Logics of Rational Interaction

Decisions, Games and Logic Workshop

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Introduction and Motivation

We are interested in reasoning about rational agents interacting in *social* situations.

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- Philosophy (social philosophy, epistemology)
- Game Theory
- Social Choice Theory
- AI (multiagent systems)

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What is a rational agent?

- maximize expected utility (instrumentally rational)
- react to observations
- revise beliefs when learning a *surprising* piece of information
- understand higher-order information
- plans for the future
- ????

J. van Benthem. *Rational Animals: What is 'KRA'?*. invited lecture Malaga ESSLLI Summer School 2006.

Introduction and Motivation

We are interested in reasoning about rational agents interacting in *social* situations.

There is a jungle of formal systems!

- logics of informational attitudes (knowledge, beliefs, certainty)
- logics of action & agency
- temporal logics/dynamic logics
- logics of motivational attitudes (preferences, intentions)

(*Not to mention various game-theoretic/social choice models and logical languages for reasoning about them*)

Introduction and Motivation

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There is a jungle of formal systems!

- How do we compare different logical systems studying the same phenomena?
- How *complex* is it to reason about rational agents?
- (How) should we *merge* the various logical systems?
- What do the logical frameworks contribute to the discussion on rational agency?

s

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Plan

- General comments about logics of rational agency
- Navigating the jungle of formal systems
- Modeling the dynamics of information in social situations
- Summary and conclusions

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- Logics *of* rational agents in social situations.

  vs.

  Logics *about* rational agents in social situations.

Navigating the jungle of formal systems (by example)

1. Background: logics of informational attitudes

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3. From temporal to strategy logics

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2. Actions and agency
3. From temporal to strategy logics
4. General issues

Single-Agent Epistemic Logic

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$K(P \to Q)$: "Ann knows that P implies Q"

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LP: "P is an epistemic possibility"

KLP: "Ann knows that she thinks P is possible"

Example

Suppose there are three cards: 1, 2 and 3.

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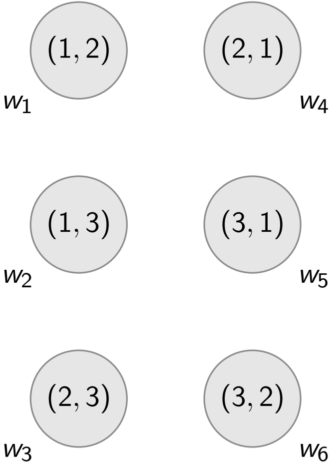

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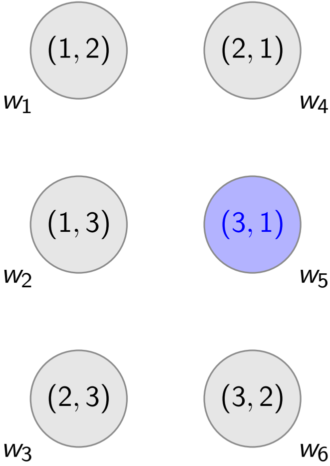
Ann is dealt one of the cards, one of the cards is placed face down on the table and the third card is put back in the deck.

Ann receives card 3 and card 1 is put on the table



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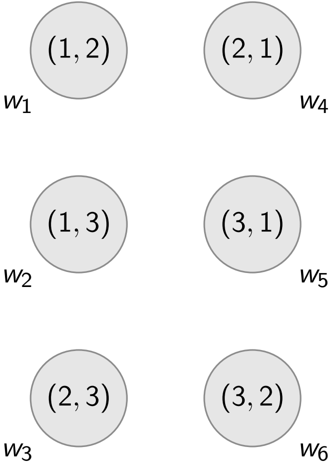

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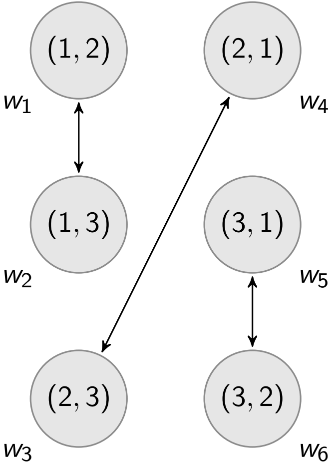

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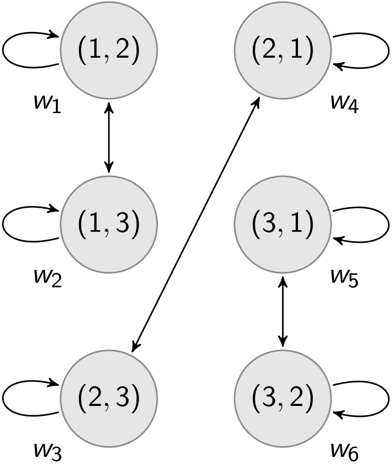

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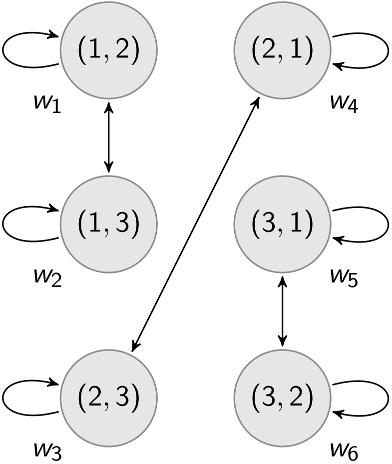
Ann is dealt one of the cards, one of the cards is placed face down on the table and the third card is put back in the deck.

Suppose H_i is intended to mean "Ann has card i"

T_i is intended to mean "card i is on the table"

Eg., $V(H_1) = \{w_1, w_2\}$



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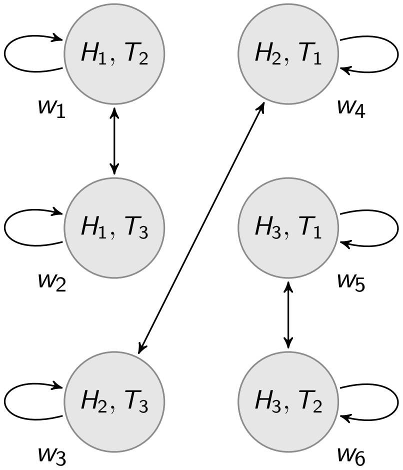

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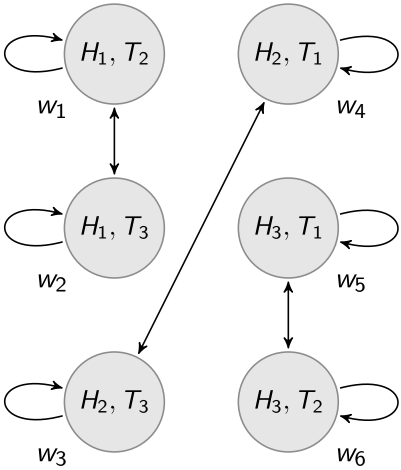

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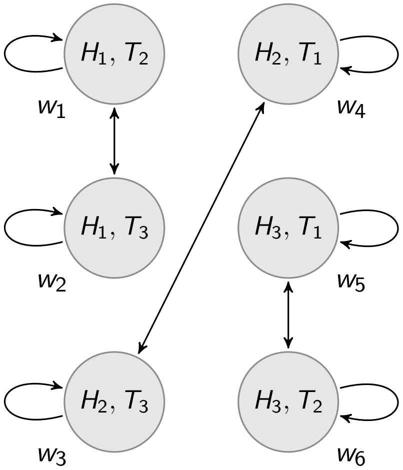

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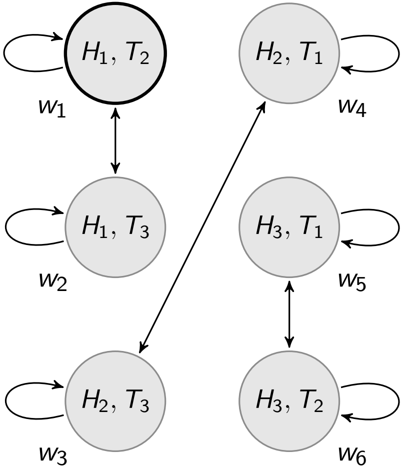

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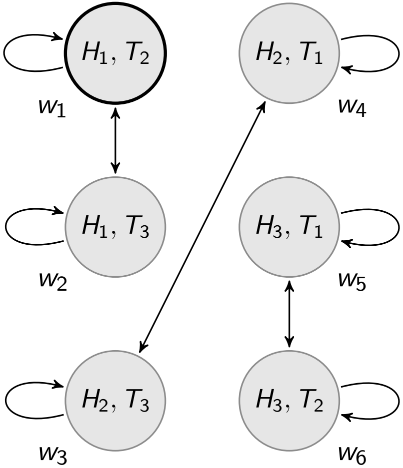

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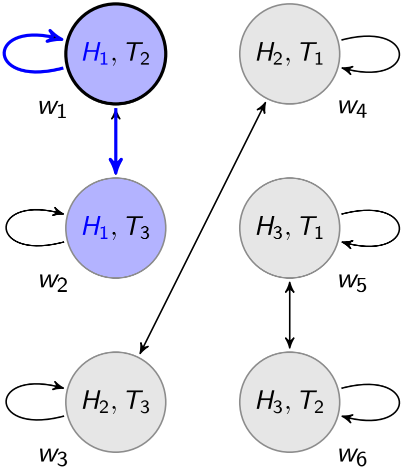

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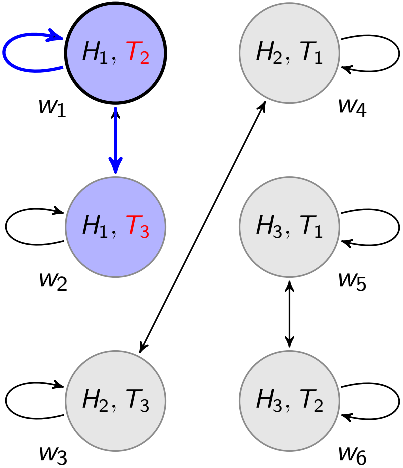

