Project Groups

<table>
<thead>
<tr>
<th>Project</th>
<th>Students - A</th>
<th>Students - B</th>
<th>Students - C</th>
<th>Students - D</th>
<th>Students - E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Humanoid</td>
<td>Valerie</td>
<td>Sean</td>
<td>Megan</td>
<td></td>
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<tr>
<td>Assembly</td>
<td>Akram</td>
<td>Ken</td>
<td>Kathleen</td>
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<tr>
<td>Offshore</td>
<td>Yutian</td>
<td>Abdoul</td>
<td>Chinmay</td>
<td>Bo</td>
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<tr>
<td>Service</td>
<td>Andrew</td>
<td>Kevin</td>
<td>Carolyn</td>
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<tr>
<td>Sport</td>
<td>Sergio</td>
<td>Courtney</td>
<td>Bryce</td>
<td>Rohan</td>
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Menu

Kinematics
Dynamics
Jacobians
Inverses
Task
Representations
Equations of
Motion
Operational
Space Control
Dynamic
Models
Control
Modalities
Compliance
Force Control
Redundant
Robots
Posture
Null Space
Dynamic
Behavior
Whole-Body
Control

Collision Avoidance
And Elastic Planning
Motion Planning

C-Obstacle
Computational complexity grows exponentially in the number of DOF.

Randomized Motion Planning
Avoids Explicit C-Obstacle computation
Artificial Potential Field

Artificial Potential Field
Artificial Potential Field

\[ U_\phi(x) = \begin{cases} 
\frac{1}{2} \eta \left( \frac{1}{f(x)} - \frac{1}{f(x_0)} \right)^2 & \text{if } f(x) \leq f(x_0); \\
0 & \text{if } f(x) > f(x_0). 
\end{cases} \]

\[ U_\phi(x) = \begin{cases} 
\frac{1}{2} \eta \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right)^2 & \text{if } \rho \leq \rho_0; \\
0 & \text{if } \rho > \rho_0. 
\end{cases} \]
nEllipsoids: Analytical Representation

\[
\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1
\]

nEllipsoids: Analytical Representation

\[
\left(\frac{x}{a}\right)^{2n} + \left(\frac{y}{b}\right)^{2n} + \left(\frac{z}{c}\right)^{2n} = 1
\]
**nEllipsoids: Analytical Representation**

\[
\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^{2n} = 1
\]

**nEllipsoids: Artificial Potential Field**
nEllipsoids: Artificial Potential Field
Control Architecture

Elastic Planning
Real-time collision-free path modification

Connecting
Reactive Local Avoidance with
Global Motion Planning
Elastic Band

Elastic Band
Free-Space Representation

Free-Space Tunnel
Elastic Strip
for a Mobile-Manipulator Robot
Elastic Strip
Whole-body Control

Task & Posture Decomposition

Task Dynamics and Control

Task Dynamics

\[ \Lambda \ddot{x} + \mu + p = F \]

Task Control

\[ HF = \hat{\Lambda}(-\nabla V_{\text{Task}}) + \hat{\mu} + \hat{p} \]

\[ \Gamma = J^T F \]
Task Dynamics – Branching Structures

\[ x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & \cdots & \Lambda_{1L} \\ \Lambda_{21} & \Lambda_{22} & \cdots & \Lambda_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ \Lambda_{L1} & \Lambda_{L2} & \cdots & \Lambda_{LL} \end{pmatrix} \]

\[ x \Rightarrow J \Rightarrow \Lambda \]

\[ \Lambda \ddot{x} + \mu + p = F \]

Posture Space Control

Joint-Space Dynamics

\[ N^T \]

Task Control:

\[ J^T \]

Posture Control:

\[ N^T \]

\[ \bar{J}^T \]

\[ \Lambda \ddot{x} + \mu + p = F \]

\[ \Gamma \]
Control Structure

\[ \Gamma = J_{task}^T F_{task} + N_{task}^T \Gamma_{posture} \]

Decomposition of torque vector

Dynamic Consistency \[ \Rightarrow \ddot{x}_{task} = 0 \]

Posture Potential Energy
Whole-Body Control: Dynamics

\[ \Gamma = \Gamma_{\text{Task}} + \Gamma_{\text{Posture}} \]

\[ \Gamma_{\text{Task}} = J^T F \]

\[ \Gamma_{\text{Task}} = J^T \left[ \hat{A} \left( -\nabla V_{\text{Task}} \right) + \hat{\mu} + \hat{p} \right] \]

\[ \Gamma_{\text{Posture}} = N^T \Gamma_{\text{Desired–Posture}} \]

\[ \Gamma_{\text{Posture}} = N^T \left( -\nabla V_{\text{Desired–Posture}} \right) \]

Dynamics in Posture Space
Posture Dynamics & Control

Posture sub-task: \( x_p \rightarrow J_p \)
\[
\delta x_{p|t} = J_p (N, \delta q)
\]

