Carrier Phase Techniques

Michael P. Vitus

August 30, 2005

1 Introduction

This paper provides an explanation of the methods as well as the work performed on the GPS project under Professor Ng during the summer of 2005. The reason behind the project is to develop a way for centimeter level accuracy using GPS when the helicopter performs aerobatic manuevers such as rolls.

2 Carrier Phase

The mathematical model for the carrier phase measurements in units of cycles

$$\phi\left(t\right) = \frac{1}{\lambda}\left[r\left(t\right) - I + T\right] + f\left(\delta t_u - \delta t^s\right) + N + \varepsilon_\phi \tag{1}$$

where ϕ is the partial carrier phase cycle measured by the receiver. The carrier wavelength is λ and f is the carrier frequency. The L1 carrier frequency is being used in which the wavelength is 0.1904 meters and the frequency is 1575.42 MHz. The geometric range between the receiver and the satellite is r, I is the ionospheric advance, and T is the tropospheric delay, which are all expressed in units of meters. The receiver and satellite clock biases are δt_u and δt^s, respectively, which are expressed in units of seconds. N is the integer ambiguity, which is the total number of full cycles between the receiver and the satellite. The integer ambiguity cannot be measured and has to be estimated, but the integers remain constant as long as the carrier tracking loop maintains lock.

3 Relative Positioning using Carrier Phase

In this model, there are two receivers that collect measurements from multiple satellites. The reference receiver is stationary and is used as a reference location in which all measurement estimates will be made from. The following notation that is used is that subscripts are used to refer to different receivers while superscripts are used to refer to different satellites. Carrier phase measurements from the kth satellite from the user receiver be represented as:

$$\phi_u^k\left(t\right) = \frac{1}{\lambda}\left[r_u^k\left(t\right) - I_u^k + T_u^k\right] + f\left(\delta t_u - \delta t^k\right) + N + \varepsilon_{\phi,u}^k \tag{2}$$

A similar expression can be written for the stationary reference receiver.

$$\phi_r^k\left(t\right) = \frac{1}{\lambda}\left[r_r^k\left(t\right) - I_r^k + T_r^k\right] + f\left(\delta t_r - \delta t^k\right) + N + \varepsilon_{\phi,r}^k \tag{3}$$

Now that the model of the carrier phase measurements for two receivers and the satellites has been established, it can be mathematically manipulated to eliminate some of unknown nuisance terms.

3.1 Single Difference

To form a single difference there needs to be two receivers and at least one satellite. The single difference is formed by subtracting the reference receiver's carrier phase equation from the user's.

$$\phi_{ur}^k(t) = \phi_u^k(t) - \phi_r^k(t) \tag{4}$$

$$\phi_{ur}^k(t) = \frac{1}{\lambda}\left[(r_u^k - r_r^k) - (I_u^k - I_r^k) + (T_u^k - T_r^k)\right] + f(\delta t_u - \delta t_r) + (N_u^k - N_r^k) + (\varepsilon_{\phi,u}^k - \varepsilon_{\phi,r}^k) \tag{5}$$

$$\phi_{ur}^k(t) = \frac{1}{\lambda}\left[r_{ur}^k - I_{ur}^k + T_{ur}^k\right] + f\delta t_{ur} + N_{ur}^k + \varepsilon_{\phi,ur}^k \tag{6}$$

In Eqn. (6), the satellite's clock bias term has cancelled out of the single differencing equation. In our experiments the distance between the user and reference receivers is "short", the ionospheric and tropospheric effects are small compared to the noise in the receiver as well as multipath, and therefore they will be dropped from the single difference equation. The final form of the single difference equation is shown in Eqn.(7).

$$\phi_{ur}^k(t) = \frac{1}{\lambda}r_{ur}^k + f\delta t_{ur} + N_{ur}^k + \varepsilon_{\phi,ur}^k \tag{7}$$

The geometry of the single difference is shown in Figure 1[1]. From this geometry, the relative position vector can be formed using the difference of the distance from the receivers to the satellite, r_{ur}^k, and the unit vector in the direction of the kth satellite, which is shown in Eqn. (8).

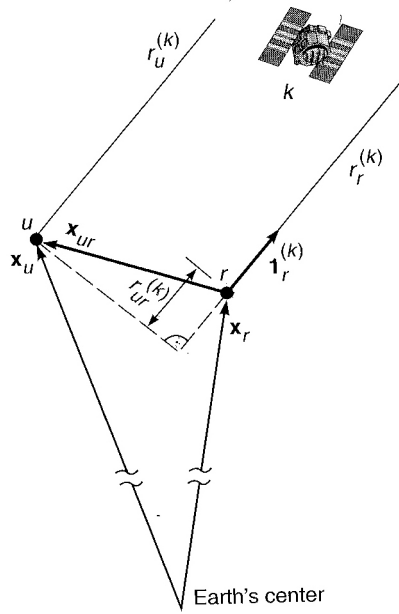$$r_{ur}^k = r_u^k - r_r^k = -\mathbf{1}_r^k \cdot \mathbf{x}_{ur} \tag{8}$$

Figure 1: Single Difference Geometry [1]

If there are K satellites, then single differences can be formed to each satellite, which is shown in Eqn.(9).