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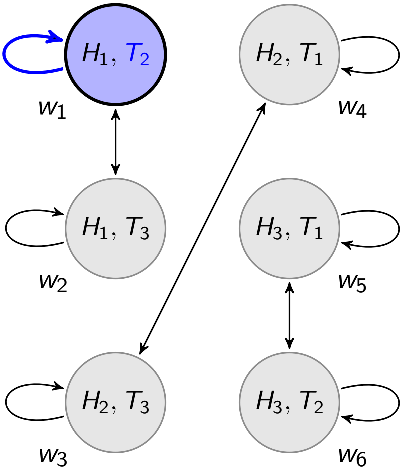
$\mathcal{M}, w_1 \models LT_2$



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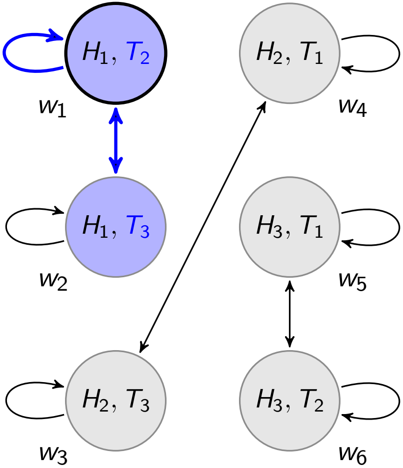
$\mathcal{M}, w_1 \models K(T_2 \vee T_3)$



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- $K_A K_B \varphi$: "Ann knows that Bob knows φ"
- $K_A(K_B\varphi \vee K_B\neg\varphi)$: "Ann knows that Bob knows whether φ
- $\neg K_B K_A K_B(\varphi)$: "Bob does not know that Ann knows that Bob knows that φ"

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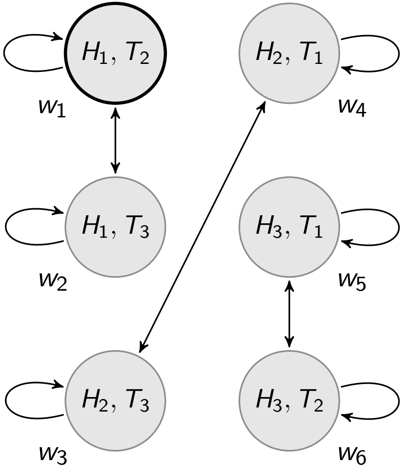

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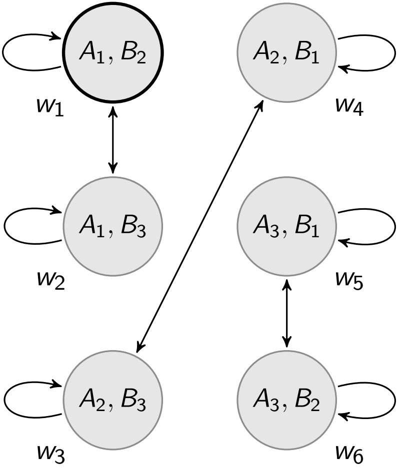

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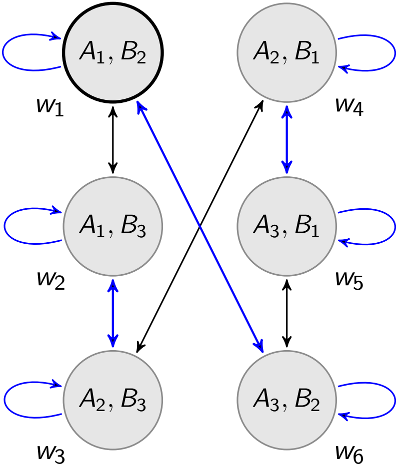

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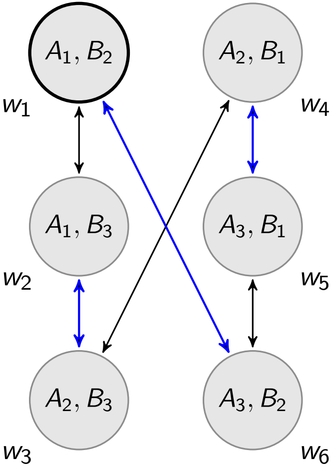

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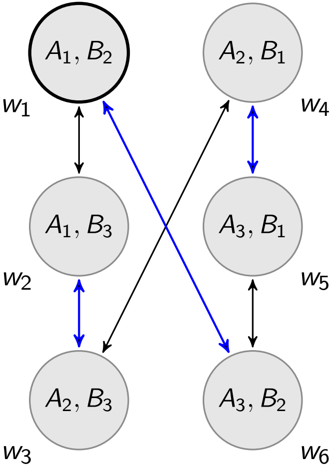

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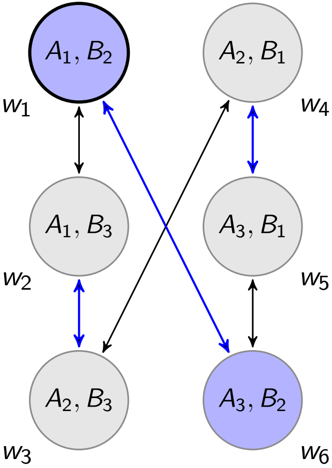

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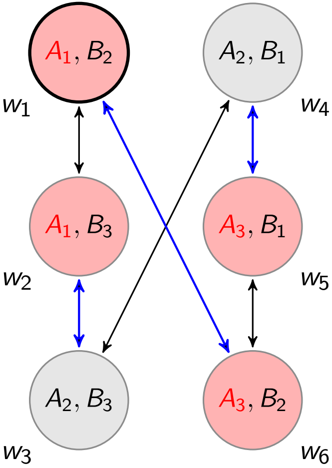

Actions and Ability

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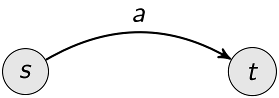

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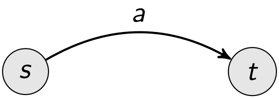

Propositional Dynamic Logic

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid [\alpha]\varphi$$

$$\alpha := a \mid \alpha;\alpha \mid \alpha \cup \alpha \mid \alpha^* \mid \varphi?$$

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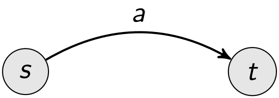

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where p is a propositional variable and a is an atomic action.

The formula $[\alpha]\varphi$ means "after executing action α, φ is true".

Actions and Ability

An early approach to interpret PDL as logic of actions was put forward by Krister Segerberg.

Segerberg adds an "agency" program to the PDL language δA where A is a formula.

K. Segerberg. *Bringing it about*. JPL, 1989.

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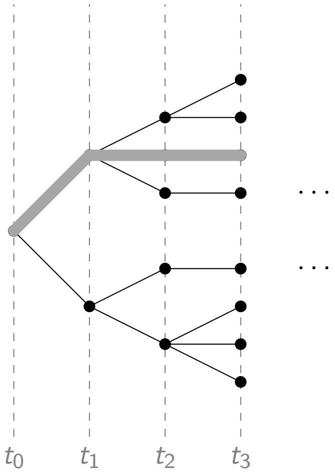
The axioms:

1. $[\delta A]A$
2. $[\delta A]B \rightarrow ([\delta B]C \rightarrow [\delta A]C)$

Logics of Action and Agency

Alternative accounts of agency do not include explicit description of the actions:



STIT

- Each node represents a choice point for the agent.
- A **history** is a maximal branch in the above tree.
- Formulas are interpreted at history moment pairs.
- At each moment there is a choice available to the agent (partition of the histories through that moment)
- The key modality is $[stit]\varphi$ which is intended to mean that the agent i can "see to it that φ is true".
  - $[stit]\varphi$ is true at a history moment pair provided the agent can choose a (set of) branch(es) such that every future history-moment pair satisfies φ

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$\Diamond[stit]\varphi$: "the agent has the ability to bring about φ.

Example Consider the example of an agent (call her Ann) throwing a dart. Suppose Ann is not a very good dart player, but she just happens to throw a bull's eye. Intuitively, we do not want to say that Ann has the *ability* to throw a bull's eye even though it happens to be true. That is, the following principle should be falsifiable:

$$\varphi \rightarrow \Diamond[stit]\varphi$$

STIT

Example Continuing with this example, suppose that Ann has the ability to hit the dart board, but has no other control over the placement of the dart. Now, when she throws the dart, as a matter of fact, it will either hit the top half of the board or the bottom half of the board. Since, Ann has the ability to hit the dart board, she has the ability to either hit the top half of the board or the bottom half of the board.

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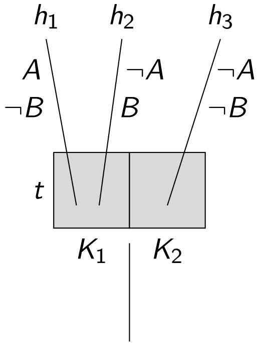
However, intuitively, it seems true that Ann does not have the ability to hit the top half of the dart board, and also she does not have the ability to hit the bottom half of the dart board. Thus, the following principle should be falsifiable:

$$\Diamond[\mathit{stit}](\varphi \vee \psi) \rightarrow \Diamond[\mathit{stit}]\varphi \vee \Diamond[\mathit{stit}]\psi$$

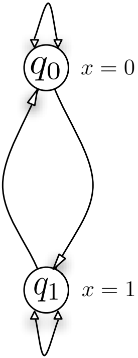
STIT

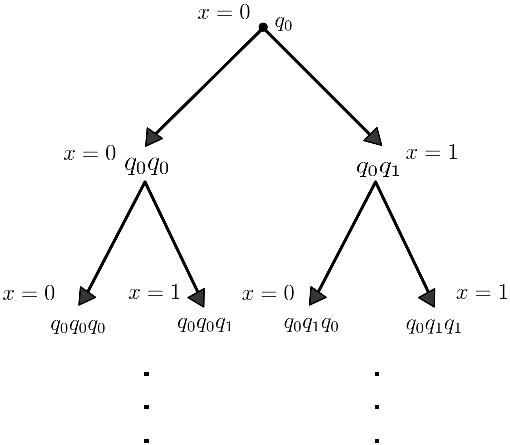
The following model will falsify both of the above formulas:



