Experimental Robotics

CS225A

Lecture 5

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Project Proposals
Experimental Robotics

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Spring2021 Projects

Sports
Environment
Cooking
Groceries
Human Robot Interaction
Medical
Service

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Menu

- Kinematics Dynamics
  - Jacobians Inverses
  - Task Representations
  - Equations of Motion
- Operational Space Control
  - Dynamic Models
  - Control Modalities
  - Compliance Force Control
- Redundant Robots
  - Posture Null Space
  - Dynamic Behavior
  - Whole-Body Control

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Project Proposals
Experimental Robotics

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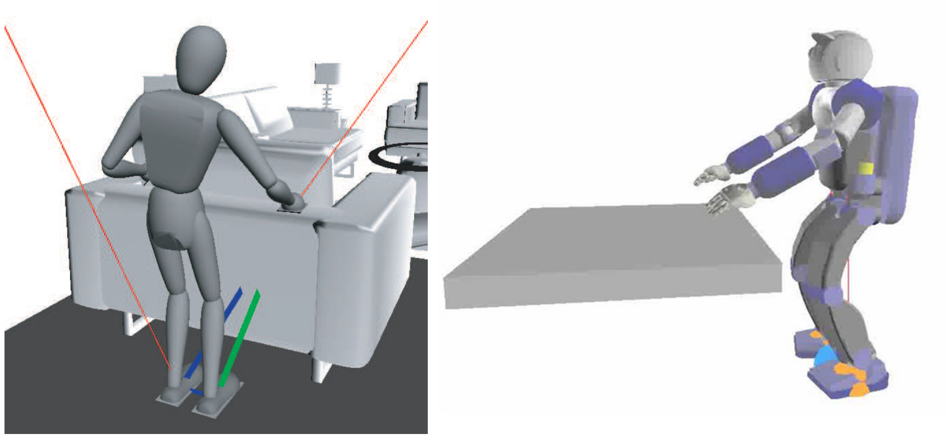
Operational Space Framework



.. motion in contact

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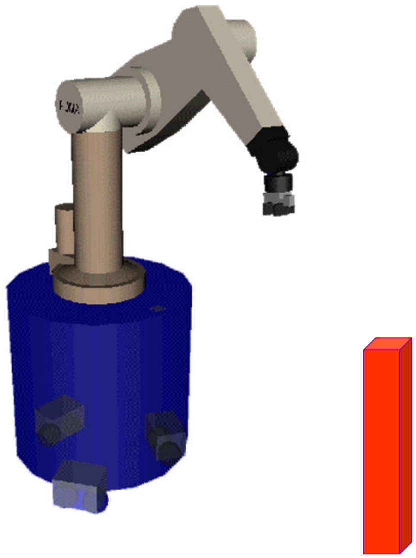
Joint Space Control



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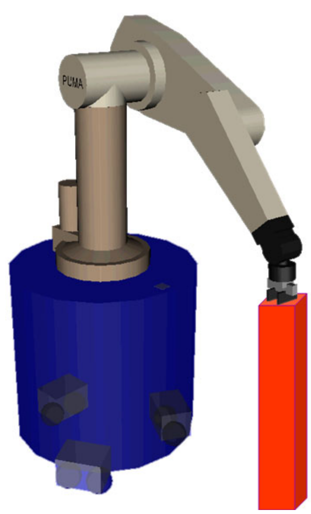
Joint Space Control



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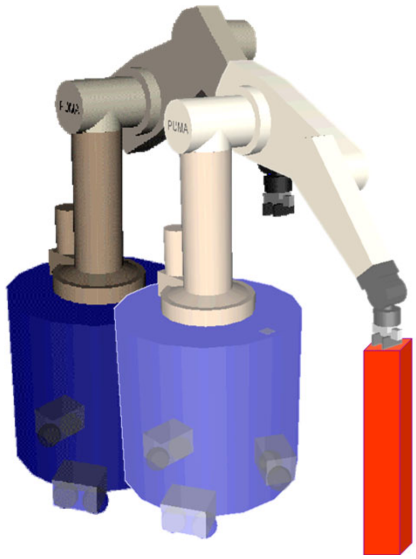
Joint Space Control