$$
\begin{bmatrix} \phi_{ur}^1 \\ \phi_{ur}^2 \\ \vdots \\ \phi_{ur}^K \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} (-\mathbf{1}_r^1)^T \\ (-\mathbf{1}_r^2)^T \\ \vdots \\ (-\mathbf{1}_r^K)^T \end{bmatrix} \mathbf{x}_{ur} + f\delta t_{ur} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + \begin{bmatrix} N_{ur}^1 \\ N_{ur}^2 \\ \vdots \\ N_{ur}^K \end{bmatrix} + \varepsilon_{\phi,ur} \tag{9}
$$

3.2 Double Difference

The single difference formula still has 2 unknown terms: the integer ambiguity, N, and the receiver's clock bias, δt_{ur}. To eliminate the receiver's clock bias term, two single difference equations can be subtracted to yield the double difference equation. To form the double difference equation, two satellites are needed.

$$
\begin{array}{rcl}
\phi_{ur}^{kl} &=& \left(\phi_u^k - \phi_r^k\right) - \left(\phi_u^l - \phi_r^l\right) \tag{10}\\
\phi_{ur}^{kl} &=& \phi_{ur}^k - \phi_{ur}^l \tag{11}\\
\phi_{ur}^{kl} &=& \frac{1}{\lambda}\left(r_{ur}^k - r_{ur}^l\right) + f\left(\delta t_{ur} - \delta t_{ur}\right) + \left(N_{ur}^k - N_{ur}^l\right) + \left(\varepsilon_{ur}^k - \varepsilon_{ur}^l\right) \tag{12}\\
\phi_{ur}^{kl} &=& \frac{1}{\lambda}r_{ur}^{kl} + N_{ur}^{kl} + \varepsilon_{ur}^{kl} \tag{13}
\end{array}
$$

As one can note in Eqn. (12), the receiver's clock bias terms cancel which reduces the number of unknowns to the 3 position coordinates and $(K-1)$ integer unknowns, where K is the number of satellites. The geometry of the double difference is shown in Figure 2[1]. From the geometry, the relative position vector can be formed using the difference of the distance from the receivers to the satellites, r_{ur}^k and r_{ur}^l, and the unit vectors in the direction of the kth and l^{th} satellites, which is shown in Eqn. (16).

$$
\begin{array}{rcl}
r_{ur}^{kl} &=& \left(r_u^k - r_r^k\right) - \left(r_u^l - r_r^l\right) \tag{14}\\
r_{ur}^{kl} &=& -\mathbf{1}_r^k \cdot \mathbf{x}_{ur} + \mathbf{1}_r^l \cdot \mathbf{x}_{ur} \tag{15}\\
r_{ur}^{kl} &=& -\left(\mathbf{1}_r^k - \mathbf{1}_r^l\right) \cdot \mathbf{x}_{ur} \tag{16}
\end{array}
$$

Substituting Eqn. (16) into Eqn. (13) yeilds the final form of the double differencing equation which is shown in Eqn. (17) in its matrix form for all K satellites. As one can note, if there are K satellites, then $(K-1)$ double difference equations can be formed.

$$
\begin{bmatrix} \phi_{ur}^{21} \\ \phi_{ur}^{31} \\ \vdots \\ \phi_{ur}^{K1} \end{bmatrix} = \frac{1}{\lambda} \begin{bmatrix} -(\mathbf{1}_r^2 - \mathbf{1}_r^1)^T \\ -(\mathbf{1}_r^3 - \mathbf{1}_r^1)^T \\ \vdots \\ -(\mathbf{1}_r^K - \mathbf{1}_r^1)^T \end{bmatrix} \mathbf{x}_{ur} + \begin{bmatrix} N_{ur}^{21} \\ N_{ur}^{31} \\ \vdots \\ N_{ur}^{K1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{\phi,ur}^{21} \\ \varepsilon_{\phi,ur}^{31} \\ \vdots \\ \varepsilon_{\phi,ur}^{K1} \end{bmatrix} \tag{17}
$$

4 Formulation

The double differencing technique will be used because it eliminates all of the unknowns except for the integer ambiguity terms. If the integer ambiguities are known, then there needs to be at least 4 satellites in view to solve for the three unknowns. Manipulating Eqn. (17), as well as dropping the error terms, yields Eqn. (18).

$$
\begin{bmatrix} -(\mathbf{1}_r^2 - \mathbf{1}_r^1)^T \\ -(\mathbf{1}_r^3 - \mathbf{1}_r^1)^T \\ \vdots \\ -(\mathbf{1}_r^K - \mathbf{1}_r^1)^T \end{bmatrix} \mathbf{x}_{ur} = \lambda \left(\begin{bmatrix} \phi_{ur}^{21} \\ \phi_{ur}^{31} \\ \vdots \\ \phi_{ur}^{K1} \end{bmatrix} - \begin{bmatrix} N_{ur}^{21} \\ N_{ur}^{31} \\ \vdots \\ N_{ur}^{K1} \end{bmatrix} \right) \tag{18}
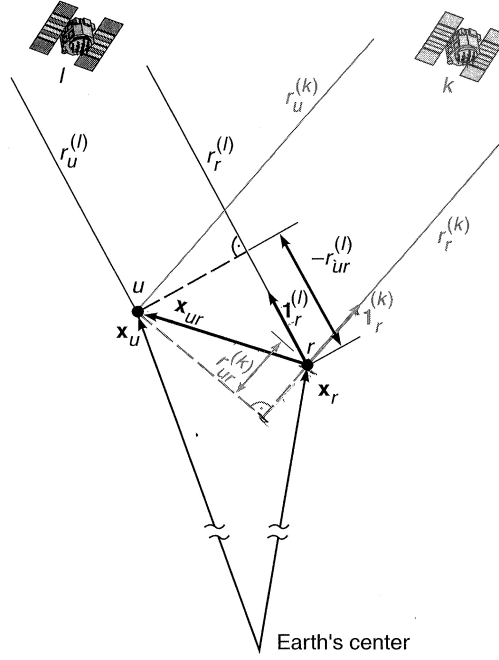$$

