# Experimental Robotics

CS225A

Lecture 7

Oussama Khatib

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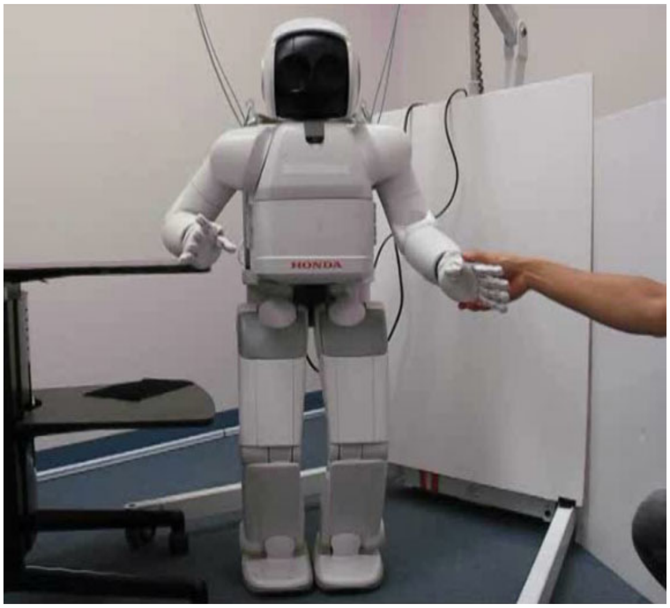
## Project Groups

| **Project** | **Students - A** | **Students - B** | **Students - C** | **Students - D** | **Students -E** |
|---|---|---|---|---|---|
| Humanoid | Valerie | Sean | Megan | | |
| Assembly | Akram | Ken | Kathleen | | |
| Offshore | Yutian | Abdoul | Chinmay | | |
| Service | Andrew | Kevin | Carolyn | | |
| Sport - I | Sergio | Courtney | Bryce | | |
| Sport -II | | Rohan | | | |

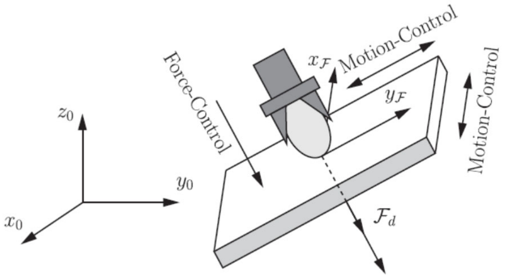
# Menu

- Kinematics Dynamics
  - Jacobians Inverses
  - Task Representations
  - Equations of Motion
- Operational Space Control
  - Dynamic Models
  - Control Modalities
  - Compliance Force Control
- Redundant Robots
  - Posture Null Space
  - Dynamic Behavior
  - Whole-Body Control

# Redundancy



# Unified Motion/Force Control



- Generalized Selection Matrix
- Dynamic Model (Homogeneity)

$$\Lambda_0(x)\dot{\vartheta}+\mu_o\left(x,\vartheta\right)+p_0(x)+F_{contact}=F_0$$

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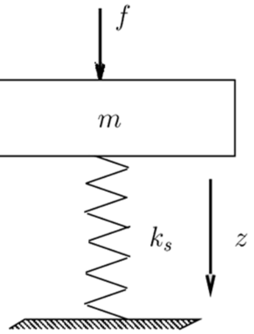
# A Mass Spring System

System

$$m\ddot{z}+k_s z=f$$

$$f_s=k_s z$$

$$m\frac{1}{k_s}\ddot{f}_s+f_s=f$$



System

$$m\frac{1}{k_s}\ddot{f}_s + f_s = f$$

Control

$$f = f_s + m\ f_{comp}$$

$$f = f_s - m\left[k_f\left(f_s - f_d\right) + k_{v_f}\dot{f}_s\right]$$

Control-loop System

$$\ddot{f}_s + k_s k_{v_f}\dot{f}_s + k_s k_f\left(f_s - f_d\right) = 0$$

Static Equilibrium

$$f_s = f_d$$

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# End-Effector/Sensor System

