneously, while choosing those solutions which comply with existing constraints and priorities of execution. For such a purpose, the task-prioritization framework introduced in [7] could be used. However, that framework was well-suited only for prioritization of tasks in form of equalities, but not inequalities. Avoiding slipping and maintaining contacts sustained is an inequality type task. One of the most recent generalizations of this framework [8], [9] has shown how inequalities can be included in the framework.

In this paper a method for checking if the motion is feasible together with the conditions that contact wrenches have to fulfill in order to maintain contacts sustained is presented. After incorporating that into a generalized task prioritization framework [8] complex multi-contact dynamically balanced whole body motions could be created.

II. CDBM - Composite dynamic balance matrix

Humanoid robots are most of time in contact with the environment, and the pure nature of the contact imposes constraints on contact wrenches. Thus, properly managing the contact wrenches is of highest importance when planning dynamically balanced humanoid motion. For that reason the constraints on contact wrenches will be derived first.

When contact between the robot and its surrounding is in one point and Columb friction acts between the bodies in contact, the contact force must be inside friction cone. In order to linearize that constraint, friction cone is approximated by s-sided pyramid, so the constraint can be written as:

$$\mathbf{S}_{\mu\ s\times 3}\mathbf{F}_{3\times 1} \succ \mathbf{0}_{s\times 1}, \tag{1}$$

where $\mathbf{S}_{\mu\ s\times 3}$ is the matrix defining the friction pyramid. This condition encompasses both unilaterality of the contact as well as prevention of the sliding.

Robotic feet are usually rectangular and contact between them and the ground surface is planar. In order to obtain constraint on contact wrench, that kind of contact will be modeled by considering four point contacts placed at the corners of the foot. Total wrench for the referent point on the foot P created by four point contacts is:

$$\begin{bmatrix}\mathbf{F}_P\\ \mathbf{M}_P\end{bmatrix} = \begin{bmatrix}\sum_{i=1}^4 \mathbf{F}_i\\ \sum_{i=1}^4 \mathbf{r}_i^P \times \mathbf{F}_i\end{bmatrix} = \mathbf{G}_P\bar{\mathbf{F}}, \tag{2}$$

where $\bar{\mathbf{F}} = \begin{bmatrix}\mathbf{F}_1^T\ \mathbf{F}_2^T\ \mathbf{F}_3^T\ \mathbf{F}_4^T\end{bmatrix}^T$, and $\mathbf{G}_P$ represents contact matrix. Each of four contact forces needs to be within their respective friction pyramids. By combining eqns (1) and (2), constraint on contact wrench calculated for referent point P can be derived:

$$\bar{\mathbf{S}}\mathbf{G}_P^+\begin{bmatrix}\mathbf{F}_P\\ \mathbf{M}_P\end{bmatrix} = \mathbf{Z}\begin{bmatrix}\mathbf{F}_P\\ \mathbf{M}_P\end{bmatrix} \succ \mathbf{0}. \tag{3}$$

Matrix $\mathbf{Z} = \bar{\mathbf{S}}\mathbf{G}_P^+$ is called *Dynamic balance matrix* (DBM) and it is a product of composite friction pyramid matrix $\bar{\mathbf{S}}$ (diagonal block matrix with four friction pyramid matrices

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$\mathbf{S}_\mu$ as diagonal elements) and Moore-Penrose inverse of contact matrix $\mathbf{G}_P$. As long as the requirement (3) remains fulfilled, the planar contact between the foot and the ground would remain sustained, meaning that there will be neither separation nor the sliding between the foot and the ground. It is important to note that constraint (3) is derived for a local coordinate frame of the contact.

When multiple spatial contacts between the robotic system and the environment exist, it is easy to generalize condition (3). Such a system is depicted in Fig. 1, where planar contacts exist between the sloped ground and the feet and between one of the robot's hands and horizontal surface. All three surface contacts have to comply with constraint (3). If all contact forces are expressed in the global coordinate frame, they must be pre-multiplied with corresponding rotation matrices in order to be expressed in local coordinate frame, in which condition (3) must hold. Hence, it can be written:

$$\begin{bmatrix} \mathbf{Z}_L & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_R & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Z}_H \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \mathbf{R}_L^T \mathbf{F}_L \\ \mathbf{R}_L^T \mathbf{M}_L \end{bmatrix} \\ \begin{bmatrix} \mathbf{R}_R^T \mathbf{F}_R \\ \mathbf{R}_R^T \mathbf{M}_R \end{bmatrix} \\ \begin{bmatrix} \mathbf{R}_H^T \mathbf{F}_H \\ \mathbf{R}_H^T \mathbf{M}_H \end{bmatrix} \end{bmatrix} = \bar{\mathbf{Z}} \mathbf{F}_{ext} \succ \mathbf{0}. \quad (4)$$