J. Horty. *Agency and Deontic Logic*. 2001.

Computational vs. Behavioral Structures





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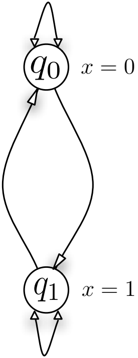

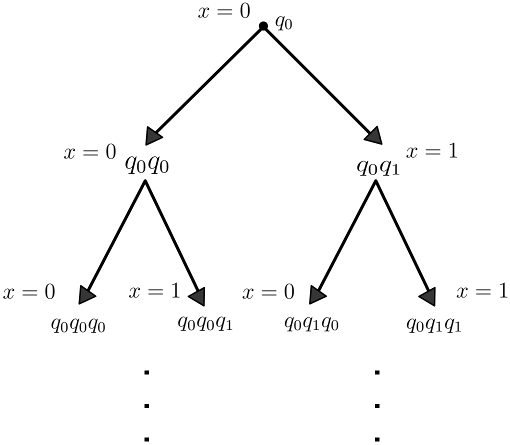

$\exists\Diamond P_{x=1}$

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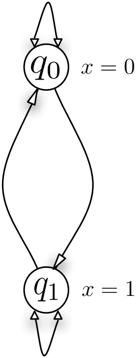

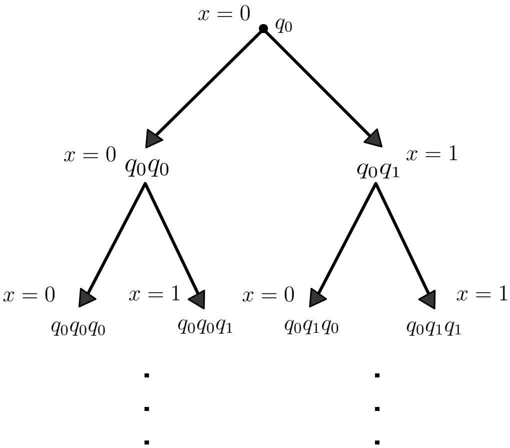

$\neg\forall\Diamond P_{x=1}$

Alternating Transition Systems

The previous model assumes there is *one* agent that “controls” the transition system.

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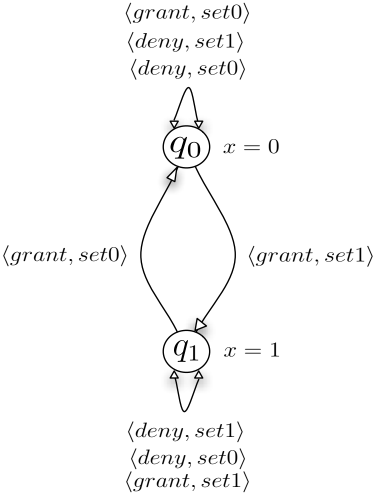
Multi-agent Transition Systems



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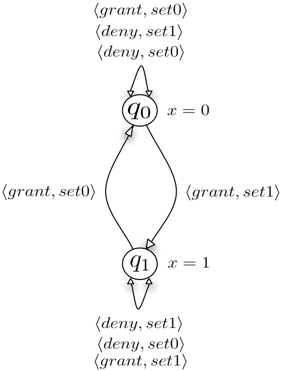

$$(P_{x=0} \rightarrow [s]P_{x=0}) \wedge (P_{x=1} \rightarrow [s]P_{x=1})$$

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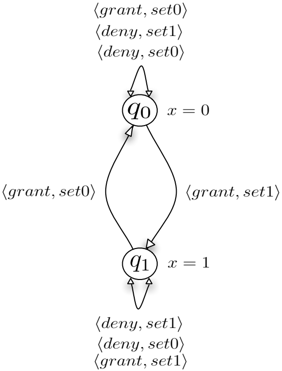

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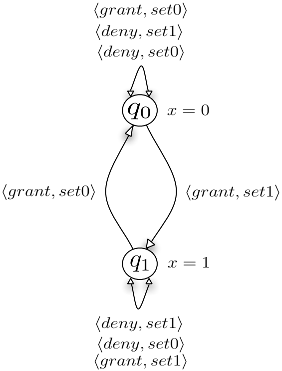

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From Temporal Logic to Strategy Logic

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- *Linear Time Temporal Logic*: Reasoning about computation paths:

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- *Branching Time Temporal Logic*: Allows quantification over paths:

$\exists\Diamond\varphi$: there is a path in which φ is eventually true.

E. M. Clarke and E. A. Emerson. *Design and Synthesis of Synchronization Skeletons using Branching-time Temproal-logic Specifications*. In *Proceedings Workshop on Logic of Programs*, LNCS (1981).

From Temporal Logic to Strategy Logic

- *Alternating-time Temporal Logic*: Reasoning about (local and global) group power:

  $\langle\langle A\rangle\rangle\Box\varphi$: The coalition A has a joint strategy to ensure that φ will remain true.

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- *Coalitional Logic*: Reasoning about (local) group power (fragment of **ATL**).

  $[C]\varphi$: coalition C has a joint strategy to bring about φ.

  M. Pauly. *A Modal Logic for Coalition Powers in Games*. *Journal of Logic and Computation* **12** (2002).

Other Motivational Attitudes

Stemming from Bratman's planning theory of intention a number of logics of rational agency have been developed:

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Some common features

- Underlying temporal model
- Belief, Desire, Intention, Plans, Actions are defined with corresponding operators in a language

J.-J. Meyer and F. Veltman. *Intelligent Agents and Common Sense Reasoning*. Handbook of Modal Logic, 2007.

General Issues

Once a semantics and language are fixed, then standard questions can be asked: eg. develop a proof theory, completeness, decidability, model checking.

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- Comparing different frameworks: eg. PDL vs. STIT, STIT vs. ATL, The Situation Calculus vs. Epistemic (temporal) Logic.

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- "Epistemizing" logics of action and ability: *knowing how to achieve* φ vs. *knowing that you can achieve* φ

Question: How do we model (and reason about) *information change* in a social situation?

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Plan for the rest of the tutorial:

- Method 1: Epistemic Temporal Logic (ETL)
- Method 2: Dynamic Epistemic Logic (DEL)
- Comparing DEL and ETL
- Some further questions

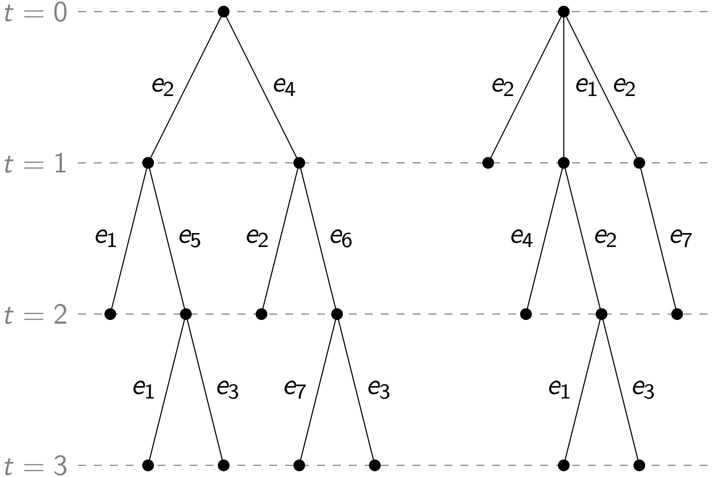
J. van Benthem, J. Gerbrandy, T. Hoshi and E. Pacuit. *Merging Frameworks for Interaction*. manuscript.

Epistemic Temporal Logic

R. Parikh and R. Ramanujam. *A Knowledge Based Semantics of Messages*. *Journal of Logic, Language and Information*, 12: 453 – 467, 1985, 2003.

FHMV. Reasoning about Knowledge. MIT Press, 1995.

The ‘Playground’



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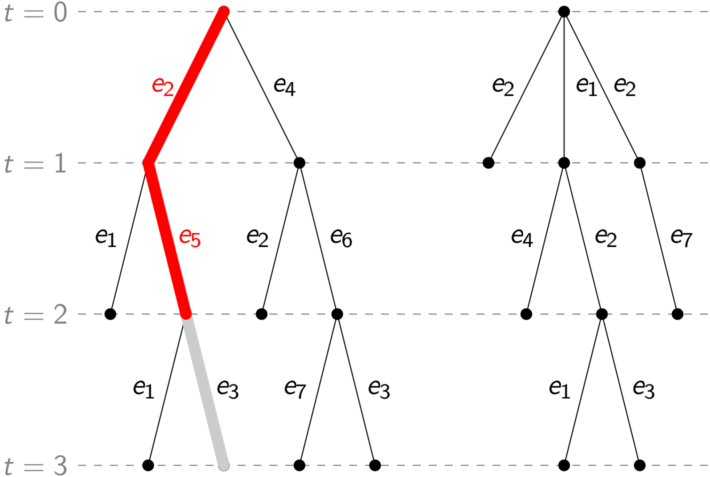

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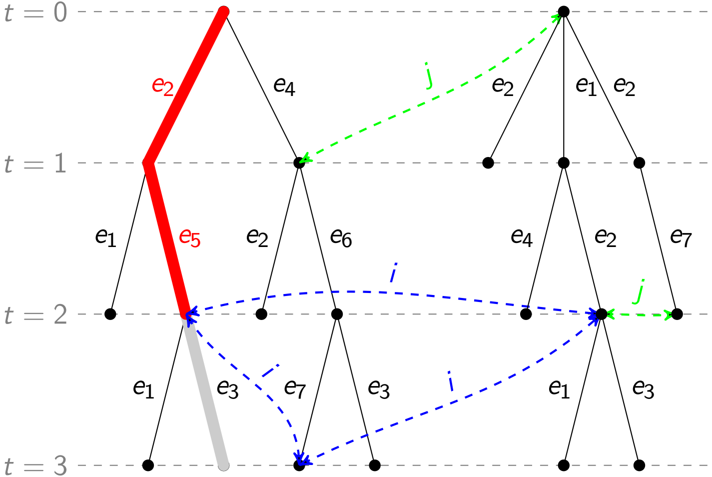

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- ϵ is the empty string and $\mathsf{FinPre}_{-\epsilon}(\mathcal{H}) = \mathsf{FinPre}(\mathcal{H}) - \{\epsilon\}$.

