☆ THE ART OF COMPUTER PROGRAMMING ☆

☆☆☆ ERRATA TO VOLUME 4A (after 2021) ☆☆☆

This document is a transcript of the notes that I have been making in my personal copy of *The Art of Computer Programming*, Volume 4A (20th printing), since it was first printed in 2022. Previous errata are recorded in another file '`all4a-pre.ps`'.

Four levels of updates — "errors," "amendments," "plans," and "improvements" — appear, indicated by four different typographic conventions:

▸**Page 666** line 1 ——— 04 Jul 1776

Technical or typographical errors (aka bugs) are the most critical items, so they are flagged with a '▸' preceding the page number. The date on which I first was told about the bug is shown; this is the effective date on which I paid the finder's fee. The necessary corrections are indicated in a straightforward way. If, for example, the book says 'n' where it should have said '$n+1$', the change is shown thus:

$n \mathrel{\not\searrow\!\!\!\mapsto} n+1$

Page 666 line 2 ——— 14 Jul 1789

Amendments to the text appear in the same format as bugs, but without the '▸'. These are things I wish I had known about or thought of when I wrote the original text, so I added them later. The date is the date I drafted the new text.

Page 666 line 3 . . . . . 20 Nov 1917

Plans for the future represent a third kind of item. In such notes I sketched my intentions about things that I wasn't ready to flesh out further when I wrote them down. You can identify these items because they're written in slanted type, and preceded by a bunch of dots '. . . . . ' leading to the date on which I recorded the plan in my files.

Page 666 line 4 ——— 10 Jan 1938

The fourth and final category — indicated by page and line number in smaller, slanted type — consists of minor corrections or improvements that most readers don't want to know about, because they are so trivial. You wouldn't even be seeing these items if you hadn't specifically chosen to print the complete errata list in all its gory details. Are you sure you wanted to do that?

The ultimate, glorious, future editions of The Art of Computer Programming are works in progress. Please let me know of any improvements that you think I ought to make. Send your comments either by snail mail to D. E. Knuth, Computer Science, Gates Building 1B, Stanford University, Stanford CA 94305-9015, or by email to `taocp@cs.stanford.edu`. *(Use email for book suggestions only, please — all other correspondence is returned unread to the sender, or discarded, because I have no time to read ordinary email.) Although*

I'm working full time on Volume 4 these days, I will try to reply to all such messages within a year of receipt. Current news is posted on

`http://www-cs-faculty.stanford.edu/~knuth/taocp.html`

and updated regularly.

—Don Knuth, January 2011

If there happen to bee any iarre or dissonance, blame not the Printer,
who (I doe assure thee) through his great paines and diligence,
doth heere deliuer to thee a perfect and true Coppie.
If . . . there bee any fault by mee committed,
I desire the skilfull, eyther with courtesie to let the same bee concealed,
or in friendly sort to bee thereof admonished:
and at the next Impression he shall find error reformed:
remembring alwaies, that it is more easie to finde a fault then to amend it.

— WILLIAM BYRD, *Psalmes, Sonets, & songs of sadnes and pietie* (1588)

He did a lot of editing. That's how he liked to work.
Sometimes he even made alterations between printings.

— P. D. JAMES, *The Lighthouse* (2005)

☆☆ COMBINATORIAL ALGORITHMS, PART 1 ☆☆

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Last updated 9 October 2023

Most of these corrections have already been made in recent printings.

Page v lines −6 and −5 — 15 Jun 2022

because, in fact, ... by 1. ↯ because the size of a combinatorial problem can increase more than 100,000-fold when a parameter n increases by just 1.

Page ix new paragraph to follow line 4 — 09 Oct 2023

Many of the computer programs that I wrote while preparing this material are listed Internet on the following webpage:

`http://www-cs-faculty.stanford.edu/~knuth/programs.html`

In particular, you can download BDD14 and BDD15, which are the experimental toolkits that I played with when studying BDDs and ZDDs, respectively, in Section 7.1.4. Another example is a program called SIMPATH, for exercise 7.1.4–225.