Figure 2: Double Difference Geometry [1]

where $\mathbf{1}^k = \begin{bmatrix} \cos E^k \sin Az^k & \cos E^k \cos Az^k & \sin E^k \end{bmatrix}^T$ in which E^k and Az^k are the elevation and azimuth angles of the kth satellite, respectively.

Using Eqn. (18), an estimate of the roaming receiver's position is obtained by taking the pseudoinverse of the left-hand side of the equation.

$$\mathbf{x}_{ur} = \lambda \begin{bmatrix} -(\mathbf{1}_r^2 - \mathbf{1}_r^1)^T \\ -(\mathbf{1}_r^3 - \mathbf{1}_r^1)^T \\ \vdots \\ -(\mathbf{1}_r^K - \mathbf{1}_r^1)^T \end{bmatrix}^{\dagger} \left(\begin{bmatrix} \phi_{ur}^{21} \\ \phi_{ur}^{31} \\ \vdots \\ \phi_{ur}^{K1} \end{bmatrix} - \begin{bmatrix} N_{ur}^{21} \\ N_{ur}^{31} \\ \vdots \\ N_{ur}^{K1} \end{bmatrix} \right) \tag{19}$$

where $(\ )^{\dagger}$ represents the pseudoinverse.

5   Results

5.1   7/26/05 - Test 3

The following is the procedure that was carried out during the experiment. The roaming receiver was placed at a known distance away from the reference receiver for a given amount of time and then moved.

1. 4cm apart for 20 seconds

2. 21.59cm apart for 20 seconds

3. 43.18cm apart for 20 seconds

4
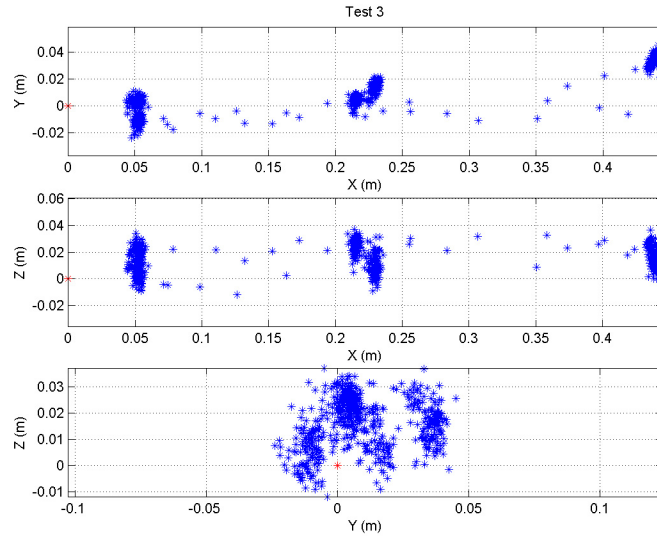
4. 21.59cm apart for 20 seconds

5. 4cm apart for 20 seconds

Figure 3: Test 3 - XYZ Plot

5.2   7/26/05 - Test 4 - this test is a repeat of test 3

The following is the procedure that was carried out during the experiment. The roaming receiver was placed at a known distance away from the reference receiver for a given amount of time and then moved.

