ILP provides probabilistic upper and lower bounds that improve over time and are often tighter than variational methods.

Discrete Integrals (e.g., expectations, partition function, quadrature)

- We are given:
  - A set of 2^n items
  - Non-negative weights w
  - Goal: compute total weight

- Compactly specified weight function (e.g., graphical model)

Generally intractable (e.g., 100 dimensions, sum over 2^{100} ~ 10^{30} items)

Connections with coding theory

Integer Linear Programming for MAP inference subject to parity constraints

Formulate the NP-hard optimization max w(x) subject to A x = b (mod 2) as an integer linear program

- Effective strategy for decoding low-density parity check codes
- Compact encoding for parity constraints A x = b (mod 2) [Yannakakis, 91]
- Upper and lower bounds

1) LP relaxations provide polynomial time (probabilistic) upper bounds on the partition function
2) Branch and bound will eventually find an optimal integer solution (lower bound matches upper bound) Provably within a constant factor of the true partition function [ICML-13]

Inducing sparsity to improve the relaxations

Problems with sparse A x = b are empirically easier to solve (similar to LDPC codes)

1) Reduce A x = b to row-echelon form using Gauss-Jordan elimination
2) Generate sparse matrices A. Still provides probabilistic lower bounds (but no upper bounds)

Experimental results – Partition function via LP relaxations

ILP provides probabilistic upper and lower bounds that improve over time and are often tighter than variational methods.