The matrix $\bar{\mathbf{Z}}$ is called *Composite Dynamic Balance Matrix* (CDBM). It is compiled as a diagonal block matrix, where diagonal elements are DBMs for left foot ($\mathbf{Z}_L$), right foot ($\mathbf{Z}_R$) and robot's hand ($\mathbf{Z}_H$) multiplied by diagonal block matrix consisting of transposes of corresponding rotation matrices ($\mathbf{R}_L$, $\mathbf{R}_R$ and $\mathbf{R}_H$) for each contact. The vector of generalized contact forces acting on the robot expressed in the global coordinate frame is denoted by $\mathbf{F}_{ext} = \begin{bmatrix} \mathbf{F}_L^T & \mathbf{M}_L^T & \mathbf{F}_R^T & \mathbf{M}_R^T & \mathbf{F}_H^T & \mathbf{M}_H^T \end{bmatrix}^T$.

III. MOTION FEASIBILITY

In complex environments, such as one shown in Fig. 1 it is extremely important to check if the intended motion is feasible, before robot initiates it's execution. For example, a robot can not lift his hand from the surface while maintaining position of the center of mass (CM) of the system, because the robot will lose dynamical balance and ultimately fall. Also, if the surface in contact with the hand has low friction coefficient, the robot will be unable to maintain even the static posture, since the hand will slide and the body will not be able to maintain its posture.

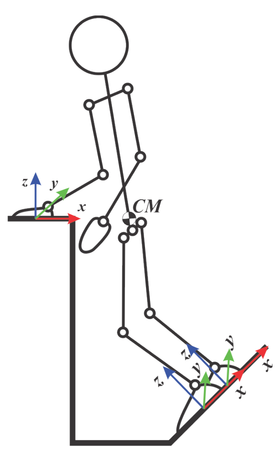

Fig. 1. System with multiple contacts

Total wrench for the system from Fig. 1, that all contact wrenches create for the CM is:

$$\begin{bmatrix} \mathbf{F}_{CM} \\ \mathbf{M}_{CM} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_L + \mathbf{F}_R + \mathbf{F}_H \\ \mathbf{r}_L^{CM} \times \mathbf{F}_L + \mathbf{M}_L + \mathbf{r}_R^{CM} \times \mathbf{F}_R + \\ + \mathbf{M}_R + \mathbf{r}_H^{CM} \times \mathbf{F}_H + \mathbf{M}_H \end{bmatrix} = \\ = \mathbf{G}_{CM} \mathbf{F}_{ext}. \quad (5)$$

The matrix $\mathbf{G}_{CM}$ represents contact matrix for the CM. It relates all contact wrenches acting on the body with total wrench acting on the CM. If robot with mass m needs to perform a motion with desired acceleration of COM $\mathbf{a}_{CM}^{des}$ and desired rate of change of angular momentum $\dot{\mathbf{L}}_{CM}^{des}$, contact forces need to counterbalance effects of gravitational and inertial forces:

$$\begin{bmatrix} m\mathbf{a}_{CM}^{des} \\ \dot{\mathbf{L}}_{CM}^{des} \end{bmatrix} = \begin{bmatrix} \mathbf{F}_{CM} \\ \mathbf{M}_{CM} \end{bmatrix} + \begin{bmatrix} m\mathbf{g} \\ \mathbf{0} \end{bmatrix}. \quad (6)$$

Since contact wrenches are bounded, total wrench for the CM can not assume arbitrary values, meaning that robot is not able to perform arbitrary desired motion. By combining eqns (4), (5) and (6) it is easy to derive linear program whose feasibility tells us if the motion is possible:

$$\begin{aligned} \text{find} \quad & \mathbf{F}_{ext}. \\ \text{such that:} \quad & \mathbf{G}_{CM}\mathbf{F}_{ext} = \begin{bmatrix} m\left(\mathbf{a}_{CM}^{des} - \mathbf{g}\right) \\ \dot{\mathbf{L}}_{CM}^{des} \end{bmatrix} \\ & \bar{\mathbf{Z}}\mathbf{F}_{ext} \succ \mathbf{0}. \end{aligned} \quad (7)$$

If there is a solution to the previous program, the desired motion is feasible, otherwise desired motion is not feasible. Although it looks simple, this program is a very powerful tool when determining if some intended motion can be performed or not. It can also give an answer if additional contact will make movement feasible (additional contacts change vector $\mathbf{F}_{ext}$ and matrices $\mathbf{G}_{CM}$ and $\bar{\mathbf{Z}}$). Another option is to check if CM needs to be moved to some other place (this changes only matrix $\mathbf{G}_{CM}$), or some of the existing contacts had to be placed elsewhere (changing matrices $\mathbf{G}_{CM}$ and $\bar{\mathbf{Z}}$). Also, when desired motion is feasible, it can be easily checked if some contact can be broken so that desired motion remains feasible.