1. 4cm apart for 20 seconds

2. 21.59cm apart for 20 seconds

3. 43.18cm apart for 20 seconds

4. 21.59cm apart for 20 seconds

5. 4cm apart for 20 seconds

5.3   7/26/05 - Test 5

This test was a trace of a box which is about 27 cm in width and 45 cm in length. While the box has those dimensions, we were not certain that when we traced the box that it hit each corner exactly. Also, there will be some discrepancy to the dimensions of the box because the center of the receiver's was not directly over the corners.
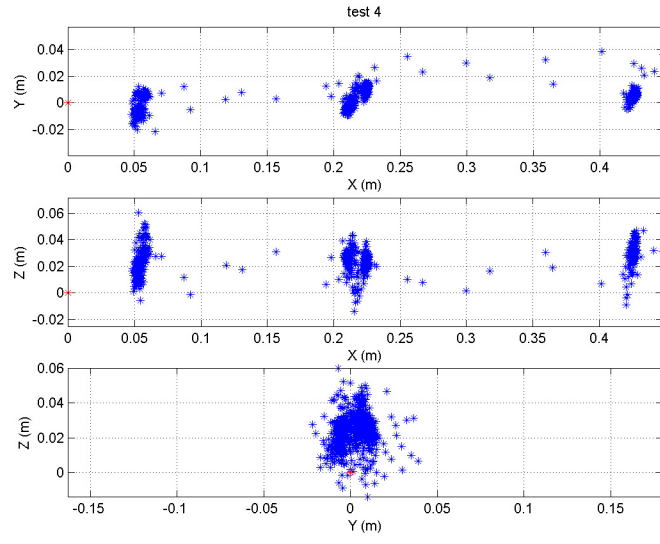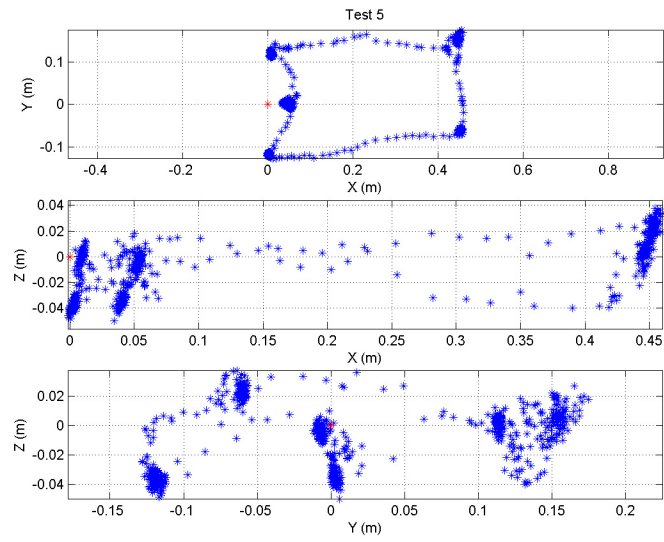
Figure 4: Test 4 - XYZ Plot

Figure 5: Test 5 - XYZ Plot

6

6 Cycle Slips

A cycle occurs when the receiver isn't able to keep track of the number of half-cycles that it receives from a certain satellite. If a cycle slip occurs for a satellite, then the measurements for that satellite during that epoch aren't valid and cannot be used. After the cycle slip, the measurements can be used for that satellite, but the integer ambiguity has to be restimated. The following is an algorithm for dealing with cycle slips and restimating the integer ambiguity.

1. Check for a cycle slip. (The Novatel SuperStar II tries to predict when a cycle slip occurs and increments a counter in message ID #23.)

2. If a cycle slip occurs, estimate $\mathbf{x}_{ur}$ without using that satellite's measurements.

3. If a cycle slip occured during the previous epoch but not during the current epoch, then estimate $\mathbf{x}_{ur}$ without using that satellite's measurements. Then use the estimate of $\mathbf{x}_{ur}$ to obtain a value for the integer ambiguity for that satellite. Reinitialize the ICP for the reference satellite as well as for the satellite that had the cycle slip. Use the integer ambiguity and the satellite to obtain a better estimate of $\mathbf{x}_{ur}$.

4. If no cycle slip occurs, estimate $\mathbf{x}_{ur}$ using all of the satellites.

7 Dilution Of Precision (DOP)

Dilution of Precision provides a method in which to characterize the error associated with the estimate. The covariance of the error depends on two factors: the covariance of the actual receiver and the satellite geometry, which is shown in Eqn. (20).

$$Cov \left[\begin{array}{c} \delta \mathbf{X} \\ \delta b \end{array} \right] = \sigma^2 \left(J^T J \right)^{-1} \tag{20}$$

where $J = \left[\begin{array}{cc} \left(-\mathbf{1}^1\right)^T & 1 \\ \left(-\mathbf{1}^2\right)^T & 1 \\ \vdots & \\ \left(-\mathbf{1}^K\right)^T & 1 \end{array} \right]$ and σ^2 is the covariance of the receiver's measurements.

Equation (21) relates the covariance matrix to the position dilution of precision, namely the East DOP, North DOP, Vertical DOP, and Time DOP.

$$Cov \left[\begin{array}{c} \delta \mathbf{X} \\ \delta b \end{array} \right] = \left[\begin{array}{cccc} EDOP^2 & \bullet & \bullet & \bullet \\ \bullet & NDOP^2 & \bullet & \bullet \\ \bullet & \bullet & VDOP^2 & \bullet \\ \bullet & \bullet & \bullet & TDOP^2 \end{array} \right] \tag{21}$$

The dilution of position can be used to determine to what precision the position estimate is to.

8 Resolving Integer Ambiguities

8.1 LAMBDA Method

To use the LAMBDA method, a float estimate of the position and integer ambiguities must first be solved for. Since the integer ambiguities are integers instead of floating point numbers, then the float solution obtain is not an exact solution and can be further refined. The ellipsoid formed from the covariance of

the integer ambiguities is extremely elongated in one direction and it would be inefficient to search over this space. Therefore, the LAMBDA method transforms the system into another in which the extremely elongated ellipse more resembles a circle, and now it is much more efficient to search over all of the possible solutions to the integer ambiguities. For a more in-depth look at the LAMBDA method please refer to articles [3], [4] and [5]

8.2 Code Measurements

Another formulation of the problem is to use the code as well as the carrier phase measurements at each epoch. The only pitfall of this approach is that to obtain correlation between the code and carrier phase measurements the *time slew* value is needed and is only outputted at 1 Hz on the Novatel SuperStar II. A brief summary of the method is to use only the measurements at the single epoch as well as a covariance of the integer ambiguities from the previous epoch. The reason that the covariance of the integer ambiguities from the previous epoch is used is because the integer ambiguities should be constant from one epoch to the next as long as the receivers have maintained phase lock on the carrier phase. Using the code and carrier phase measurements, the float solution of the position and integer ambiguity unknowns and then the LAMBDA method is used with the float solution of the integer ambiguity unknowns to find the actual integer solution of the ambiguities. This method of using the code and carrier phase at 1 Hz will speed up the initialization of the integer ambiguities, but it will not work during the operation of the helicopter because a faster position update is needed. Therefore, after the initialization of the integers, a switch to straight double differencing would be sufficient with the integer ambiguities known. For a more detailed explanation as well as the formulation of the equations, see reference [2].