History-based Frames

Definition

Let Σ be any set of events. A set $\mathcal{H} \subseteq \Sigma^* \cup \Sigma^\omega$ is called a protocol provided $\text{FinPre}_{-\epsilon}(\mathcal{H}) \subseteq \mathcal{H}$. A rooted protocol is any set $\mathcal{H} \subseteq \Sigma^* \cup \Sigma^\omega$ where $\text{FinPre}(\mathcal{H}) \subseteq \mathcal{H}$.

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An ETL frame is a tuple $\langle \Sigma, \mathcal{H}, \{\sim_i\}_{i \in \mathcal{A}} \rangle$ where Σ is a (finite or infinite) set of events, $\mathcal{H}$ is a protocol, and for each $i \in \mathcal{A}$, $\sim_i$ is an equivalence relation on the set of finite strings in $\mathcal{H}$.

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Some assumptions:

1. If Σ is assumed to be finite, then we say that $\mathcal{F}$ is **finitely branching**.
2. If $\mathcal{H}$ is a rooted protocol, $\mathcal{F}$ is a **tree frame**.

Formal Languages

- $P\varphi$ (φ is true *sometime* in the past),
- $F\varphi$ (φ is true *sometime* in the future),
- $Y\varphi$ (φ is true at *the* previous moment),
- $N\varphi$ (φ is true at *the* next moment),
- $N_e\varphi$ (φ is true after event e)
- $K_i\varphi$ (agent i knows φ) and
- $C_B\varphi$ (the group $B \subseteq \mathcal{A}$ commonly knows φ).

History-based Models

An ETL **model** is a structure $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}, V\rangle$ where $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}\rangle$ is an ETL frame and

$V : \mathsf{At} \rightarrow 2^{\mathsf{finite}(\mathcal{H})}$ is a valuation function.

Formulas are interpreted at pairs H, t:

$$H, t \models \varphi$$

Truth in a Model

- $H, t \models P\varphi$ iff there exists $t' \leq t$ such that $H, t' \models \varphi$
- $H, t \models F\varphi$ iff there exists $t' \geq t$ such that $H, t' \models \varphi$
- $H, t \models N\varphi$ iff $H, t+1 \models \varphi$
- $H, t \models Y\varphi$ iff $t > 1$ and $H, t-1 \models \varphi$
- $H, t \models K_i\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_i H'_m$ then $H', m \models \varphi$
- $H, t \models C\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_* H'_m$ then $H', m \models \varphi$.

where $\sim_*$ is the reflexive transitive closure of the union of the $\sim_i$.

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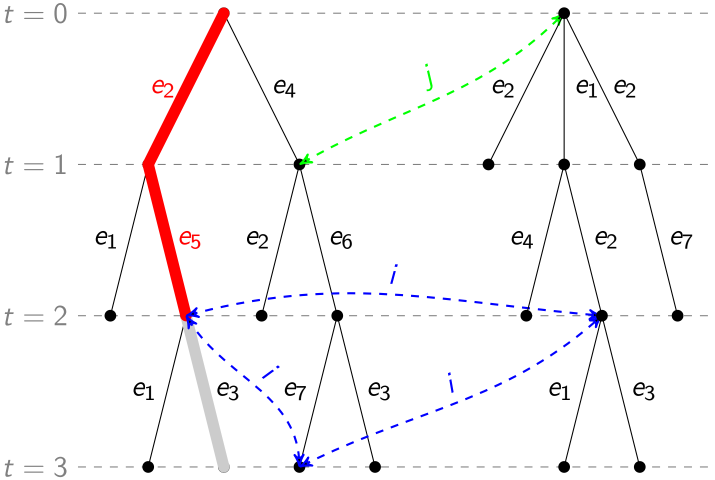
t = 0
t = 1
t = 2
t = 3
e2
e4
j
e2
e1
e2
e1
e5
e2
e6
e4
e2
e7
i
e1
e3
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i
e1
e3

An Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

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Is this procedure correct?

An Example

Yes, if

1. Ann knows about the talk.

An Example

Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.

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1. Ann knows about the talk.
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3. Ann knows that Bob knows about the talk.

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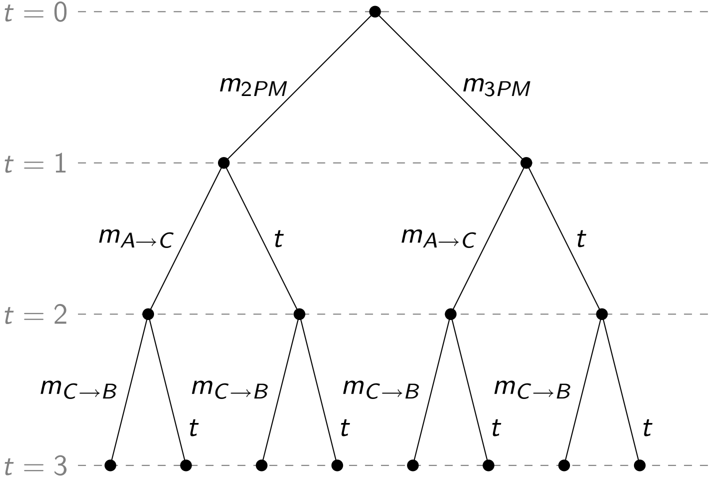
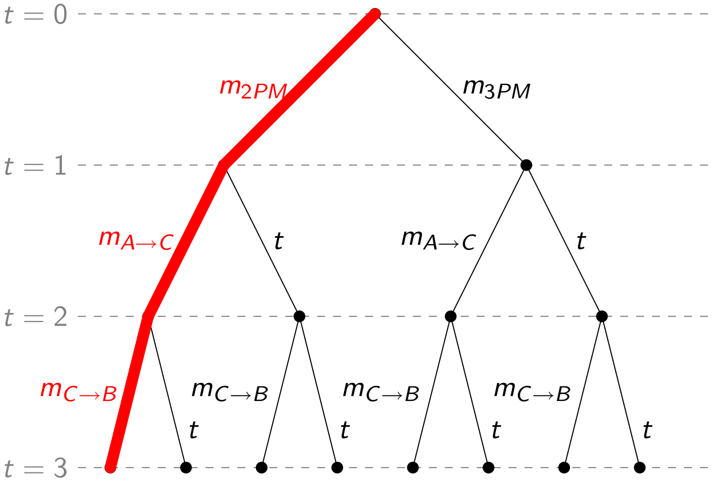
Yes, if

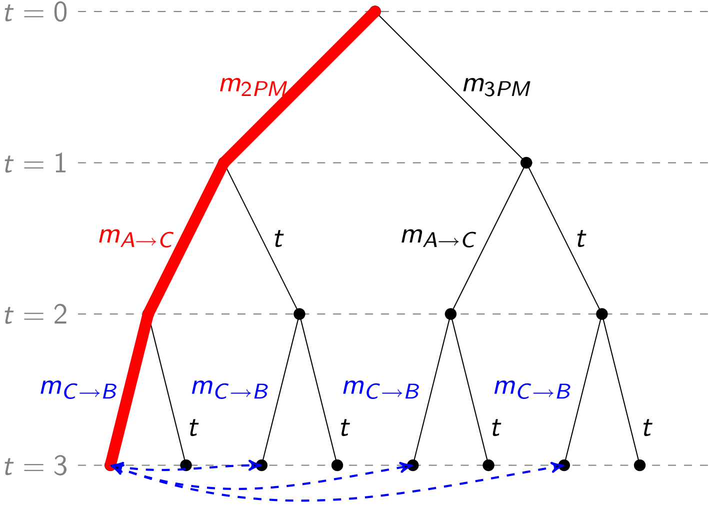
1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob *does not* know that Ann knows that he knows about the talk.
5. *And nothing else.*



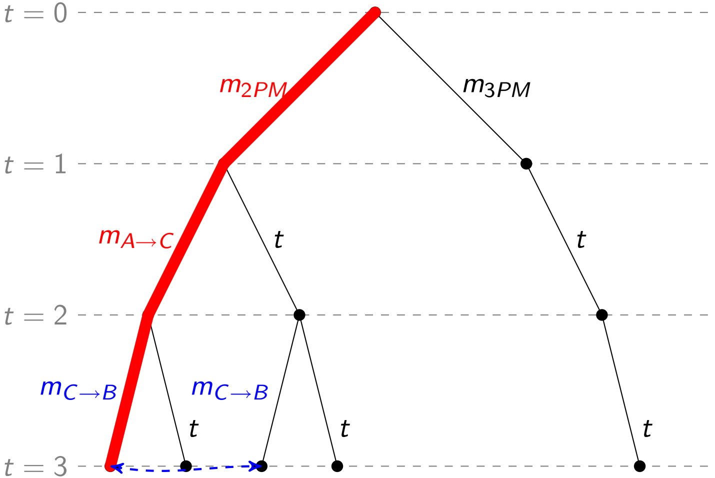
$t = 0$
$t = 1$
$t = 2$
$t = 3$
m_{2PM}
m_{3PM}
$m_{A \to C}$
t
$m_{A \to C}$
t
$m_{C \to B}$
t
$m_{C \to B}$
t
$m_{C \to B}$
t
$m_{C \to B}$
t