$$\Lambda_0\dot{\vartheta} + \mu_0(x,\vartheta) + p_0(x) + F_{contact} = F_0$$

Unified Control

$$F_0 = F_{motion} + F_{force}$$

$$F_{motion} = \hat{\Lambda}_0\,\Omega\,F^*_{motion} + \hat{\mu}_0 + \hat{P}_0$$

$$F_{force} = \hat{\Lambda}_0\,\bar{\Omega}\,F^*_{force} + F_{sensor}$$

## End-Effector/Sensor System

$$\Lambda_0 \dot{\vartheta} + \mu_0(x,\vartheta) + p_0(x) + F_{contact} = F_0$$

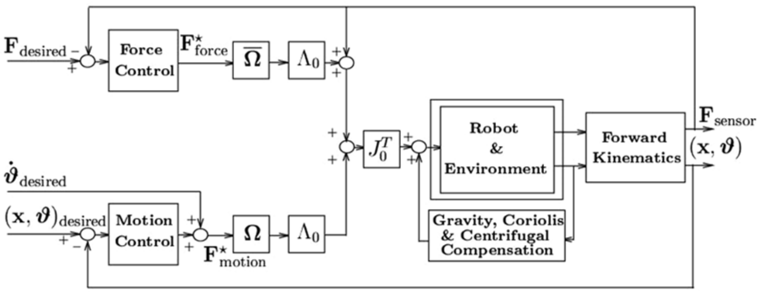
Unified Control

$$F_0 = F_{motion} + F_{force}$$

$$F_{motion} = \hat{\Lambda}_0 \, \Omega \, F^*_{motion} + \hat{\mu}_0 + \hat{P}_0$$

$$F_{force} = \hat{\Lambda}_0 \, \bar{\Omega} \, F^*_{force} + \bar{\Omega} \, F_{desired}$$