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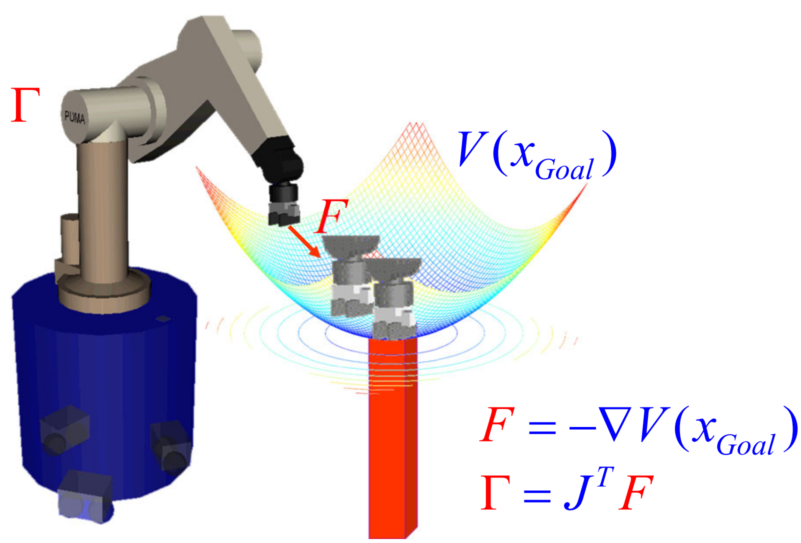
Task-Oriented Control



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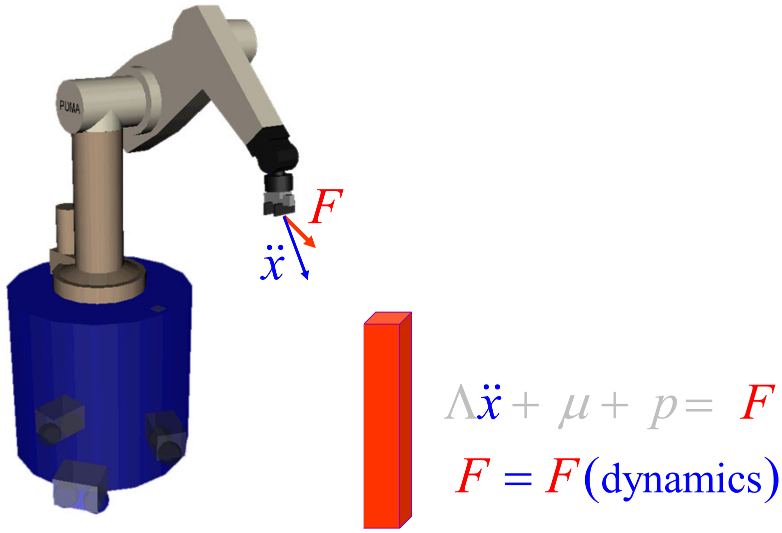
Task-Oriented Control



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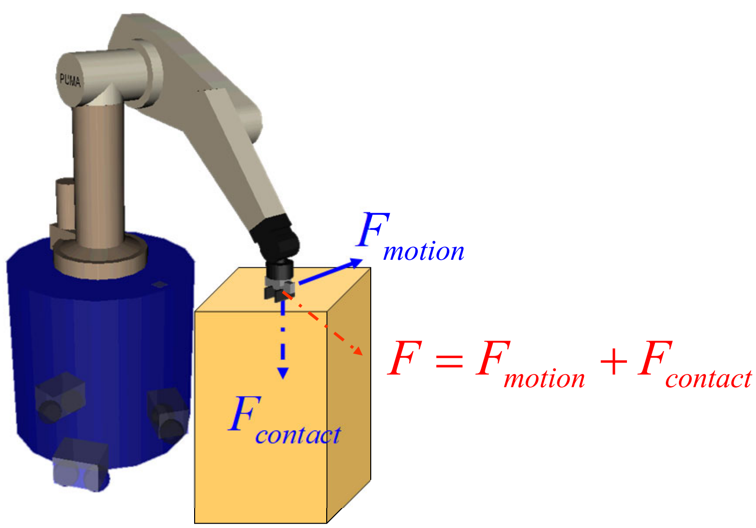
Unified Motion & Force Control



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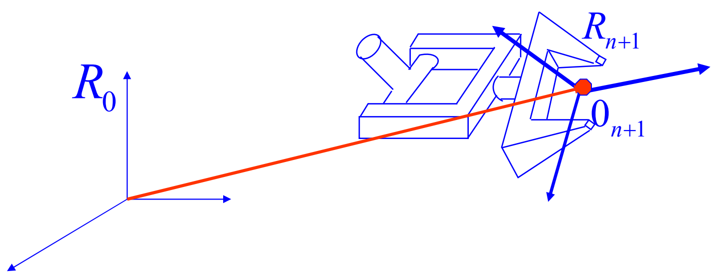
Effector Equations of Motion