▸**Page 60** line 2 — 28 Feb 2022

$l \leftarrow l + k$ ↯ $\texttt{START}(c) \leftarrow l$, $l \leftarrow l + k$

Page 60 new steps C3 and C5 — 30 May 2023

C3. [Done with loop?] If $c = \Lambda$, return to C2. Otherwise set $c' \leftarrow \texttt{PREV}(c)$, $q \leftarrow \texttt{CONCLUSION}(c)$, and go to C6 if $\texttt{TRUTH}(q) = 1$. Otherwise set $l \leftarrow \texttt{START}(c)$ and $k \leftarrow \texttt{COUNT}(c) - 1$.

C5. [Deduce $\texttt{CONCLUSION}(c)$.] Set $\texttt{TRUTH}(q) \leftarrow 1$, $S_s \leftarrow q$, $s \leftarrow s + 1$.

Page 112 new copy to follow line 3 — 15 Jul 2023

[An even better upper bound, replacing $3 \lg n$ by $\lg n + \lg\lg n$, has been proved by S. A. Lozhkin, *Moscow University Mathematics Bulletin* **77** (2022), 144–153.]

▸**Page 142** line 6 — 24 May 2022

64 substrings ↯ 64 truncated shifts

Page 299 line 1 — 24 May 2022

the sequence ↯ the digits change by ±1 and the sequence

▸**Page 334** line 21 — 20 Nov 2022

[*Inf. Proc. Letters* **17** (1983), 231–234] ↯ [*Inf. Proc. Letters* **17** (1983), 231–233]

Page 363 first line of step R4 — 24 May 2022

$c_j = c_{j-1} + 1$ ↯ $c_{j-1} = c_j - 1$

Page 379 second line of exercise 1 — 08 Oct 2023

$\infty \cdot n - t$ ↯ $\infty \cdot (n - t)$

Page 389 in exercise 110 — 24 May 2022

line 4: where one card ... choice of k: ↝ where a player holds $\{c_1, c_2, c_3, c_4\}$ and card c_5 is called the *starter*. The score is the sum of points computed as follows, for each subset S of C:
lines 11 and 12: If $s = 4$ ... same suit as ↝ If $S = \{c_1, c_2, c_3, c_4\}$ and all cards of S have the same suit, score $4 + [c_5$ has the same suit as
line 13: c_k ↝ c_5 (twice)
line −2: combinations and starter choices ↝ combinations

▸**Page 425** line 14 — 20 Nov 2022

Duke Math. J. **25** (1957), 29–43 ↝ *Duke Math. J.* **25** (1958), 29–43

Page 428 insert new algorithm in the middle of the page — 05 May 2023

line 3 before (53): A nice way ↝ An interesting way
line 5 after (53): $1/\varpi_n$. ↝ $1/\varpi_n$. We can generate M by using 3.4.1–(3); it tends to be approximately $n/\xi = e^{\xi}$ (see exercise 67).

now replace the following paragraph by new copy:

There's also an all-integer method, based on Peirce's triangle:

Algorithm P (*Uniformly random set partitions*). Given $N \geq 1$, this algorithm generates the blocks of a random partition of $\{1, \ldots, N\}$, using ideas explained in exercise 26. Three auxiliary stacks, S, T, and B, hold auxiliary data.

P1. [Initialize.] Set $n \leftarrow N$, $k \leftarrow 1$, $S \leftarrow \{1, \ldots, N\}$, and $T \leftarrow B \leftarrow \emptyset$.

P2. [Begin a block.] Pop $x \Leftarrow S$ and push $B \Leftarrow x$. (See 2.2.1–(2) and 2.2.1–(1).)

P3. [Done with block?] If $n = k$, output B as a block of the partition, and set $n \leftarrow n - 1$, $k \leftarrow 1$, $S \leftarrow T$, $T \leftarrow B \leftarrow \emptyset$; terminate if $n = 0$, else go to P2.

P4. [Add to block?] Pop $x \Leftarrow S$ and let U be a random integer, $0 \leq U < \varpi_{nk}$. If $U < \varpi_{(n-1)k}$, push $B \Leftarrow x$ and set $n \leftarrow n - 1$; otherwise push $T \Leftarrow x$ and set $k \leftarrow k + 1$. Return to P3. ∎

Page 462 new paragraph before the Spanning trees subsection — 01 Nov 2022

E. F. Harding [*Advances in Appl. Prob.* **3** (1971), 44–77] has shown how to generate the kth oriented *binary* tree with n vertices (see exercise 2.3.4.4–6).