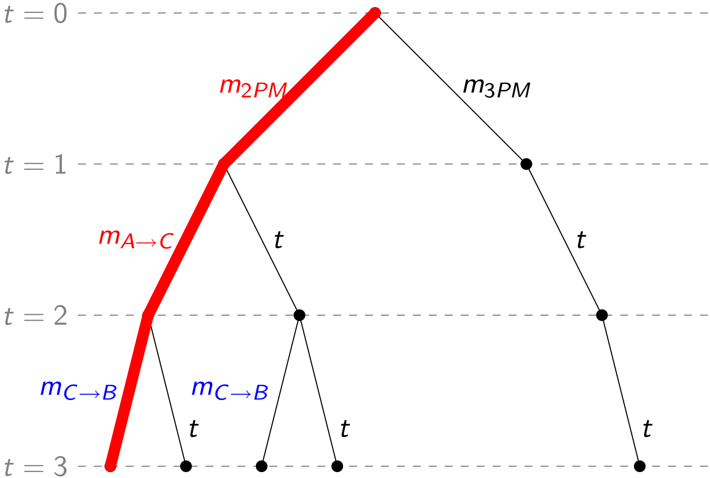
$$H, 3 \models \varphi$$



Bob's uncertainty: $H, 3 \models \neg K_B P_{2PM}$



Bob's uncertainty + 'Protocol information': $H, 3 \models K_B P_{2PM}$



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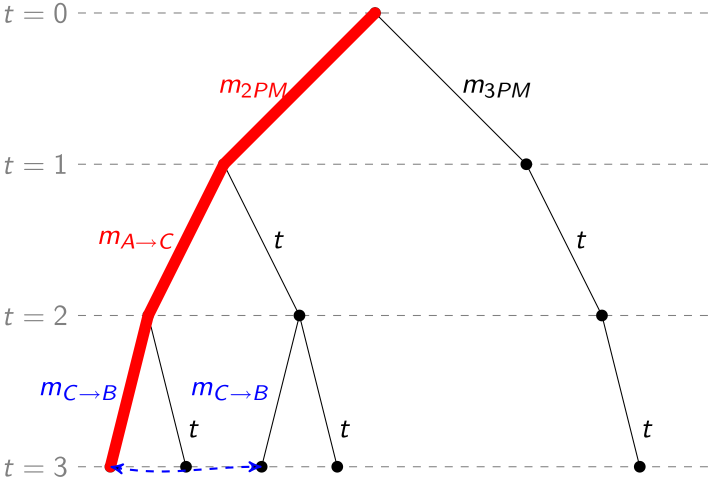

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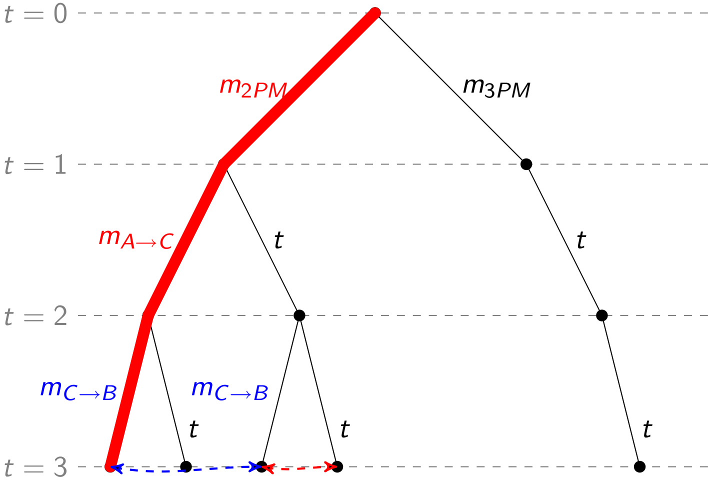

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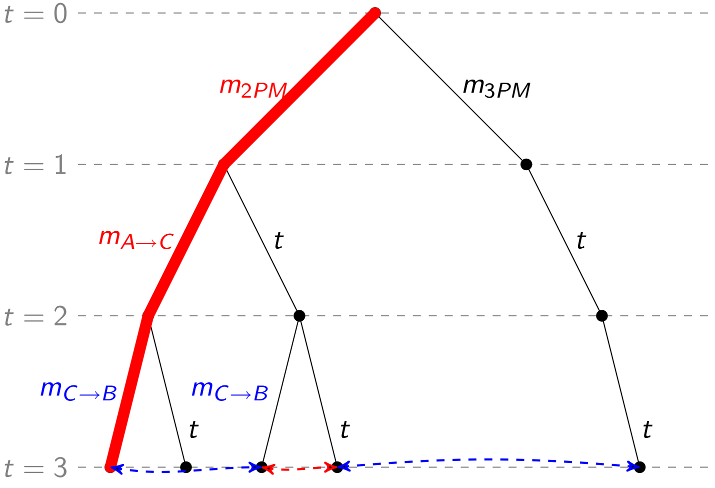

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3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Do they agents' know what time it is?

Agent Oriented Properties:

- **No Miracles**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $H \sim_i H'$ then $He \sim_i H'e$.
- **Perfect Recall**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $He \sim_i H'e$ then $H \sim_i H'$.
- **Synchronous**: For all finite histories $H, H' \in \mathcal{H}$, if $H \sim_i H'$ then $\mathrm{len}(H) = \mathrm{len}(H')$.

Ideal Agents

Assume there are two agents

Theorem

The logic of ideal *agents with respect to a language with common knowledge and future is highly undecidable (for example, by assuming perfect recall).*

J. Halpern and M. Vardi.. *The Complexity of Reasoning abut Knowledge and Time*. *J. Computer and Systems Sciences*, 38, 1989.

J. van Benthem and EP. *The Tree of Knowledge in Action*. Proceedings of AiML, 2006.

Two Methodologies

ETL methodology: when describing a social situation, first write down all possible sequences of events, then at each moment write down the agents' uncertainty, from that infer how the agents' knowledge changes from one moment to the next.

Two Methodologies

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Dynamic Epistemic Logic

Returning to the Example: DEL

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$$(\mathcal{M} \otimes E_1) \otimes E_2$$

Returning to the Example: DEL

$$(\mathcal{M} \otimes E_1) \otimes E_2$$

The initial model (Ann and Bob are ignorant about P_{2PM}).

Private announcement to Ann about the talk.

Abstract Description of the Event

Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.

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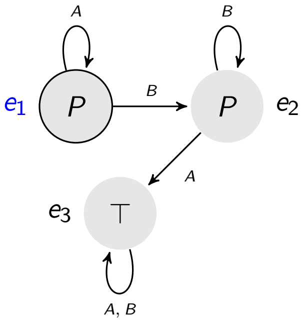

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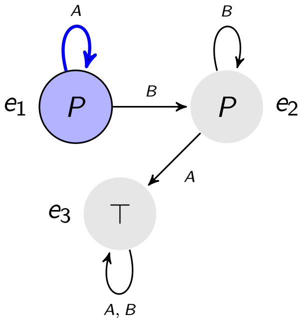
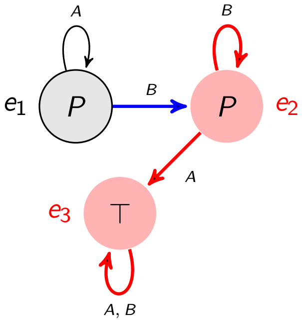


Ann knows which event took place.

Abstract Description of the Event

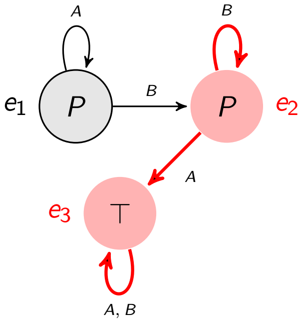
Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.



Bob thinks a different event took place.

Abstract Description of the Event

Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.

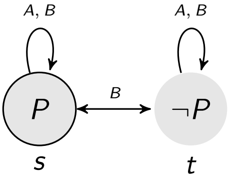


That is, Bob learns the time of the talk, but Ann learns nothing.

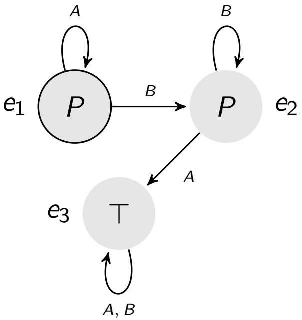
Product Update

Product Update

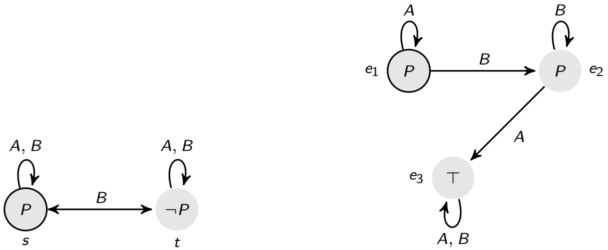
$\mathcal{M} \otimes E_1$

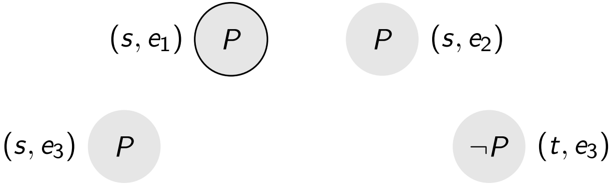


E_2



Product Update





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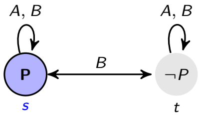

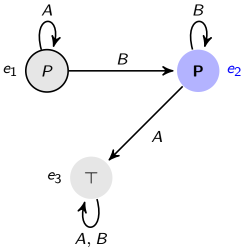

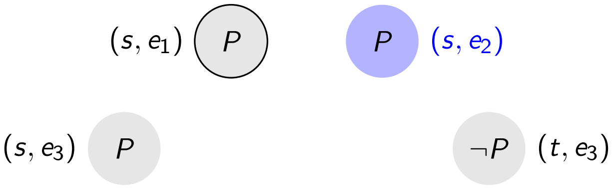

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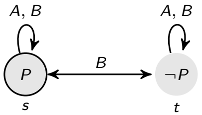
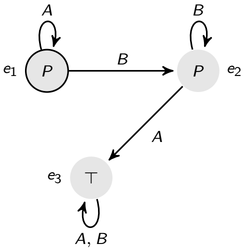
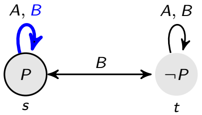
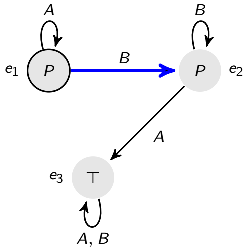




$(s, e_1) \models \neg K_B K_A K_B P$

Product Update





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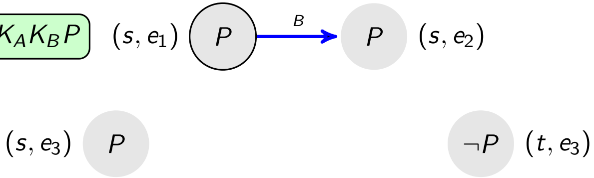