Non-Redundant Manipulator ; $n = m_0$

$$x = \left(x_1\, x_2 \ldots x_{m_0}\right)^T$$

$$q = \left(q_1\, q_2 \ldots q_n\right)^T$$

$$x = G\left(q\right)$$

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Lagrange Equations in Operational Space

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F$$

Kinetic Energy

Lagrangian $L = T - V$ Potential Energy (Gravity)

Since $V = V_{Gravity}(q)$

$$\Rightarrow \underbrace{\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial T}{\partial x}}_{\text{Inertial forces}} + \frac{\partial V_{Gravity}}{\partial x} = F$$

Gravity vector

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Operational Space Dynamics

$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F$$

$x:$ End-Effector Position and Orientation

$\Lambda(x):$ End-Effector Kinetic Energy Matrix

$\mu(x,\dot{x}):$ End-Effector Centrifugal and Coriolis forces

$p(x):$ End-Effector Gravity forces

$F:$ End-Effector Generalized forces

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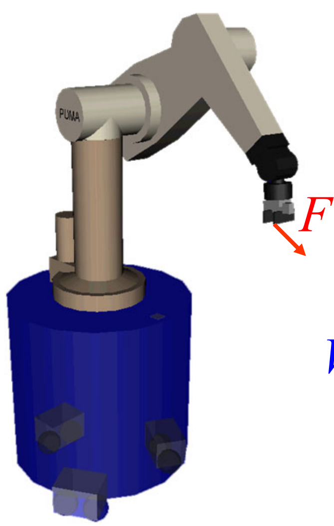
Operational Space Control



$$\Gamma = J^T(q)F$$

$$F = -\nabla V(x_{Goal})$$

$$V_{Goal} = \frac{1}{2}k_p\left(x - x_{Goal}\right)^T\left(x - x_{Goal}\right)$$

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Passive Systems

$$V_{Goal} = \frac{1}{2} k_p \left(x - x_{Goal} \right)^T \left(x - x_{Goal} \right)$$

System $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial \left(T - V_{Gravity} \right)}{\partial x} = F$

$$\Downarrow \ F = -\frac{\partial}{\partial X}\left(V_{Goal} - \hat{V}_{Gravity} \right)$$

Conservative Forces

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial \left(T - V_{Goal} \right)}{\partial x} = 0$$

Stable

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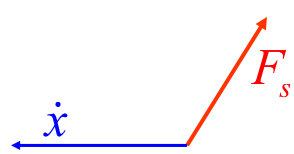
Asymptotic Stability

a system $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) - \frac{\partial \left(T - V_{Goal} \right)}{\partial x} = F_s$

is asymptotically stable if

$$\boxed{F_s^T \dot{x} < 0 \quad ; \ for \ \dot{x} \neq 0}$$



$$F_s = -k_v \dot{x} \ \rightarrow \ k_v > 0$$

Control

$$F = -k_p \left(x - x_{Goal} \right) - k_v \dot{x} + \hat{p}$$

$\hat{p}$ Estimate of p

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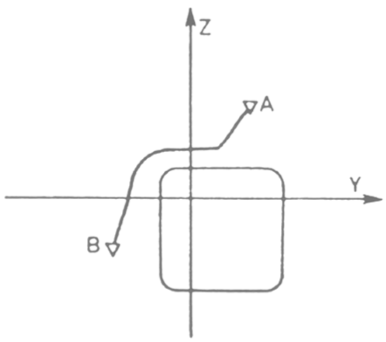