**Task-consistent posture Jacobian:**
\[
J_{p|t} = J_p N_t
\]

**Dynamics:**
\[
\Lambda_{p|t} \ddot{x}_{p|t} + \mu_{p|t} + p_{p|t} = F_{p|t}
\]

**Control:**
\[
\Gamma_{\text{posture}} = J_{p|t}^T [\hat{\Lambda}_{p|t} (-\nabla V_{\text{posture}}) + \hat{\mu}_{p|t} + \hat{p}_{p|t}]
\]

with Dynamics in Posture Space
Constraints

\[ \Gamma = \mathbf{J}^T \mathbf{F}_{\text{crash}} + \mathbf{N}_{\text{task}}^T \left( \mathbf{F}_{\text{task}} + N^T \mathbf{\Gamma}_{\text{posture}} \right) \]
Self Collision

\[ \Gamma = J_{\text{cnst}}^T F_{\text{cnst}} + N_{\text{cnst}}^T (J_{\text{task}}^T F_{\text{task}} + N_{\text{task}}^T \Gamma_{\text{posture}}) \]

Obstacles

\[ \Gamma = J_{\text{cnst}}^T F_{\text{cnst}} + N_{\text{cnst}}^T (J_{\text{task}}^T F_{\text{task}} + N_{\text{task}}^T \Gamma_{\text{posture}}) \]
Obstacles

\[ \Gamma = J_{\text{enst}}^T F_{\text{enst}} + N_{\text{enst}}^T \left( J_{\text{task}}^T F_{\text{task}} + N_{\text{task}}^T \Gamma_{\text{posture}} \right) \]
Multi-Contact Whole-body Control
Integration of Whole-Body Control & Locomotion

Under-actuated Balance Reaction forces

Multi-Contact Whole-body Control
Integration of Whole-Body Control & Walking

\[
\begin{align*}
\Gamma &= \text{contact task} \\
\mathbf{F} &= \text{contact task} \\
\mathbf{N} &= \text{contact task} \\
\end{align*}
\]

\[
\mathbf{J}^T (\mathbf{J} \mathbf{F} + \mathbf{N}^T) \text{ task, posture}
\]
Balanced Supporting Contacts
*Internal Force Control – Virtual Linkage*

Unified Whole-Body Control Framework
*Task, Posture, Constraints, Multiple Contacts, and Balance*
posture consistent with { }
task consistent with { }
contact consistent with { }
internal constraints
self collision
local obstacles consistent with { }
balance 

interact with the world, cooperate, and manipulate

\[ \Gamma = J^T_{\text{constraint}} F_{\text{constraint}} + N^T_{\text{constraint}} (J^T_{\text{task}} F_{\text{task}} + N^T_{\text{task}} \Gamma_{\text{posture}}) \]
\[ \Gamma = J^T_{\text{constraint}} F_{\text{constraint}} + N^T_{\text{constraint}} (J^T_{\text{task}} F_{\text{task}} + N^T_{\text{task}} J^T_{\text{posture}} F_{\text{posture}}) \]
\[ \Gamma = J^T_{\text{constraint}} F_{\text{constraint}} + (J^T_{\text{task}} N_{\text{constraint}})^T F_{\text{task}} + (J^T_{\text{posture}} N_{\text{constraint}})^T F_{\text{posture}} \]
\[ \Gamma = J^T_{\text{constraint}} F_{\text{constraint}} + J^T_{\text{task}} F_{\text{task}} + J^T_{\text{posture}} F_{\text{posture}} \]
\[
\Gamma = J^T_c F_c + J^T_{t/c} F_t + J^T_{p/t} F_p
\]
Unified Framework

\[ \Gamma = J^T_c F_c + J^T_{t|c} F_t + J^T_{p|t|c} F_p \]

\[ \mathcal{g}_\otimes \triangleq \begin{pmatrix} \mathcal{g}_c \\ \mathcal{g}_{t|c} \\ \mathcal{g}_{p|t|c} \end{pmatrix}; F_\otimes \triangleq \begin{pmatrix} F_c \\ F_{t|c} \\ F_{p|t|c} \end{pmatrix}; J_\otimes \triangleq \begin{pmatrix} J_c \\ J_{t|c} \\ J_{p|t|c} \end{pmatrix} \]

\[ \Lambda_\otimes \ddot{\mathcal{g}} + \mu_\otimes + p_\otimes = F_\otimes \]

\[ \Gamma = J^T_\otimes F_\otimes \]

Constraint-Consistent Operational Space

\[ \mathcal{v}_\otimes = \begin{pmatrix} \mathcal{v}_c \\ \mathcal{v}_{f|c} \\ \mathcal{v}_{m|f|c} \\ \mathcal{v}_{p|m|f|c} \end{pmatrix} \]

\[ \mathcal{v}_\otimes \in CcOspace \]

interact with the world, cooperate, and manipulate
Constraint-Consistent Operational Space

Unified Whole-Body Control Framework

Dynamics

\[ \Lambda \dot{\varphi} + \mu + p + F_f = F \]

Control

\[ F = \hat{\Lambda} F^* + \hat{\mu} + \hat{p} \]

Torques

\[ \Gamma = J^T F \]
Experimental Validation
Experimental Validation

Unified Whole-Body Control Framework

Task, Posture, Constraints, Multiple Contacts, and Balance