8.3 Multiple Epochs

For a single epoch, there are 3 position unknowns and $(K-1)$ integer ambiguities, where K is the number of satellites, and there are $(K-1)$ equations. Therefore, there are 3 more unknowns than equations for a single epoch and not all of the unknowns can be solved for uniquely. If two epochs are used, then there are 6 position unknowns, if they aren't assumed to be the same, and $(K-1)$ integer ambiguities, and there are $2(K-1)$ equations. If there are 7 or more satellites in view, then there will be at least as many equations as unknowns. Equation (22) shows the relation between the number of epochs and satellites needed if it is assumed that the position isn't the same from epoch to epoch.

$$
\begin{aligned}
i(K-1) &\geq 3i + (K-1) \\
i(K-4) &\geq (K-1) \\
i &\geq \frac{K-1}{K-4}
\end{aligned}
\tag{22}
$$

where i is the number of epochs and K is the number of satellites. If it was assumed that the position was the same during each epoch, the receiver was stationary, then the constraint would change to Eqn. (23).

$$
\begin{aligned}
i(K-1) &\geq 3 + (K-1) \\
i(K-1) &\geq (K+2) \\
i &\geq \frac{K+2}{K-1}
\end{aligned}
\tag{23}
$$

If the problem is formulated with the position changing from epoch to epoch, then the governing equation is (24).

$$\lambda \begin{bmatrix} \phi_{ur}^1 \\ \phi_{ur}^2 \\ \vdots \\ \phi_{ur}^i \end{bmatrix} = \begin{bmatrix} G_{AB}^1 & 0 & \cdots & 0 & \lambda I \\ 0 & \ddots & \ddots & \vdots & \lambda I \\ \vdots & \ddots & \ddots & 0 & \lambda I \\ 0 & \cdots & 0 & G_{AB}^i & \lambda I \end{bmatrix} \begin{bmatrix} \mathbf{x}_{ur}^1 \\ \vdots \\ \mathbf{x}_{ur}^i \\ N_{ur}^{21} \\ \vdots \\ N_{ur}^{K1} \end{bmatrix} \tag{24}$$

where G_{AB}^i is $\begin{bmatrix} -(\mathbf{1}_A^2 - \mathbf{1}_A^1)^T \\ -(\mathbf{1}_A^3 - \mathbf{1}_A^1)^T \\ \vdots \\ -(\mathbf{1}_A^{K_{AB}} - \mathbf{1}_A^1)^T \end{bmatrix}$, ϕ_{ur}^i is $\begin{bmatrix} \phi_{ur}^{21} \\ \phi_{ur}^{31} \\ \vdots \\ \phi_{ur}^{K1} \end{bmatrix}$, λI is of size $(K-1) \times (K-1)$, and i represents each

epoch. If the matrix on the righthand side is examined, it will reveal that there are 3 linearly dependent columns if the matrix G_{AB}^i does not change from epoch to epoch. This is shown in the example below.

$$\begin{bmatrix} 3 & 4 & 5 & 0 & 0 & 0 & 1 & 0 & 0 \\ 6 & 7 & 8 & 0 & 0 & 0 & 0 & 1 & 0 \\ 9 & 10 & 11 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 & 4 & 5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 6 & 7 & 8 & 0 & 1 & 0 \\ 0 & 0 & 0 & 9 & 10 & 11 & 0 & 0 & 1 \end{bmatrix}$$

Add columns 1 and 4 together to obtain: add together 3 times column 7, 6 times column 8, and 9 times

column 10 to yield. $\begin{bmatrix} 3 \\ 6 \\ 9 \\ 3 \\ 6 \\ 9 \end{bmatrix}$. Add together 3 times column 7, 6 times column 8, and 9 times column 10 to

yield. $\begin{bmatrix} 3 \\ 6 \\ 9 \\ 3 \\ 6 \\ 9 \end{bmatrix}$. This can be done for the 2nd and 5th columns as well as the 3rd and 6th columns. Therefore,

there are 3 linearly dependent columns in this example and depicts what happens if the satellite geometry doesn't change from one epoch to the next. This causes a problem because when performing the least squares solution for the unknowns using the pseudo-inverse, it doesn't estimate the correct values.