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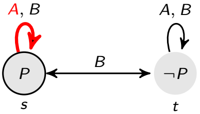

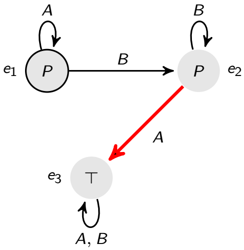

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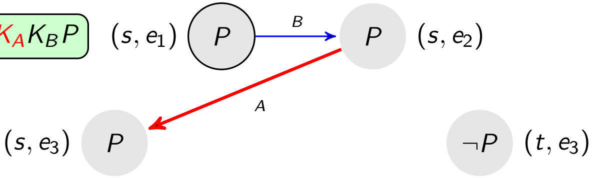

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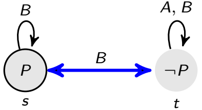

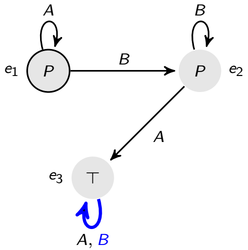

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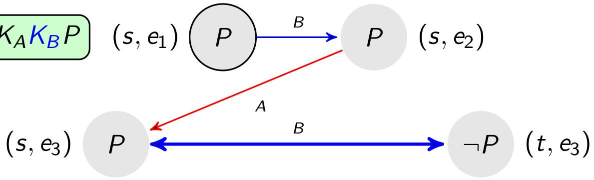

Product Update Details

Let $\mathbb{M} = \langle W, R, V \rangle$ be a Kripke model.

An event model is a tuple $\mathbb{A} = \langle A, S, Pre \rangle$, where $S \subseteq A \times A$ and $Pre : \mathcal{L} \rightarrow \wp(A)$.

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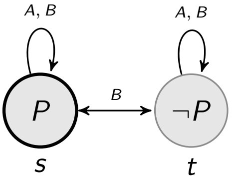
$\mathcal{M}, w \models [\mathrm{A}, a]\varphi$ iff $\mathcal{M}, w \models Pre(a)$ implies $\mathcal{M} \otimes A, (w, a) \models \varphi$.

Literarture

A. Baltag and L. Moss. *Logics for Epistemic Programs*. 2004.

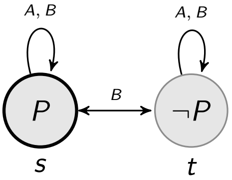
W. van der Hoek, H. van Ditmarsch and B. Kooi. *Dynamic Episetmic Logic*. 2007.

Example: Public Announcement



P means "The talk is at 2PM".

Example: Public Announcement



What happens if Ann publicly announces P?

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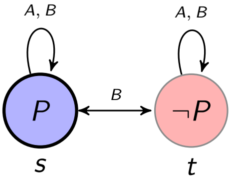

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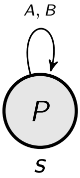

What happens if Ann publicly announces P? $s \models CP$

Example: Public Announcement Logic

J. Plaza. *Logics of Public Communications*. 1989.

J. Gerbrandy. *Bisimulations on Planet Kripke*. 1999.

J. van Benthem. *One is a lonely number*. 2002.

Example: Public Announcement Logic

The Public Announcement Language is generated by the following grammar:

$$p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i\varphi \mid C\varphi \mid [\psi]\varphi$$

where $p \in \mathsf{At}$ and $i \in \mathcal{A}$.

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- $[\psi]\varphi$ is intended to mean "After publicly announcing ψ, φ is true".

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- $[P]K_iP$: "After publicly announcing P, agent i knows P"

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where $p \in \mathsf{At}$ and $i \in \mathcal{A}$.

- $[\neg K_i P]K_i P$: "after announcing i does not know P, then i knows P. "

Example: Public Announcement Logic

Suppose $\mathcal{M} = \langle W, \{R_i\}_{i \in \mathcal{A}}, V \rangle$ is a multi-agent Kripke Model

$$\mathcal{M}, w \models [\psi]\varphi \text{ iff } \mathcal{M}, w \models \psi \text{ implies } \mathcal{M}|_{\psi}, w \models \varphi$$

where $\mathcal{M}|_{\psi} = \langle W', R', V' \rangle$ with

- $W' = W \cap \{w \mid \mathcal{M}, w \models \psi\}$
- $R' = R \cap W' \times W'$
- for all $p \in \mathsf{At}$, $V'(p) = V(p) \cap W'$

Example: Public Announcement Logic

$$[\psi]p \quad \leftrightarrow \quad (\psi \rightarrow p)$$

Example: Public Announcement Logic

$$\begin{array}{rcl} [\psi]p & \leftrightarrow & (\psi \rightarrow p) \\ [\psi]\neg\varphi & \leftrightarrow & (\psi \rightarrow \neg[\psi]\varphi) \end{array}$$

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[\psi]\neg\varphi & \leftrightarrow & (\psi \rightarrow \neg[\psi]\varphi) \\
[\psi](\psi \wedge \chi) & \leftrightarrow & ([\varphi]\psi \wedge [\varphi]\chi)
\end{array}
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Theorem Every formula of Public Announcement Logic is equivalent to a formula of Epistemic Logic.

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The situation is more complicated with common knowledge.

J. van Benthem, J. van Eijk, B. Kooi. *Logics of Communication and Change*. 2006.

Some Questions

- How do we relate the ETL-style analysis with the DEL-style analysis?
- In the DEL setting, what are the underlying assumptions about the reasoning abilities of the agents?
- Can we axiomatize interesting subclasses of ETL frames?

J. van Benthem, J. Gerbrandy, EP. *Merging Frameworks for Interaction: DEL and ETL*. TARK 2007.

J. van Benthem, J. Gerbrandy, T. Hoshi, EP. *Merging Frameworks for Interaction*. manuscript.

DEL *and* ETL

Observation: By repeatedly updating an epistemic model with event models, the machinery of DEL creates ETL models.

DEL *and* ETL

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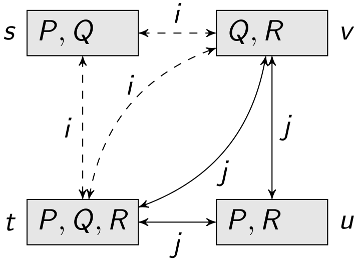
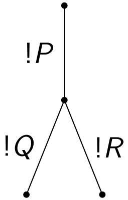
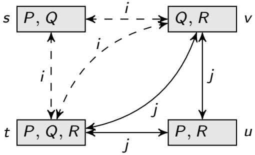
Let M be an epistemic model, and P a DEL protocol (tree of event models). The ETL model generated by M and P, forest(M, P), represents all possible evolutions of the system obtained by updating M with sequences from P.