Page 464 line 2 of Algorithm S becomes two lines — 15 Dec 2022

trees. ↝ trees in a revolving-door Gray code order.

▸**Page 507** line −5 — 13 Apr 2023

Zahlen durch ↝ *Zahlen, durch*

Page 514 replacement for the first answer 3 — 26 Jul 2022

3. See H. Poincaré, *Rendiconti del Circolo Matematico di Palermo* **18** (1904), 45–110; R. H. Bing, *Annals of Math.* (2) **68** (1958), 17–37; G. Perelman, arXiv:math/0211159 [math.DG] (2002), 39 pages; 0303109 and 0307245 [math.DG] (2003), 22+7 pages.

Page 516 last line of answer 9 — 09 Dec 2022

132 rows ↝ 132 options

▸**Page 521** in answer 47 — 10 Nov 2022

Geometrie der Gewebe (1937) ↝ *Geometrie der Gewebe* (1938)

▸**Page 522** in answer 54 — 10 Nov 2022

Darryl Francis, Philip Cohen, and A. Ross Eckler in *Word Ways* **19** (1976), 241; **20** (1977), 8. ↝ Darryl Francis and Philip Cohen in *Word Ways* **9** (1976), 241; **10** (1977), 46–47.

Page 672 line 3 of answer 241(d) — 01 Jun 2023

Once again, ↝ See (C6) and (C7). Once again,

▸**Page 679** in answer 10 — 10 Nov 2022

De Viribus Quantititatis ↝ *De Viribus Quantitatis*

Page 707 append a new sentence at end of answer 27 — 05 Mar 2023

F. Stappers has found `WRITE` + `WRITE` + `WRITE` + `WRITE` + `WRITE` + `WRITE` = `BOOKS`; `NOTES` + `SOUND` + `TONES` = `MUSIC`.

Page 717 new paragraph before answer 89(e) — 17 Sep 2022

[But when applied to (48), this procedure actually *increases* the value of M in (b).]

Page 724 lines 1 and 2 of answer 1 — 08 Oct 2023

by listing the distinct elements first ↝ first listing its distinct elements (in increasing order)

▸**Page 735** line 1 of answer 55(b) — 24 May 2022

$t > 0$ ↝ $0 < t \le 16$

Page 735 replacement for the final reference in answer 56 — 08 Sep 2022

A. Merino, O. Mička, and T. Mütze, *SICOMP* **51** (2022), 379–423.

Page 746 first line of answer 110 — 24 May 2022

A nice ↝ There are $5\binom{52}{5} = 52\binom{51}{4}$ cases, because each card of a 5-combination C can play the role of starter. A nice

Page 746 line −5 of answer 110 — 24 May 2022

A four-card flush is not allowed in the "crib." ↝ A four-point flush is not counted in the so-called "crib hand." Then the

▸**Page 751,** line 9 of answer 24 — 10 Nov 2022

4 (1953) ↝ **4** (1954)

Page 824 line −4 — 24 May 2022

saturated subtraction ↝ saturating subtraction

Page 829 new entry in the rightmost column — 05 May 2023

Algorithm 7.2.1.5P, 428.

▸**Page 831** in the Clique entry — 19 Jan 2023

$\sim G$ ↝ $\overline{G}$

Page 834 and following 01 Jan 2011

Miscellaneous changes to the existing index of Volume 4A are collected here, including corrections and amendments to the old entries as well as new entries that are occasioned by the new material. Thus, the lines of the full index that have changed serve also as an index to the present document. However, when a correction or amendment has caused an old index entry to be deleted, the deletion is usually not indicated.

ϵ (the empty string), 266, 272, 366.
al-Samaw'al (= as-Samaw'al), ibn Yaḥyā ibn Yahūda al-Maghribī (ابن يحيى بن يهودا المغربي السموءل, modern form السموأل), 493, 812.
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Menon, Vairelil Vishwanath (വൈരേലിൽ വിശ്വനാഥ മേനോൻ), 525.
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