## Unified Motion & Force Control

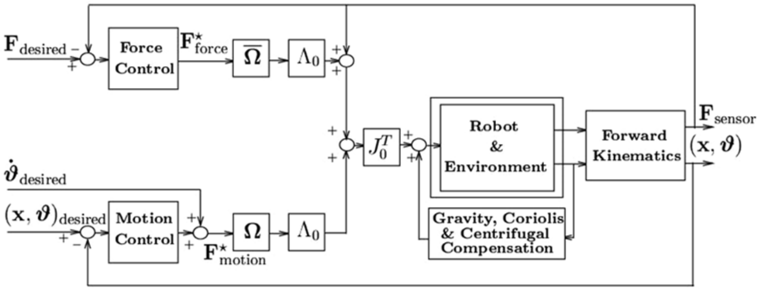


Two decoupled Subsystems

$$\Omega \, \dot{\vartheta} = \Omega \, F^*_{motion}$$

$$\bar{\Omega} \, \dot{\vartheta} = \bar{\Omega} F^*_{force}$$

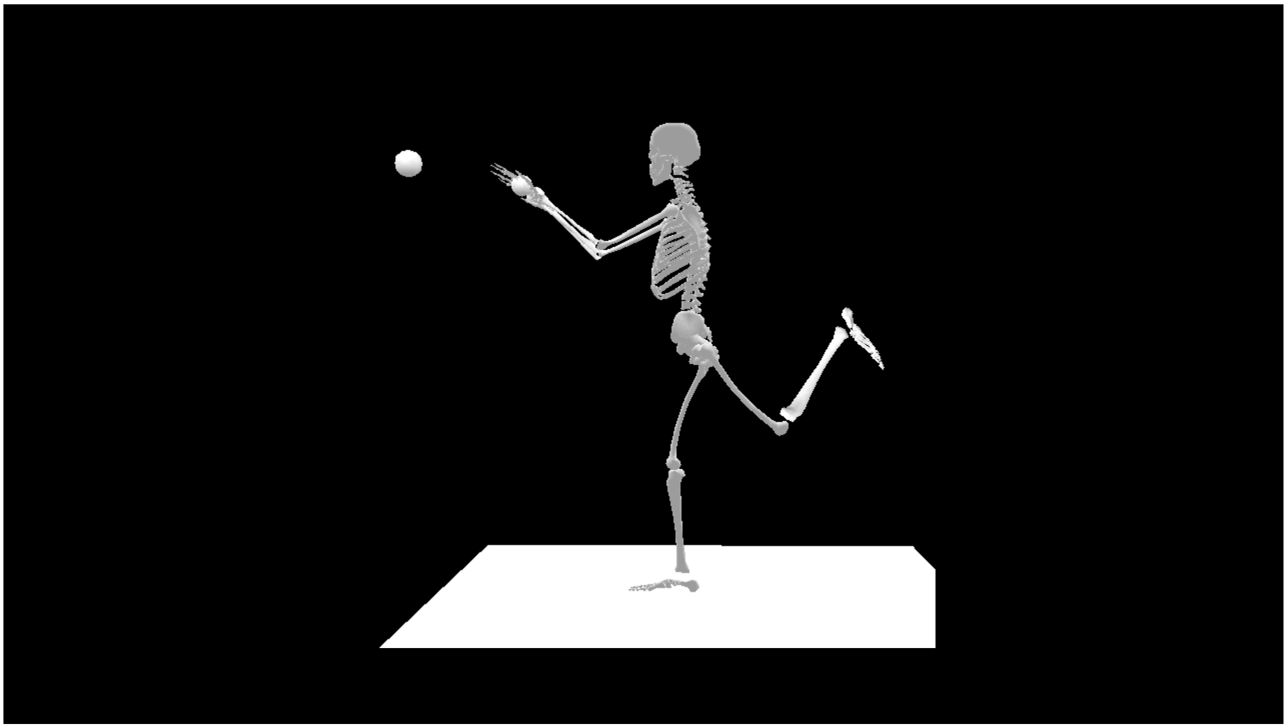
# Unified Motion & Force Control

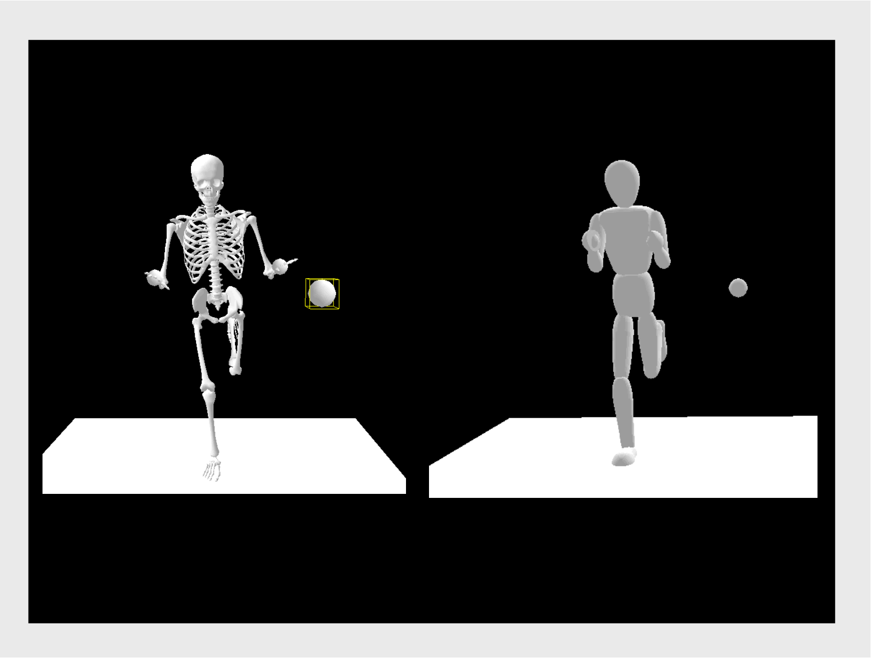


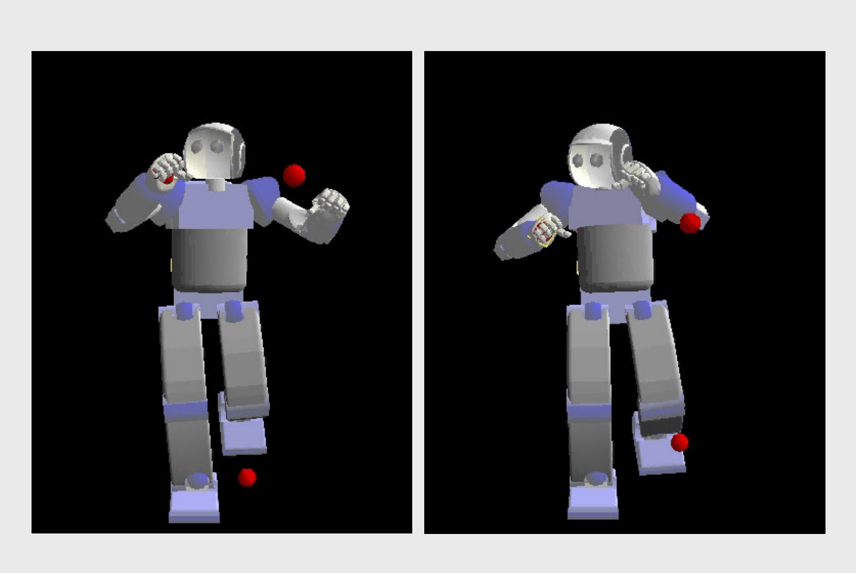
Two decoupled Subsystems

$$\Omega \dot{\vartheta} = \Omega F^{*}_{motion}$$

$$\bar{\Omega} \dot{\vartheta} = \bar{\Omega} F^{*}_{force}$$







ASIMO

# Equations of Motion

Joint Space

$$A(q)\ddot{q} + b(q,\dot{q}) + g(q) = \Gamma$$

Operational Space

$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F$$

Relationships

$$\Gamma = J^T F$$

$$(A\ddot{q} + b + g) = J^T(\Lambda\ddot{x} + \mu + p)$$

$$(A\ddot{q} + b) = J^T(\Lambda\ddot{x} + \mu)$$ Inertial forces