9 Multiple Receivers

Figure 6 shows the geometry of the multiple receivers. Receiver A is the base station receiver, receiver B and C are the roaming receivers, and D is a point in space whose position is desired.
When using one roaming receiver the double difference relative positioning relations is shown in Eqn. (25).

$$\lambda \phi_{AB} = G_{AB} \vec{X}_{AB} + \lambda N_{AB} \tag{25}$$

where G_{AB} is $\begin{bmatrix} -(\mathbf{1}_A^2 - \mathbf{1}_A^1)^T \\ -(\mathbf{1}_A^3 - \mathbf{1}_A^1)^T \\ \vdots \\ -(\mathbf{1}_A^{K_{AB}} - \mathbf{1}_A^1)^T \end{bmatrix}$.
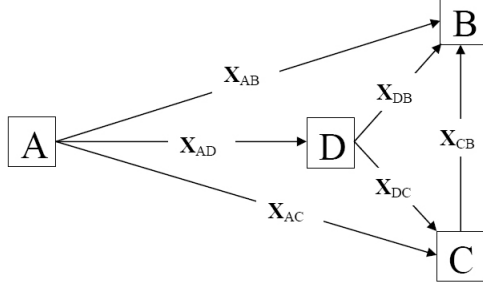
Figure 6: Multiple Receiver Geometry

Adding another receiver and forming two double differences for each receiver B and C yields Eqn. (26). The constraint of knowing the displacment and orientation of the two receivers is also imposed.

$$
\begin{bmatrix} \lambda \phi_{AB} \\ \lambda \phi_{AC} \\ \vec{X}_{CB} \end{bmatrix} = \begin{bmatrix} G_{AB} & & 0 & & & \\ 0 & & G_{AC} & & & \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \vec{X}_{AB} \\ \vec{X}_{AC} \end{bmatrix} + \lambda \begin{bmatrix} N_{AB} \\ N_{AC} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{26}
$$

where ϕ_{AB} is a $(K_{AB} - 1) \times 1$ vector in which K_{AB} is the number of satellies that are common between receiver A and B, ϕ_{AC} is an $(K_{AC} - 1) \times 1$ vector in which K_{AC} is the number of satellites that are common between receiver A and C, and $\vec{X}_{CB}$ is a 3×1 vector. This yields a total possible number of equations of $(K_{AB} - 1) + (K_{AC} - 1) + 3$. The vectors $\vec{X}_{AB}$ and $\vec{X}_{AC}$ are also 3×1 and therefore, there are 6 unknowns which correspond to the receivers' positions. Therefore, the number of satellites that are neccessary between receiver B and C are $K_{AB} + K_{AC} \geq 5$.

Equation (27) forms the double difference for the two roaming receivers with the constraint of knowing the displacement vector $\vec{X}_{CB}$ in a different manner in which it uses the fact that $\vec{X}_{AC} = \vec{X}_{AB} - \vec{X}_{CB}$.

$$
\lambda \begin{bmatrix} \phi_{AB} \\ \phi_{AC} \end{bmatrix} = \begin{bmatrix} G_{AB} \\ G_{AC} \end{bmatrix} \vec{X}_{AB} + \lambda \begin{bmatrix} N_{AB} \\ N_{AC} \end{bmatrix} + \begin{bmatrix} 0 \\ -G_{AC} \end{bmatrix} \vec{X}_{CB} \tag{27}
$$

where the constraint of knowing the vector between the two roaming receivers has been imposed by augmenting the measurements from the additional roaming receiver.

The estimated location, $\vec{X}_{AB}$, can be transformed into any point D in space as long as the vector from D to B is known. The transformation is shown in Eqn. (30).

$$
\vec{X}_{AD} = \vec{X}_{AB} - \vec{X}_{DB} \tag{28}
$$

$$
\lambda \begin{bmatrix} \phi_{AB} \\ \phi_{AC} \end{bmatrix} = \begin{bmatrix} G_{AB} \\ G_{AC} \end{bmatrix} \left(\vec{X}_{AD} + \vec{X}_{DB} \right) + \lambda \begin{bmatrix} N_{AB} \\ N_{AC} \end{bmatrix} + \begin{bmatrix} 0 \\ -G_{AC} \end{bmatrix} \vec{X}_{CB} \tag{29}
$$

$$
\lambda \begin{bmatrix} \phi_{AB} \\ \phi_{AC} \end{bmatrix} = \begin{bmatrix} G_{AB} \\ G_{AC} \end{bmatrix} \vec{X}_{AD} + \lambda \begin{bmatrix} N_{AB} \\ N_{AC} \end{bmatrix} + \begin{bmatrix} 0 & G_{AB} \\ -G_{AC} & G_{AC} \end{bmatrix} \begin{bmatrix} \vec{X}_{CB} \\ \vec{X}_{DB} \end{bmatrix} \tag{30}
$$