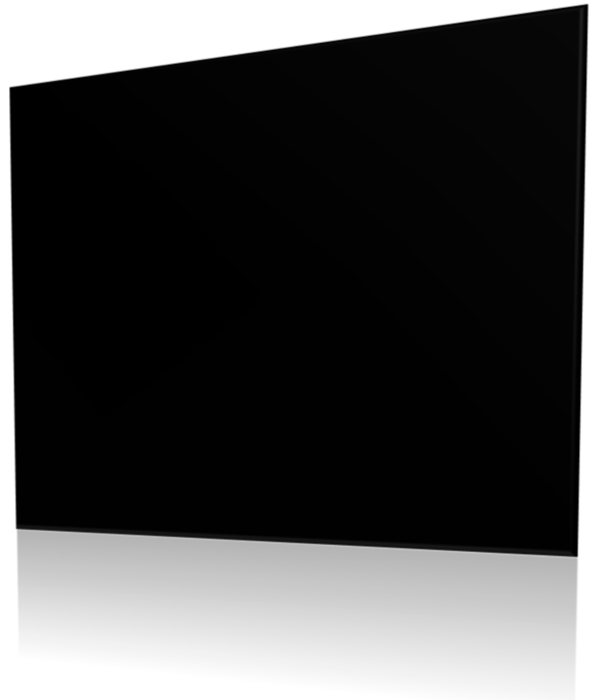

Artificial Potential Field

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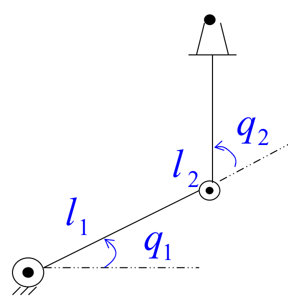
Example: 2-d.o.f arm

$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F$$

$$F = -k_p\left(x - x_g\right) - k_v\,\dot{x} + \hat{p}(x)$$



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$$\left(m_1^* c^2 12 + m_2\right)\ddot{x} + m_1^* \ddot{y} + \mu_1 = -k_p\left(x - x_g\right) - k_v \dot{x}$$

$$\left(m_1^* c^2 12 + m_2\right)\ddot{y} + m_1^* \ddot{x} + \mu_2 = -k_p\left(y - y_g\right) - k_v \dot{y}$$

Closed loop behavior

$$m_{11}(q)\ddot{x} + k_v \dot{x} + k_p\left(x - x_g\right) = -\left(m_1^* \ddot{y} + \mu_1\right)$$

$$m_{22}(q)\ddot{y} + k_v \dot{y} + k_p\left(y - y_g\right) = -\left(m_1^* \ddot{x} + \mu_2\right)$$

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Joint Space/Operational Space

Relationships

$$T_x(x, \dot{x}) \equiv T_q(q, \dot{q})$$

$$\frac{1}{2}\dot{x}^T \Lambda(X)\dot{x} = \frac{1}{2}\dot{q}^T A(q)\dot{q}$$

Using $\dot{x} = J(q)\dot{q}$

$$\frac{1}{2}\dot{q}^T J^T \Lambda\, J\, \dot{q} = \frac{1}{2}\dot{q}^T A\, \dot{q}$$

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Joint Space/Operational Space
Relationships

$$\Lambda(x) = J^{-T}(q)\,A(q)\,J^{-1}(q)$$

$$\mu(x,\dot{x}) = J^{-T}(q)\,b(q,\dot{q}) - \Lambda(q)\,h(q,\dot{q})$$

$$p(x) = J^{-T}(q)\,g(q)$$

where $h(q,\dot{q}) \doteq \dot{J}(q)\dot{q}$

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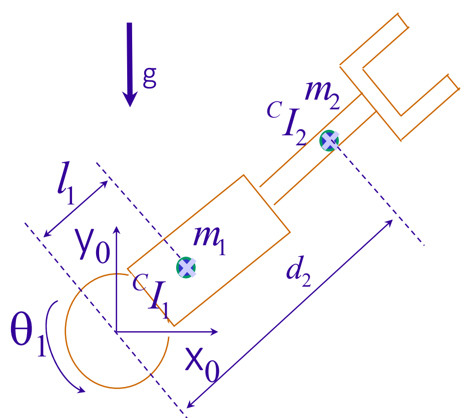
Example

$$q_2 = d_2$$

$$x = \begin{bmatrix} d_2 c1 \\ d_2 s1 \end{bmatrix}$$

$${}^0J = \begin{bmatrix} -d_2 s1 & c1 \\ d_2 c1 & s1 \end{bmatrix}$$



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$$ {}^{0}J = \begin{bmatrix} -d_2 s1 & c1 \\ d_2 c1 & s1 \end{bmatrix} $$

$$ {}^{0}J = \begin{pmatrix} c1 & -s1 \\ s1 & c1 \end{pmatrix} \overbrace{\begin{pmatrix} 0 & 1 \\ d_2 & 0 \end{pmatrix}}^{{}^{1}J} $$

$$ {}^{1}J^{-1} = \begin{pmatrix} 0 & 1/d_2 \\ 1 & 0 \end{pmatrix}; $$

$$ {}^{1}\Lambda = \begin{pmatrix} 0 & 1 \\ 1/d_2 & 0 \end{pmatrix} \begin{pmatrix} m_{11} & 0 \\ 0 & m_{22} \end{pmatrix} \begin{pmatrix} 0 & 1/d_2 \\ 1 & 0 \end{pmatrix} $$