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# Non Redundancy

$$\begin{tikzcd}
\boxed{A\ddot{q} + b + g = \Gamma} \arrow[d, bend right, "J^{-T}"'] & \text{(joint dynamics)} \\
\boxed{\Lambda\ddot{x} + \mu + p = F} \arrow[u, bend right, "J^T"'] & \text{(Task dynamics)}
\end{tikzcd}$$

## Redundancy

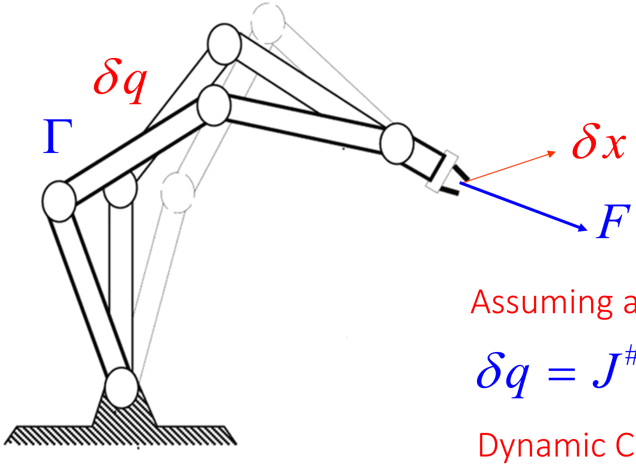
$$\begin{tikzcd} \boxed{A\ddot{q}+b+g=\Gamma} \arrow[d, shift right, "\text{projection}\ \bar{J}^T"'] & \text{(joint dynamics)} \\ \boxed{\Lambda\ddot{x}+\mu+p=F} \arrow[u, shift right, "J^T"'] & \text{(Task dynamics)} \end{tikzcd}$$

where

$$\bar{J} = A^{-1}J^T(JA^{-1}J^T)^{-1}$$

$\bar{J}$ : dynamically consistent generalized inverse where $J\bar{J} = I$

## Redundancy



Virtual Work

$$\Gamma^T\,\delta q \;\geq\; F^T\,\delta x$$

Assuming a Virtual Displacement

$$\delta q = J^{\#}\,\delta x + \left[I - J^{\#}J\right]\delta q_0$$

Dynamic Consistency:

$$\delta q = \bar{J}\,\delta x + \left[I - \bar{J}J\right]\delta q_0$$

# Dynamic Consistency

$\bar{J}(q)$ is the Dynamically Consistent Generalized Inverse

**Theorem (Consistency)**

$\bar{J}$ is unique and $\bar{J} = A^{-1} J^T \Lambda$

Non-redundant

$$\bar{J} = J^{-1}$$

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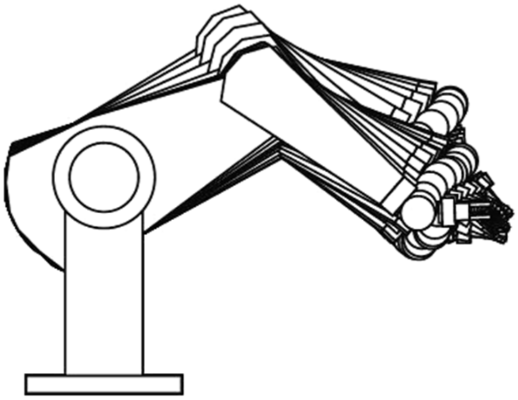
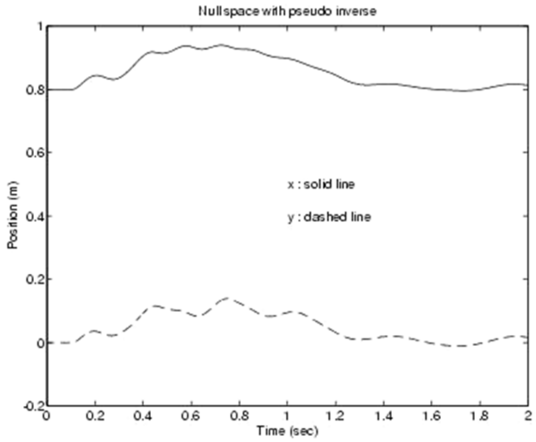
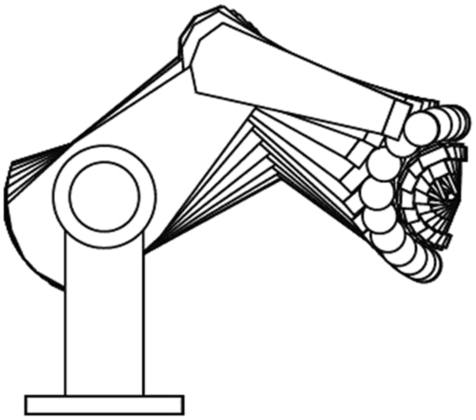
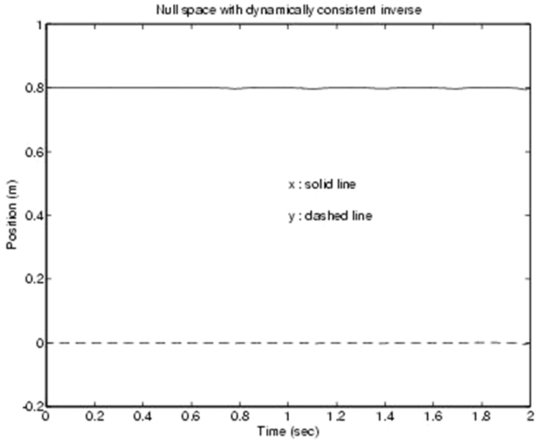
# Null Space with Pseudo Inverse





# Dynamic Consistency





# Velocity Force Duality

| | Velocity | Force |
|---|---|---|
| Non Red. | $\delta q = J^{-1}\delta x$ | $\Gamma = J^T F$ |
| Redundant | $\delta q = \bar{J}\delta x + \left[I - \bar{J}J\right]\delta q_0$ | $\Gamma = J^T F + \left[I - J^T \bar{J}^T\right]\Gamma_0$ |