In order to test the validity of the proposed solution, several tests were run using the single roaming receiver's data for receiver B and C, as well as declaring $\vec{X}_{CB} = \mathbf{0}$ and changing the distance $\vec{X}_{DB}$. Figure 7 shows

the two data sets, from Test 4 on 7/26/05, that were produced by the single receiver estimation as well as the double receiver estimation. In this example, $\vec{X}_{DB} = [-2 \ -3 \ -1]^T \ m.$ and the multiple receiver data shown in the figure has been shifted back into its original position by adding $-\vec{X}_{DB}$ to each data point. As one could note, the two data sets are identical.
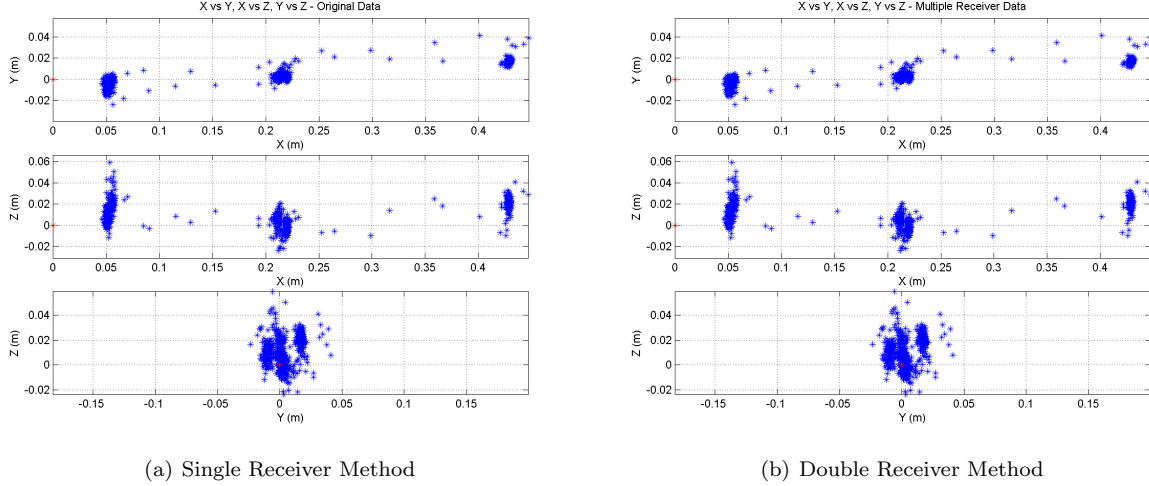

(a) Single Receiver Method   (b) Double Receiver Method

Figure 7: Single Receiver and Double Receiver Comparison

Experimentation with the erorr in the vector between receiver B and C was also evaluated. Figure 8, shows the two data sets, from Test 4 on 7/26/05, that were produced by the single receiver estimation as well as the double receiver estimation with $\vec{X}_{true_{CB}} = \mathbf{0} \, (m)$ and $\vec{X}_{experimental_{CB}} = [0.03 \ 0.01 \ 0.02]^T \, (m)$. In this example, $\vec{X}_{DB} = [-2 \ -3 \ -1]^T \, m.$ and the multiple receiver data shown in the figure has been shifted back into its original position by adding $-\vec{X}_{DB}$ to each data point. The error for each data point between the single and double receiver estimation is $Error = X_{single} - X_{double} = [-0.015 \ -0.005 \ -0.01]^T \, (m)$. As one can note, the error is exactly half the error in the vector $\vec{X}_{CB}$.

Physical experiments were conducted on 8/25/05 in which two receivers, B ans C, were placed 23 cm. apart and moved to a distance 23 cm away from the base station. Figure 9 shows the results from each individual receiver as well as the multiple receiver estimation were the middle of the two receivers was estimated. In this evaluation, all of the satellites from both roaming receivers were used, and one can note that the multiple receiver estimation is just the average of the two single receiver estimates.

Figure 10 shows the results from each individual receiver as well as the multiple receiver estimation were the middle of the two receivers was estimated. In this evaluation, only 4 of the satellites were used for each receiver with 1 common satellite between receiver B and C. Even though the data for receiver C alone is very bad, when used with the data from receiver B, the multiple receiver position estimate becomes much more accurate.

10   Novatel SuperStar II

Sometimes the base station receiver will start recording its data 0.2 seconds before the roaming receivers. Need to make sure that the times agree.
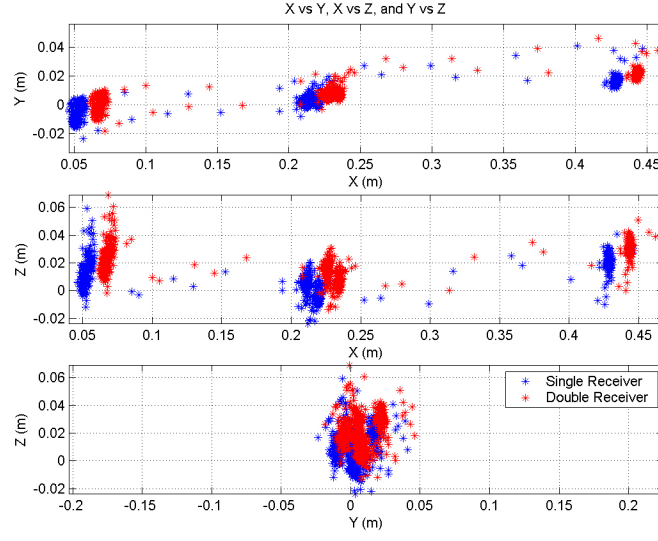
Figure 8: Comparison between Single and Double receiver estimation with a error in $\vec{X}_{CB} = [0.03\ 0.01\ 0.02]^T\ (m)$
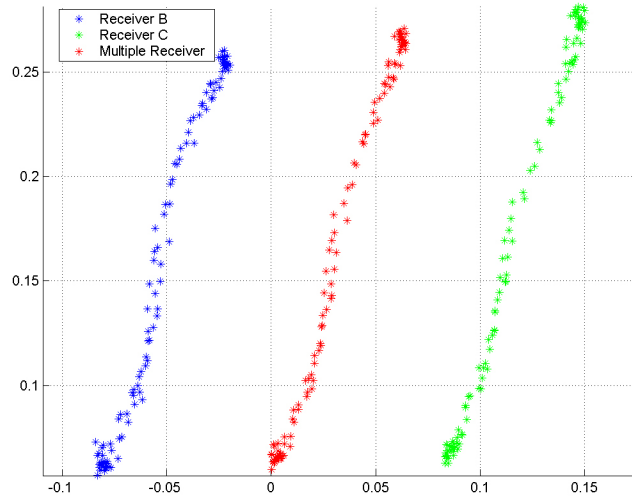