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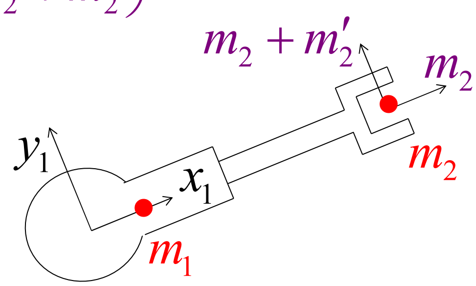
$$ {}^{1}\Lambda = \begin{pmatrix} m_2 & 0 \\ 0 & m_2 + m_2' \end{pmatrix} $$



$$ m_2' = \frac{I_{221} + I_{222} + m_1 l_1^{\,2}}{d_2^{\,2}} $$

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$$
{}^0\Lambda = \begin{pmatrix} c1 & -s1 \\ s1 & c1 \end{pmatrix} \begin{pmatrix} m_2 & 0 \\ 0 & m_2^{\ +} \end{pmatrix} \begin{pmatrix} c1 & s1 \\ -s1 & c1 \end{pmatrix}
$$

$$
m_2^{\ +} = m_2 + m_2'
$$

$$
{}^0\Lambda = \begin{pmatrix} m_2 + m_2' \, s1^2 & -m_2' s\, c1 \\ -m_2' s\, c1 & m_2 + m_2' \, c1^2 \end{pmatrix}
$$

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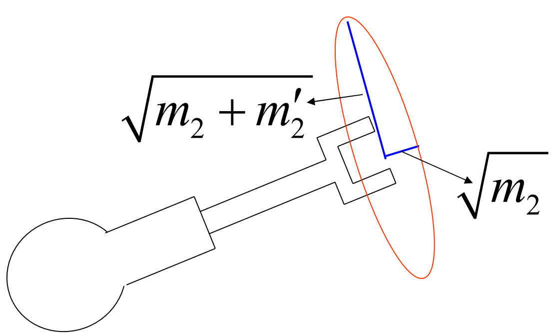

$$
{}^0\Lambda = \begin{pmatrix} m_2 + m_2' \, s1^2 & -m_2' s\, c1 \\ -m_2' s\, c1 & m_2 + m_2' \, c1^2 \end{pmatrix}
$$

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Nonlinear Dynamic Decoupling

Model

$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F$$

Control Structure

$$F = \hat{\Lambda}(x)F^{*} + \hat{\mu}(x,\dot{x}) + \hat{p}(x)$$

Decoupled System

$$I\,\ddot{x} = F^{*}$$

with $\Gamma = J^{T}F$

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Dynamic Decoupling

$$\Lambda(x)\ddot{x} + \mu(x,\dot{x}) + p(x) = F$$

$$F = \hat{\Lambda}F^{*} + \hat{\mu}(x,\dot{x}) + \hat{p}(x)$$

$$I_{m_0}\ddot{X} = \underbrace{\left(\Lambda^{-1}\hat{\Lambda}\right)}_{G(x)}F^{*} + \Lambda^{-1}\underbrace{\left(\mu - \hat{\mu}\right)}_{\tilde{\mu}_{(x,\dot{x})}} + \Lambda^{-1}\underbrace{\left(P - \hat{P}\right)}_{\tilde{P}(x)}$$

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Dynamic Decoupling: Closed Loop