IV. WHOLE-BODY MOTION SYNTHESIS

For synthesizing whole body motion of the humanoid robot with multiple spatial contacts a generalized task-prioritization framework will be employed [8]. It is derived from [7], while additional details can be found in [8], [9]. It's main advantage in comparison to the others is that it can enforce inequality type tasks on joint torques, which gives us the ability to impose constraint given by CDBM (4).

Let's consider example sketched in Fig. 1. Three planar contacts introduce constraints on the system. Both feet and one hand can neither translate nor rotate, which can be written as $\bar{\mathbf{J}}\dot{\mathbf{q}} = \mathbf{0}$, where the vectors of robot's joint coordinates, velocities and accelerations are $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$. $\bar{\mathbf{J}}$ represents composite Jacobian matrix for all contacts between the robot and the environment. For the case shown in Fig. 1 it has the form $\bar{\mathbf{J}} = \left[\mathbf{J}_L^T \; \mathbf{J}_R^T \; \mathbf{J}_H^T\right]^T$, where $\mathbf{J}_L$, $\mathbf{J}_R$ and $\mathbf{J}_H$ are Jacobian matrices for the left foot, right foot and robot's hand. Dynamics of the multi-body system with multiple contacts is given by:

$$\mathbf{H}\ddot{\mathbf{q}} + \mathbf{h_0} = \boldsymbol{\tau} + \bar{\mathbf{J}}^T\mathbf{F}_{ext}, \tag{8}$$

where $\mathbf{H}$ is the inertia matrix, $\mathbf{h}_0$ is the vector of centrifugal, Coriollis and gravitational loads, while $\boldsymbol{\tau}$ is the vector of joint torques. When $\bar{\mathbf{J}}$ has a full row rank, vector of external forces induced by the constraints is:

$$\mathbf{F}_{ext} = \left(\bar{\mathbf{J}}\mathbf{H}^{-1}\bar{\mathbf{J}}^T\right)^{-1}\left(-\dot{\bar{\mathbf{J}}}\dot{\mathbf{q}} - \bar{\mathbf{J}}\mathbf{H}^{-1}\left(\boldsymbol{\tau} - \mathbf{h}_0\right)\right). \tag{9}$$

If this external force complies with (4) all contact will remain sustained. This constraint is inequality linear in joint torques. That is the main reason why generalized task prioritization framework is employed.

Generalized task prioritization framework has to control the system, whose dynamics is given by (8) and (9) in such a way, that p tasks written in form of:

$$\mathbf{A}_i\ddot{\mathbf{q}} = \mathbf{b}_i; \; \mathbf{A}_i\ddot{\mathbf{q}} \preceq \mathbf{b}_i; \; \text{or } \mathbf{A}_i\mathbf{H}^{-1}\boldsymbol{\tau} \preceq \mathbf{b}_i \tag{10}$$

are fulfilled in a prioritized manner. That means, that any lower priority task is executed to such extent so it does not interfere with the execution of any task with higher priority. Equality tasks which are not dependent on joint accelerations, but only on joint coordinates or velocities, can be transformed in required form by differentiation. Similarly, inequality type tasks not dependent on joint accelerations can be written in required form by using Taylor series.

Algorithm for calculating control torques is shown in Algorithm 1. Torque is calculated by iteratively solving the quadratic optimization problem with both equality and inequality constraints. Task matrices post-multiplied by the inverse of root of inertia matrix $\mathbf{B}_i = \mathbf{A}_i\mathbf{H}^{-1/2}$ in order to make whole procedure dynamicaly consistent. Constraint matrix is $\mathbf{B}_0 = \bar{\mathbf{J}}\mathbf{H}^{-1/2}$ and vector $\mathbf{b}_0 = -\dot{\bar{\mathbf{J}}}\dot{\mathbf{q}}$. The modified vector of joint acceleration is $\mathbf{r} = \mathbf{H}^{1/2}\ddot{\mathbf{q}}$ while the modified vector of velocity and gravitational effects is $\mathbf{p} = \mathbf{H}^{-1/2}\mathbf{h}_0$. Matrices $\mathbf{C}_a$ and $\mathbf{C}_\tau$ and vectors $\mathbf{d}_a$ and $\mathbf{d}_\tau$ are used in order to include inequality tasks in the framework.