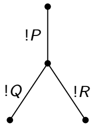
Example: Initial Model and Protocol





Example





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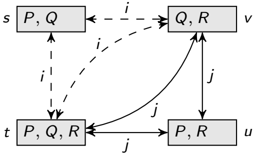
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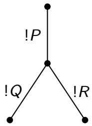
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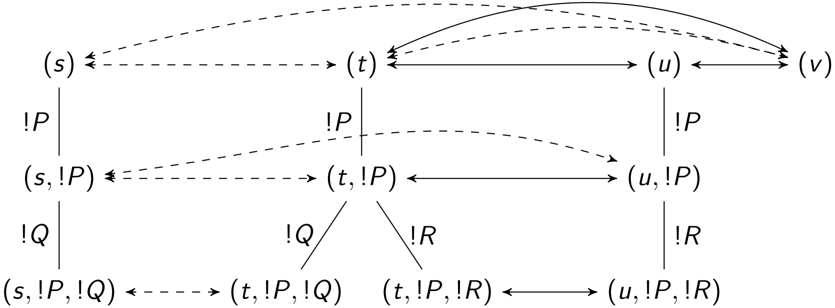
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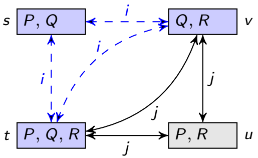
Example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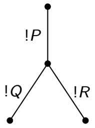
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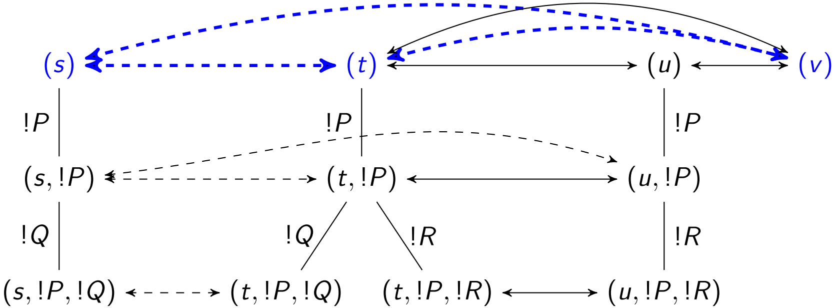
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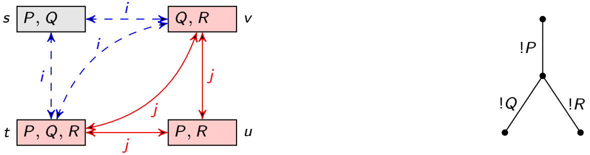
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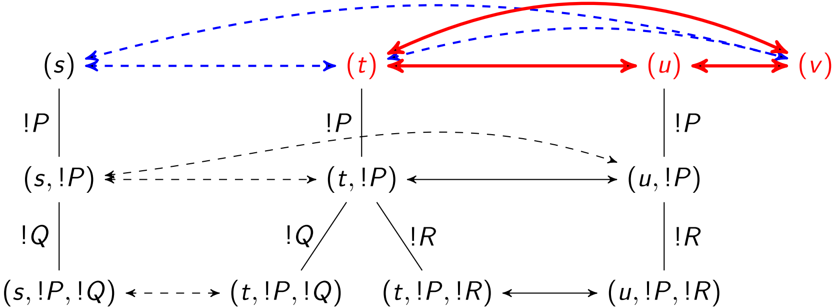
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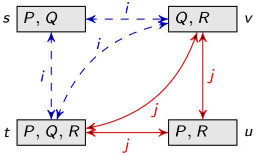
Example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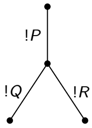
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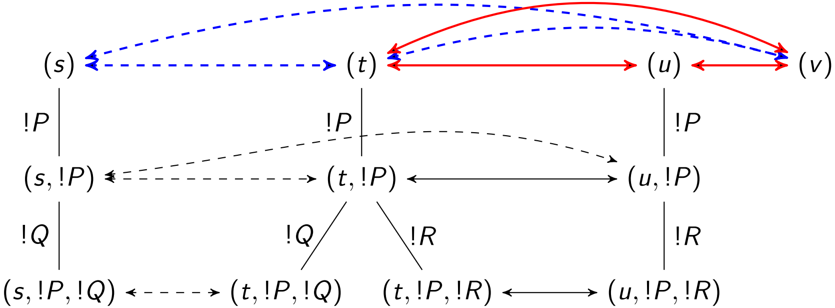
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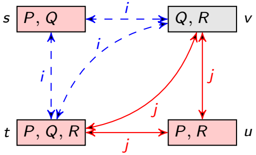
Example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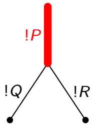
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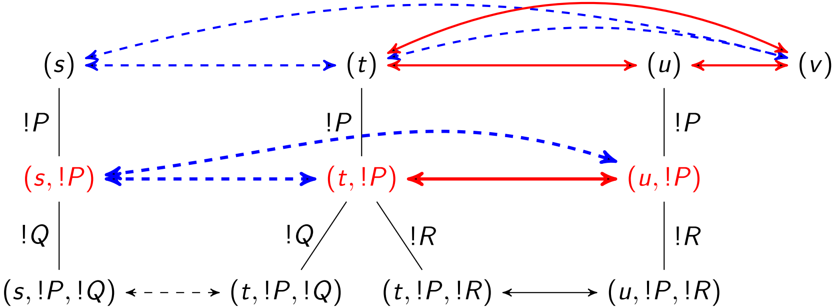
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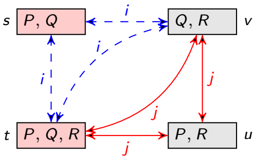
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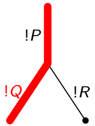
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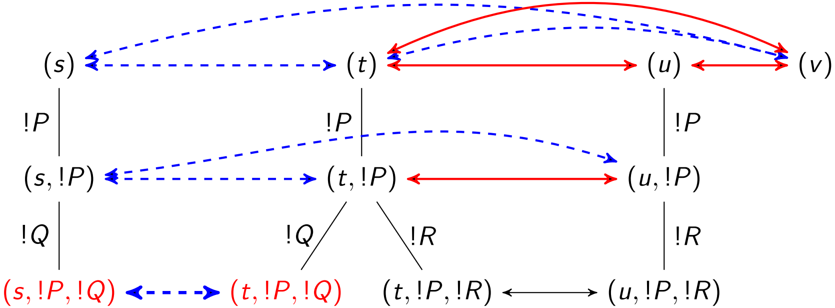
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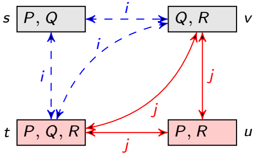
Example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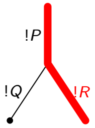
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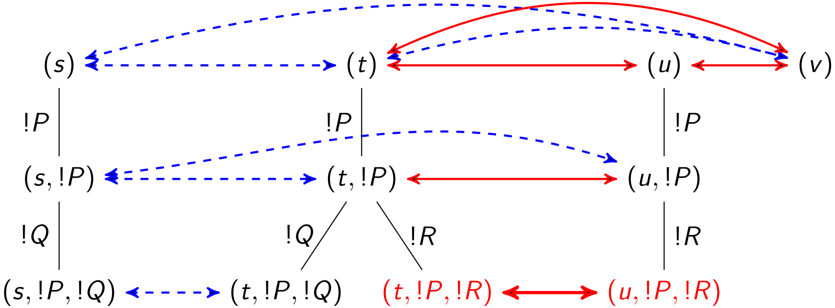
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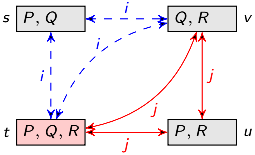
Example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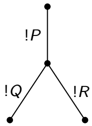
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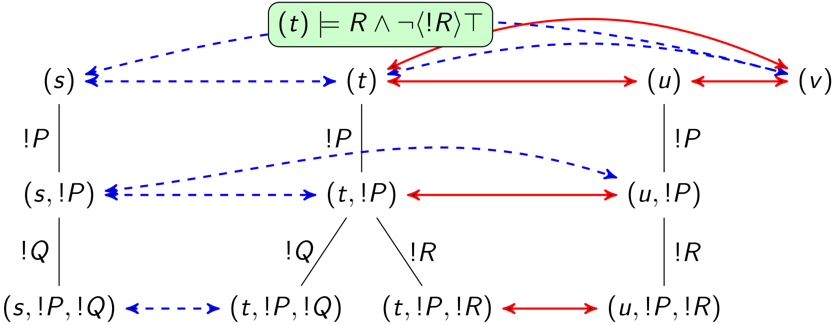
$(t) \models R \wedge \neg\langle !R\rangle\top$
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State-Dependent Protocols

The ETL models $\mathbb{F}(\mathcal{M}, \mathsf{P})$ in the previus example satisfies a rather strong *uniformity condition*: if $(\mathcal{E}, e)$ is allowable according to the protocol P then for all histories h, the epistemic action $(\mathcal{E}, e)$ can be executed at h iff pre(e) is true at h.

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Definition

State-Dependent DEL Protocol Let $\mathcal{M}$ be an epistemic model. A **state-dependent DEL protocol on** $\mathcal{M}$ is a function $p : D(\mathcal{M}) \to Ptcl(\mathbb{E})$.

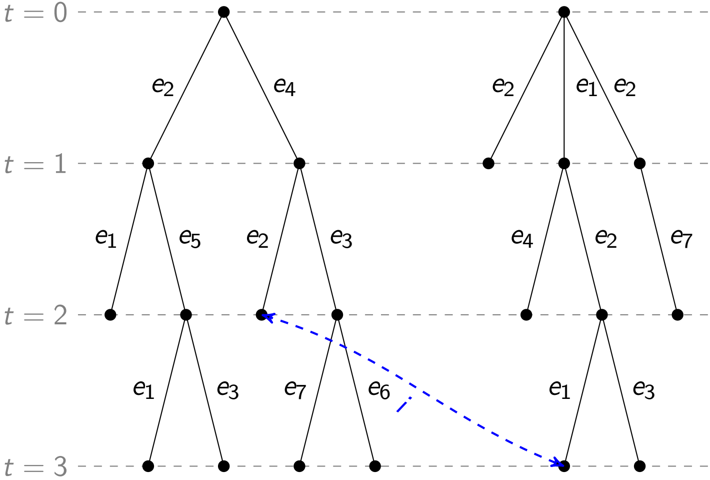
Representation Result

Given a set of DEL protocols $\mathbf{X}$, let $\mathbb{F}(\mathbf{X})$ be the class of ETL frames generated by protocols from $\mathbf{X}$.

Theorem (Main Representation Theorem)

Let Σ be a finite set of events and suppose $\mathbf{X}_{DEL}^{uni}$ is the class of uniform DEL protocols (with a finiteness condition). A model is in $\mathbb{F}(\mathbf{X}_{DEL}^{uni})$ iff it satisfies propositional stability, synchronicity, perfect recall, local no miracles, and local bisimulation invariance.

Perfect Recall



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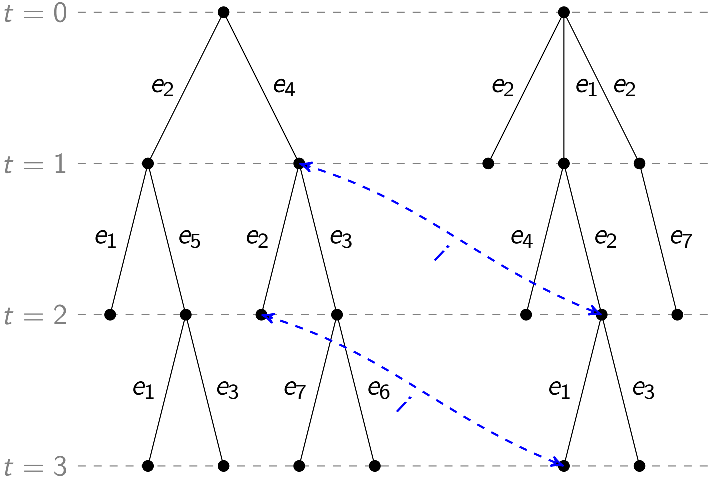

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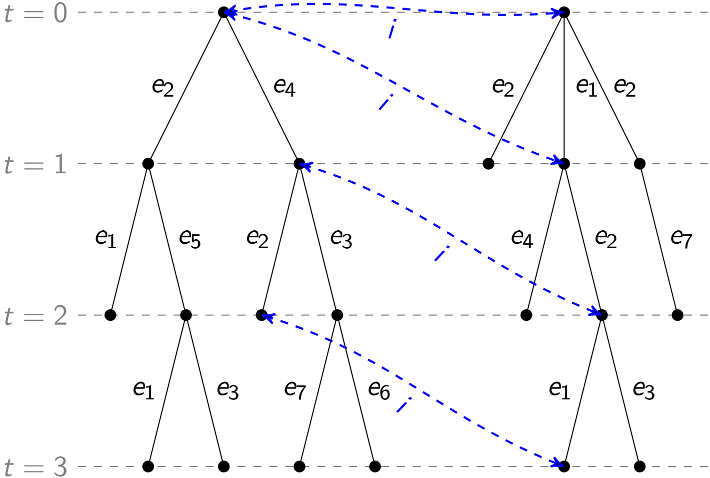
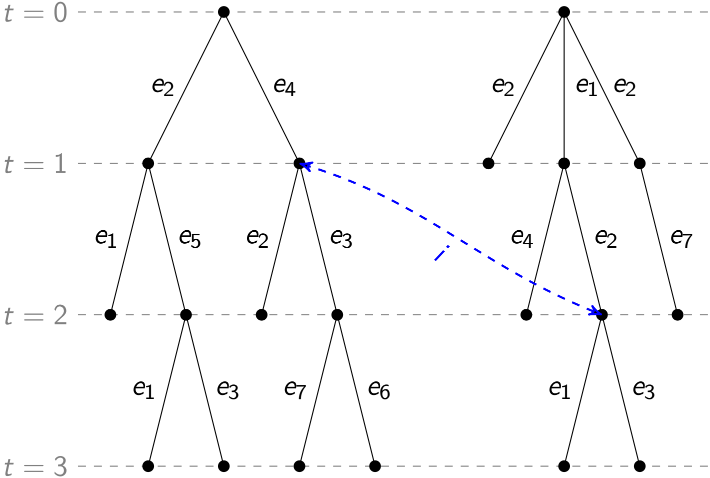