$$I_{m_0}\ddot{x} = G(x)F^* + \varepsilon(x,\dot{x}) + d(t)$$

$$G(x) = \Lambda^{-1}\hat{\Lambda} \simeq I + \varepsilon_\Lambda$$

$$\varepsilon(x,\dot{x}) = \Lambda^{-1}\left(\tilde{\mu} + \tilde{P}\right)$$

$d(t)$: unmodeled disturbances

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Perfect Estimates

$$I_{m_0}\ddot{x} = F^*$$

F^* input of decoupled end-effector

Goal Position Control

$$F^* = -k_v\dot{x} - k_p\left(x - x_g\right)$$

Closed Loop

$$I_{m_0}\ddot{x} + k_v\dot{x} + k_p x = k_p x_g$$

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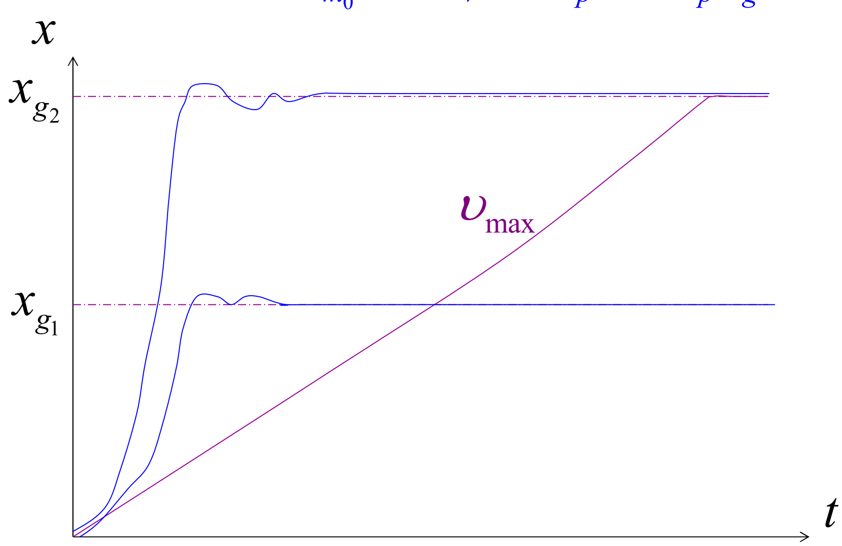
Closed Loop

$$I_{m_0}\ddot{x}+k_v\dot{x}+k_p x=k_p x_g$$



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PD Control

$$F^* = -k_v\dot{x}-k_p\left(x-x_g\right)$$

Velocity-Like Control

$$F^* = -k_v\left(\dot{x}-\frac{k_p}{k_v}\left(x_g-x\right)\right)$$

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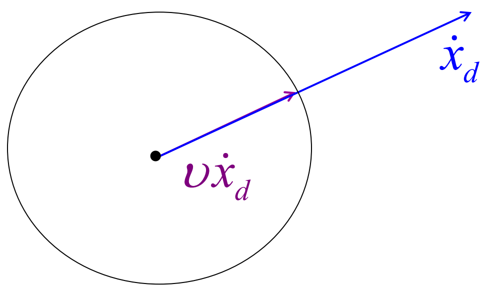

$$\dot{x}_d = \frac{k_p}{k_v}\left(x_g - x\right)$$

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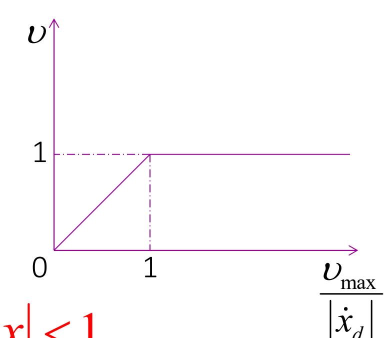
$$F^* = -k_v\left(\dot{x} - \underbrace{\frac{k_p}{k_v}\left(x_g - x\right)}_{\dot{x}_d}\right)$$

$$F^* = -k_v\left(\dot{x} - \upsilon \dot{x}_d\right)$$

with

$$\upsilon = sat\left(\frac{V_{\max}}{\left|\dot{x}_d\right|}\right)$$



$$sat\left(x\right) = \begin{cases} x & if \left|x\right| < 1 \\ sign(x) & if \left|x\right| > 1 \end{cases}$$

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Trajectory Tracking

Trajectory: $x_d\,,\ \dot{x}_d\,,\ \ddot{x}_d$

$$F^* = I_{m_0}\ddot{x}_d - k_v(\dot{x} - \dot{x}_d) - k_p(x - x_d)$$

$$(\ddot{x} - \ddot{x}_d) + k_v(\dot{x} - \dot{x}_d) + k_p(x - x_d)$$

or $\boxed{\ddot{\varepsilon}_X + k_v\dot{\varepsilon}_X + k_p\varepsilon_X = 0}$

with $\varepsilon_X = x - x_d$

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In joint space

$$\ddot{\varepsilon}_q + k_v\dot{\varepsilon}_q + k_p\varepsilon_q = 0$$

with $\varepsilon_q = q - q_d$

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