V. Simulation results

The simulated scenario is the same as one depicted in Fig. 1. The robot should move sideways, while his feet are in contact with surface angled at $45°$. Although the friction coefficient is high ($\mu = 0.8$), the robot is unable to stand on the slope without additional contact between the hand and the horizontal surface, because the ground reaction force acting on the feet will always have horizontal component which will tend to push te CM towards the wall.

During the motion, the robot was always in contact with the environment with at least three of it's limbs, while it was moving fourth limb in the direction of the motion. To perform such a pattern, the robot had to fulfill several tasks. One with the highest priority was to maintain joint torques between the predefined saturation limits. The task of second highest importance was to maintain all contacts sustained. As mentioned earlier, that is inequality type task dependent on joint torques and can be obtained in required form (10) by combining eq. (4) and eq. (9). Matrix $\bar{\mathbf{Z}}$ changes over time because of the changes in configuration of the contacts. Next task that robot had to perform is that the CM has to follow a predefined trajectory. Task for repositioning one of the limbs has second to last priority. The last task that the robot has to perform is to maintain a configuration which is as close as possible to the initial posture of the robot.

Algorithm 1 Procedure for calculating control torques

$\mathbf{N}_c \leftarrow \mathbf{I}$, $\mathbf{T}_c \leftarrow \mathbf{0}$ $\mathbf{C}_a \leftarrow []$, $\mathbf{C}_\tau \leftarrow []$ $\mathbf{d}_a \leftarrow []$, $\mathbf{d}_\tau \leftarrow []$
if system with contacts **then**
  $\mathbf{N}_c := \mathbf{I} - \mathbf{B}_0^+\mathbf{B}$, $\mathbf{T}_c := \mathbf{B}_0^+\left(\mathbf{b}_0 + \mathbf{B}_0\mathbf{p}\right)$
end if
for every task i **do**
  if inequality **then**
    if dependent on acceleration **then**
      $\mathbf{C}_a := \begin{bmatrix}\mathbf{C}_a \\ \mathbf{B}_i\end{bmatrix}$; $\mathbf{d}_a \leftarrow \begin{bmatrix}\mathbf{d}_a \\ \mathbf{b}_i\end{bmatrix}$
    else if dependant on torque **then**
      $\mathbf{C}_\tau := \begin{bmatrix}\mathbf{C}_\tau \\ \mathbf{B}_i\end{bmatrix}$; $\mathbf{d}_\tau \leftarrow \begin{bmatrix}\mathbf{d}_\tau \\ \mathbf{b}_i\end{bmatrix}$
    end if
    find $\mathbf{u}$
    s. t.: $\ddot{\mathbf{r}} + \mathbf{p} = \mathbf{T}_c + \mathbf{N}_c\mathbf{T}_{prev} + \mathbf{N}_c\mathbf{N}_{prev}\mathbf{u}$
      $\mathbf{C}_a\ddot{\mathbf{r}} \preceq \mathbf{d}_a$
      $\mathbf{C}_\tau\left(\mathbf{T}_{prev} + \mathbf{N}_{prev}\mathbf{u}\right) \preceq \mathbf{d}_\tau$
    if system feasible **then**
      continue
    end if
  else
    minimize $\|\mathbf{B}_i\ddot{\mathbf{r}} - \mathbf{b}_i\|_2^2$
    s. t. : $\ddot{\mathbf{r}} + \mathbf{p} = \mathbf{T}_c + \mathbf{N}_c\mathbf{T}_{prev} + \mathbf{N}_c\mathbf{N}_{prev}\mathbf{u}$
      $\mathbf{C}_a\ddot{\mathbf{r}} \preceq \mathbf{d}_a$
      $\mathbf{C}_\tau\left(\mathbf{T}_{prev} + \mathbf{N}_{prev}\mathbf{u}\right) \preceq \mathbf{d}_\tau$
    $\mathbf{N}_{prev} := \mathbf{N}_{prev}\left(\mathbf{I} - \left(\mathbf{B}_i\mathbf{N}_c\mathbf{N}_{prev}\right)^+\mathbf{B}_i\mathbf{N}_c\mathbf{N}_{prev}\right)$
  end if
  $\mathbf{T}_{prev} := \mathbf{T}_{prev} + \mathbf{N}_{prev}\mathbf{u}$;
end for
$\boldsymbol{\tau} \leftarrow \mathbf{H}^{1/2}\mathbf{T}_{prev}$