## Task dynamics

$$\Lambda(q)\ddot{x} + \mu(q,\dot{q}) + p(q) = F$$

$$\Lambda = \left(JA^{-1}J^{T}\right)^{-1}$$

$$\mu(q,\dot{q}) = \bar{J}^{T}b(q,\dot{q}) - \Lambda(q)\dot{J}(q)\dot{q}$$

$$p(q) = \bar{J}^{T}g(q)$$

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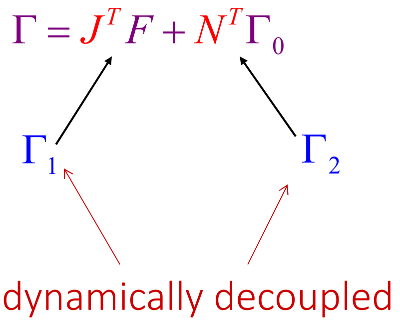
## Redundant Robot Control

Task Space: $J^{T}$

Null Space: $N^{T}$ where $N = I - \bar{J}J$

Robot Control



$$\Gamma = J^{T}F + N^{T}\Gamma_{0}$$

$\Gamma_{1}$ $\Gamma_{2}$

dynamically decoupled

## Redundant Robot Control

Task Space: $J^T$

Null Space: $N^T$ where $N = I - \bar{J}J$

Robot Control

$$\Gamma = J^T F + N^T \Gamma_0$$

$$\Gamma = J^T F + N^T (-\nabla V_{\text{Posture}})$$

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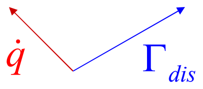
## Asymptotic Stability

$$\Gamma_{dis}^T \, \dot{q} \leq 0 \; ; \quad \text{for} \quad \dot{q} \neq 0$$



$$\Gamma_{dis} = -k_v J^T \Lambda \, \dot{x} = -k_v J^T \Lambda \, J \, \dot{q}$$

$$\dot{q}^T D(q) \, \dot{q} \geq 0 \; ; \qquad \dot{q} \neq 0$$

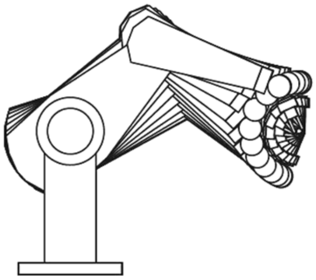
$$D(q) = k_v \left( J^T \Lambda \, J \right)$$

# Asymptotic Stability

$J^T \Lambda J$ : is a n x n matrix of rank $m_0$
it is Positive Semi-definite

The System is Stable, but not
asymptotically stable

$$\dot q^T D(q)\,\dot q = 0$$



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# Asymptotic Stability

$\Gamma^T_{dis}\,\dot q < 0$ ; for $\dot q \neq 0$

$$\Gamma_{dis} = -k_v J^T \Lambda J\,\dot q - k_v N^T A\,\dot q \qquad N = I - \bar J J$$

$$D = k_v[J^T \Lambda J + A - J^T \Lambda J\, A^{-1} A]$$

$$\Downarrow$$

$D(q) = k_v A$ Positive definite

$\dot q^T D(q)\,\dot q > 0$ for $\dot q \neq 0$

## Asymptotic Stability

$$\Gamma_{dis}^{T}\,\dot{q} < 0 \;; \qquad \text{for} \quad \dot{q} \neq 0$$

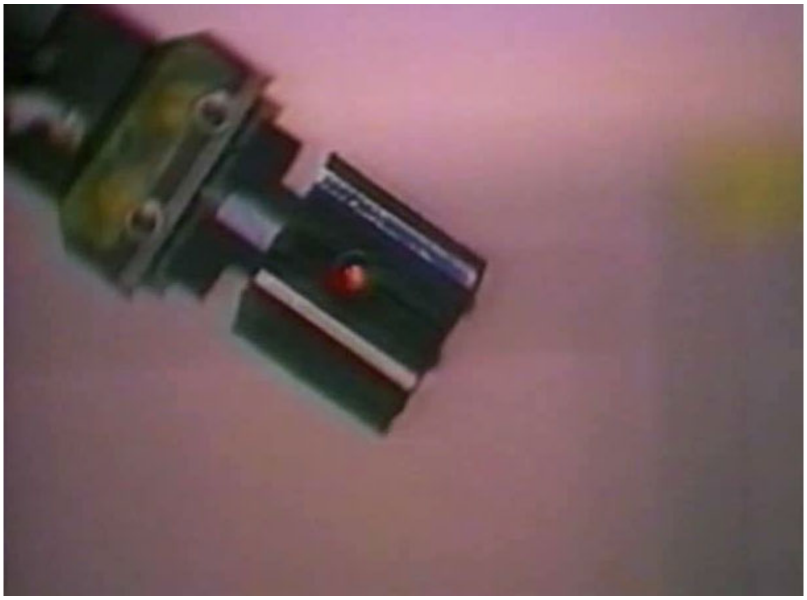
$$\Gamma_{dis} = -k_v J^T \Lambda J \dot{q} - k_v N^T A \dot{q} \qquad N = I - \bar{J}J$$