No Miracles



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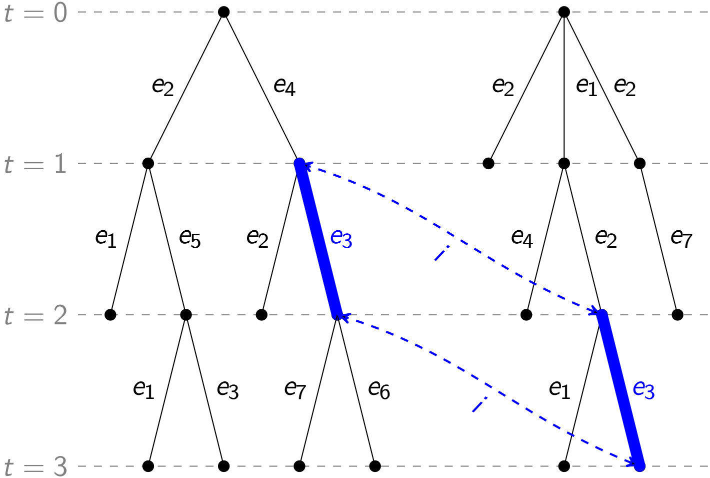
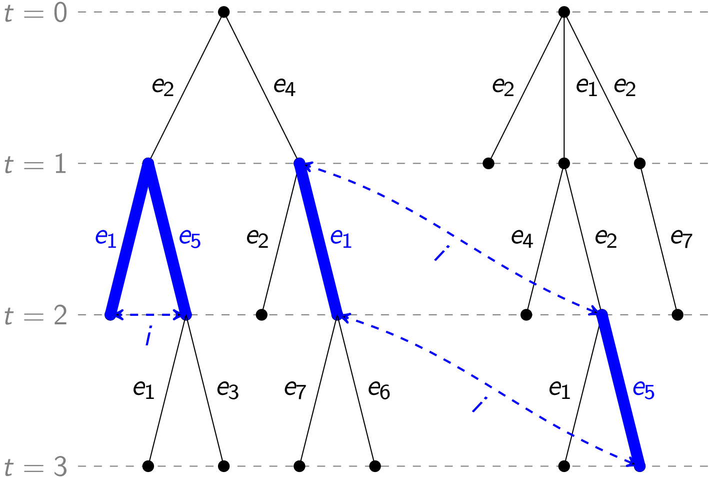
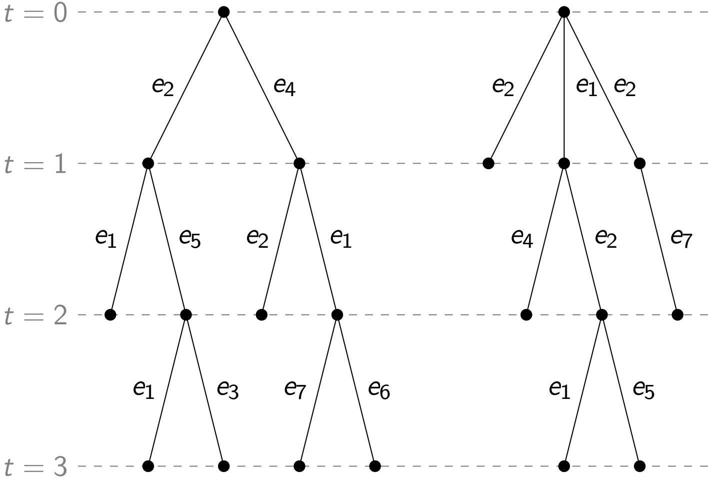


No Miracles



Bisimulation Invariance + Finiteness Condition



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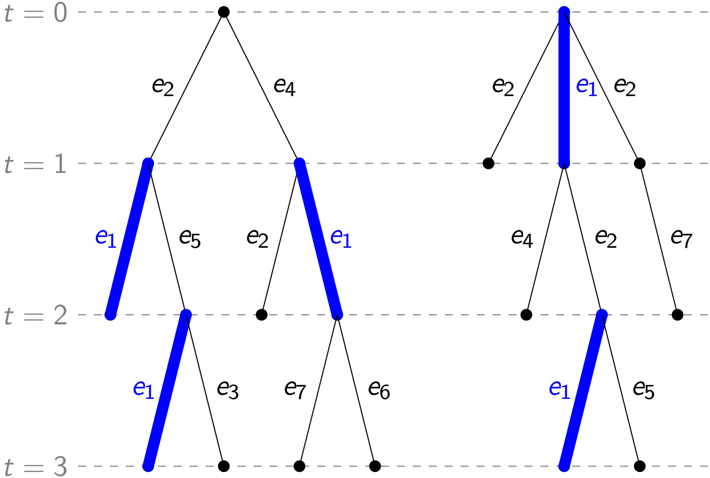

Recall that if $\mathbf{X}$ is a set of DEL protocols, we define $\mathbb{F}(\mathbf{X}) = \{\mathbb{F}(\mathcal{M}, \mathsf{P}) \mid \mathcal{M} \text{ an epistemic model and } \mathsf{P} \in \mathbf{X}\}$. This construction suggests the following natural questions:

- Which DEL protocols generate interesting ETL models?
- Which modal languages are most suitable to describe these models?
- Can we axiomatize interesting classes DEL-generated ETL models?

J. van Benthem, J. Gerbrandy, T. Hoshi, EP. *Merging Frameworks for Interaction*. manuscript.

Announcement + Protocol Information

1. $\langle A \rangle K_i P \leftrightarrow A \wedge K_i \langle A \rangle P$
2. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle \top \wedge K_i (A \rightarrow \langle A \rangle P)$
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Theorems Sound and complete axiomatizations of various generated ETL models.

Group Knowledge/Agency

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- Communication/observation + protocol information:
  study sequences of updates that do/do not lead to group knowledge.

Achieving Group Knowledge

- $\mathcal{M}, w \models C\varphi$ iff for each w', if $w \sim_* w'$ then $\mathcal{M}, w' \models \varphi$ ($\sim_*$ is the reflexive transitive closure of the union of each agent's accessibility relation)
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Theorem If every agent 'says all she knows' (i.e., 'I am in this partition cell') then distributed knowledge is turned into common knowledge.

J. van Benthem. *One is a lonely number*. 2002.

Achieving Group Knowledge

"honest" public announcement: the speaker of the announcement believes what he announces (preconditions of φ is $\varphi \wedge K_i\varphi$)

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Theorem For all $\mathcal{M}$ in which all $\sim_i$ are equivalence relations, and each φ that is purely epistemic (that is, it does not contain temporal operators) it holds that:

$$\text{Forest}(\mathcal{M}, \text{ProtocolHonest}) \models D\varphi \leftrightarrow GD\varphi$$

Achieving Group Knowledge (unreliable messages)

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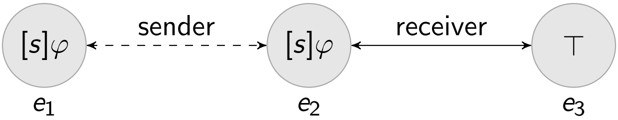

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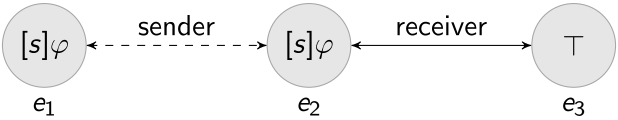

Theorem In all $S5$ models $\mathcal{M}$, it holds for all φ in which epistemic operators occur only positively:

$$\text{Forest}(\mathcal{M}, \text{ProtocolInsecure}) \models C\varphi \leftrightarrow GC\varphi$$

Many Issues!

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- What is the logic of specific protocols (in languages with group knowledge operators)?
- New notions of group knowledge?

Summary

- Surveyed various logics of rational agency: Epistemic Logics, Logics of Actions, Strategy Logics, Dynamic Epistemic Logic, Epistemic Temporal Logic
- Left out a number of issues: (dynamic) logic of preferences, belief revision (van Benthem and Degremont have analogous results)
- Compared two styles of modeling dynamics of information is social situations
- Merging the two perspectives leads to new technical and conceptual questions.

Conclusions

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What do the logical frameworks contribute to the discussion on rational agency?

- Normative vs. Descriptive
- refine and test our intuitions: provide many answers to the question *what is a rational agent*?
- (epistemic) foundations of game theory
  Logic *and* Game Theory, not Logic in place of Game Theory.
- Social Software: Verify properties of social procedures
  - *Refine existing social procedures or suggest new ones*

R. Parikh. *Social Software*. *Synthese* **132** (2002).

Conclusions

- Many types of informational attitudes: “hard” knowledge, belief, belief about the future state of affairs, “intention” based beliefs, revisable beliefs, safe beliefs. What is the relationship between these notions?

- Where does the “protocol” come from? What do the agents know about the protocol?

Logics of Rational Agency

- What's going on in the area:
  www.illc.uva.nl/wordpress
- Upcoming Workshop: Logic and Intelligent Interaciton
  ai.stanford.edu/∼epacuit/LaII
- Upcoming special issue of the Journal of Logic, Language and Information edited by J. van Benthem and EP.
- Third Indian Conference on Logic and its Applications, Chennai, India

Thank You!