When the robot moves sideways it repositions it's limbs in following order: right hand, left hand, right foot, left foot. Procedure for calculating torques described in Alg. 1 is employed, and the results of the simulation are shown in Fig. 2. The robot was able to move to the right with ease, until it puts the right hand on the oily patch (shown as pink ribbon) which is much more slippery ($\mu = 0.2$) (first image on the top row). Before trying to reposition next limb in order, the left hand, robot first checks if it could be lifted. The result is

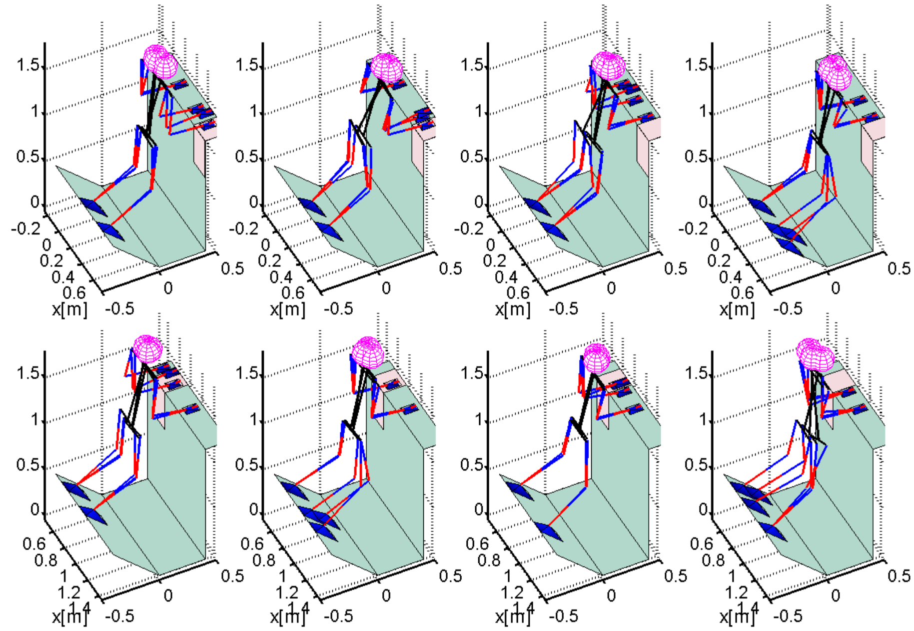

Fig. 2. System with multiple contacts

negative, since the hand on the oily patch would slide and the robot would fall. Because of that robot first repositions it's right hand out of the slippery patch. Only then he is able to continue repositioning sequence, i.e. first to move left hand, then move right leg (third and fourth image in first row).

After several cycles, the robot places its left hand on the oily patch (first figure in bottom row). But in that case, the robot is able to move it's right leg forwards with left hand on an oily patch because the right hand takes most of the load. When robot wants to move its left foot forwards, it is unable to do so, so it needs to remove the left hand from the oily patch first and then move the left foot forwards.

VI. Conclusion

This paper describes a method to check if the desired motion of humanoid robot in presence of multiple contacts is feasible. First, the constraint on contact wrenches written in the form of CDBM (4) is derived. Using that result, linear program (7), which can tell if desired motion is feasible or not, was devised. Apart from that, the same program can be employed to determine where to move the CM or how to reposition one of the contacts such that desired motion becomes feasible.

After that, the generalized task prioritization framework is briefly introduced. It is employed to calculate driving torques, such that robot fulfills multiple tasks. One of the tasks is that contact wrenches must be within the bounds defined by the CDBM. It must be emphasized that task prioritization framework can't help to synthesize the motion when previously mentioned method tells that the desired motion is infeasible. Based on the priorities of the tasks, either the wrenches will be within the bound and the CM will move in unknown and unpredictable manner, or the CM will try to follow the defined trajectory, but at some point one of the contacts will be broken or begin to slide. Both options are highly undesirable and will eventually lead to fall of the robot.

All results are illustrated by a simulation of a complex humanoid motion. Generalized task-prioritization framework was employed to synthesize dynamically balanced motion. Method for checking if the motion is feasible was employed before each phase of the motion, and when it turned out that motion is infeasible, corrective actions had to be performed.

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