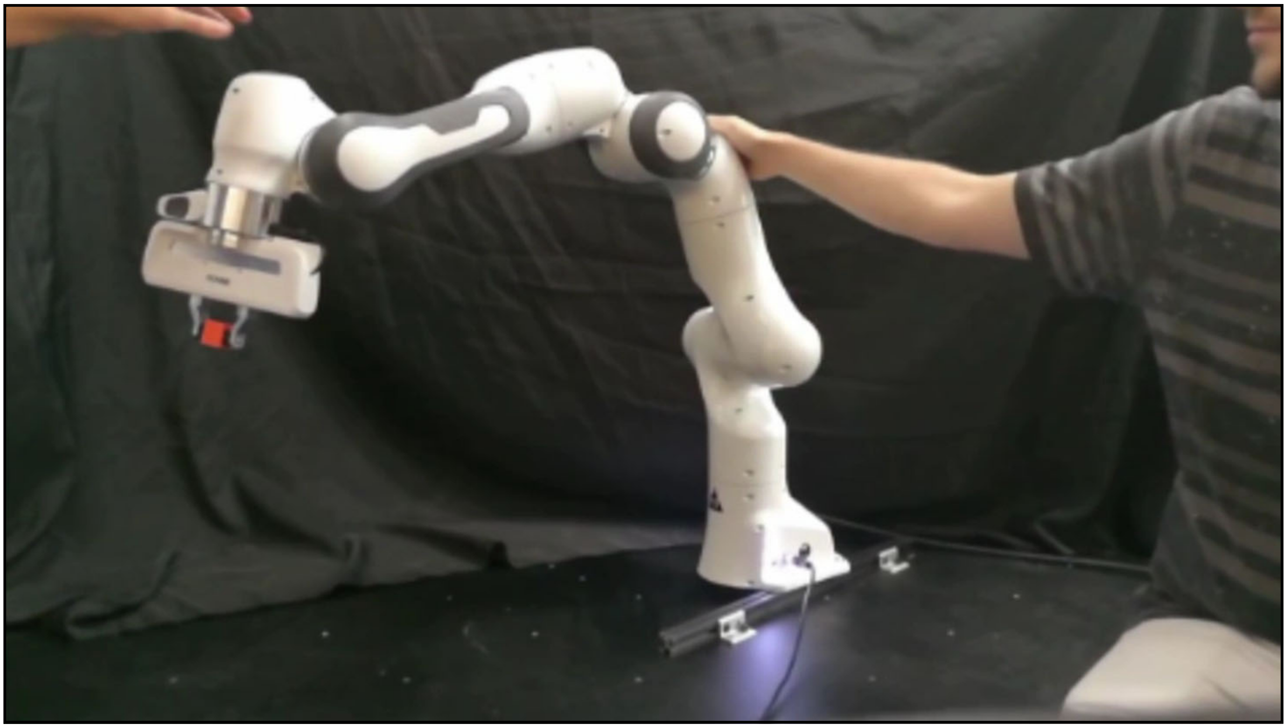
$$\Downarrow$$

$$D(q) = k_v A$$

Positive definite

$$\dot{q}^T D(q)\,\dot{q} > 0 \qquad \text{for} \quad \dot{q} \neq 0$$





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