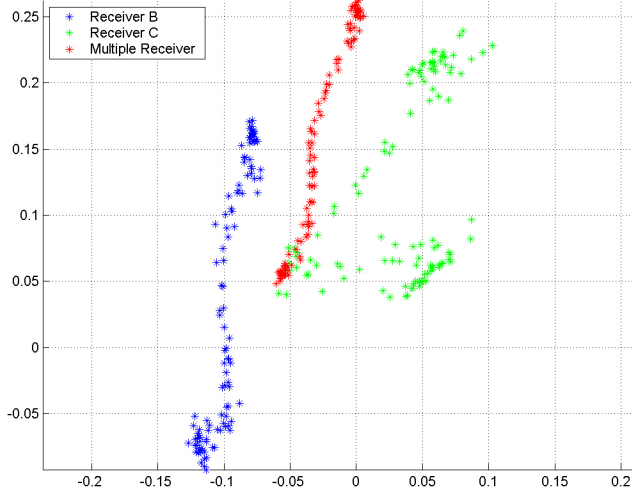Figure 9: Comparison between Single and Double receiver estimation

Figure 10: Comparison between Single and Double receiver estimation

11 Continuing Work

The following items are things that still need to be implemented or more fully developed.

1. Cycle slip on reference satellite. Currently, the cycle slip detection works for all of the satellites that aren't the reference satellite for the double differencing. If the reference satellite has a cycle slip, then you will need to pick the next highest satellite to use as the reference satellite as well as transform the integer ambiguities. To do this efficiently, remember that $N_{AB}^{21} = \left(N_B^2 - N_A^2\right) - \left(N_B^1 - N_A^1\right)$ so if you need to replace satellite 1 then all you need to do is substract another double differencing of the integer ambiguities. For example, $N_{AB}^{32} = N_{AB}^{31} - N_{AB}^{21} = \left(N_B^3 - N_A^3\right) - \left(N_B^1 - N_A^1\right) - \left[\left(N_B^2 - N_A^2\right) - \left(N_B^1 - N_A^1\right)\right] = \left(N_B^3 - N_A^3\right) - \left(N_B^2 - N_A^2\right)$. Also remember that the integers stored in N_{AB}^{32} are the initial integer ambiguities which remain the same and then ϕ term holds the change in the integer ambiguities. When a cycle slip on the reference satellite occurs, it is necessary to use the initial integer ambiguities as well as the change in integers to find the new integer ambiguities for a different reference satellite. If a cycle slip occurs on the reference satellite, then use the previous epoch's data to calculate the new integers as well as resetting the ICP_{start} variables.

$$
\begin{aligned}
N_{AB}^{32} = \ & N_{AB}^{31} - N_{AB}^{21} - \\
& \left[\left\{\left(ICP_B^3 - ICP_{start_B}^3\right) - \left(ICP_A^3 - ICP_{start_A}^3\right)\right\} - \left\{\left(ICP_B^1 - ICP_{start_B}^1\right) - \left(ICP_A^1 - ICP_{start_A}^1\right)\right\}\right] + \\
& \left[\left\{\left(ICP_B^2 - ICP_{start_B}^2\right) - \left(ICP_A^2 - ICP_{start_A}^2\right)\right\} - \left\{\left(ICP_B^1 - ICP_{start_B}^1\right) - \left(ICP_A^1 - ICP_{start_A}^1\right)\right\}\right]
\end{aligned}
\tag{31}
$$

2. Develop and implement either the carrier or code and carrier phase with LAMBDA method for resolving the initial integer ambiguities.

3. Write the multiple receiver code for the rotating case when receivers come into and out of view. Jiyun said that 15 degrees elevation of the receivers is good enough to avoid multipath issues.

References

[1] Misra P. and Enge P., *Global Positioning System: Signals, Measurements, and Performance.* Ganga-Jamuna Press, Massachusetts, 2001.

[2] Mohiuddin S. and Psiaki M., *Satellite Relative Navigation Using Carrier-Phase Differential GPS with Integer Ambiguities.* AIAA Guidance, Navigation, and Control Conference, San Francisco, 2005.

[3] de Jonge P. and Tiberius C. *The LAMBDA method for integer ambiguity estimation: implementation aspects.* Publications of the Delft Geodetic Computing Centre, 1996.

[4] Teunissen P.J.G. *Least-Squares Estimation of the Integer GPS Ambiguities.*

[5] Teunissen P.J.G., de Jonge P. and Tiberius C. *A New Way to Fix Carrier-Phase Ambiguities.* GPS World, April 1995.