Dynamic Analysis of Interactive Rationality

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Abstract

Epistemic game theory has shown the importance of informational contexts in understanding strategic interaction. We propose a general framework to analyze how such contexts may arise. The idea is to view informational contexts as the fixed-points of iterated, "rational responses" to incoming information about the agents' possible choices. We show general conditions for the stabilization of such sequences of rational responses, in terms of structural properties of both the decision rule and the information update policy.

Reasoning in Games

How should the players incorporate into their own decision making the fact that they are interacting with other (actively reasoning) rational agents?

Many models of *deliberation* in games (e.g, [1, 5, 3]).

Different frameworks, common thought: *the "rational solutions" of a game are the result of individual deliberation about the "rational" action to choose.*

Iterative Solution Concepts

Iterated removal of (weakly) dominated strategies

Two Views

1. iterative procedures narrow down or assist in the search for a equilibria

   successive stages of strategy deletion may correspond to different levels of belief

2. iterative procedures represent a rational deliberation process

   successive stages of a strategy deletion can be interpreted as tracking successive steps of reasoning that players can perform

Reasoning *to* a Context

"It is important to understand that we have two forms of irrationality in this paper...For us, a player is rational if he optimizes and also rules nothing out. So irrationality might mean not optimizing. But it can also mean optimizing while not considering everything possible." [2, pg. 314]

A player can be rationally criticized for

1. not choosing what is *best* or what is *rationally permissible*, *given one's information*.
2. not reasoning to a "proper" informational context.

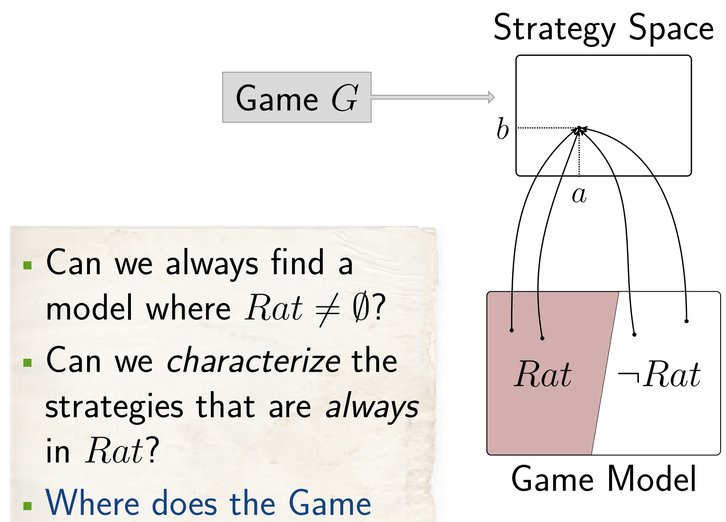
Rational Beliefs in Games

Caution/Admissibility: A player should prefer one strategy to another if the former weakly dominates the latter.

No extraneous restrictions on beliefs/privacy of "tie-breaking": Players should prefer a strategy *only if* the former weakly dominates the latter on the set of rational choices for the opponent.

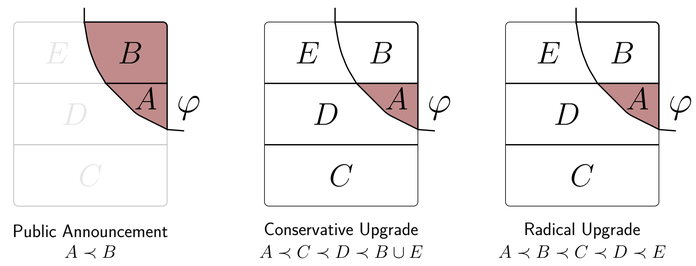
Full belief of opponent rationality: players should classify opponents' rational choices infinitely more likely than choices without this property.

Epistemic Game Theory



- Can we always find a model where $Rat \neq \emptyset$?
- Can we *characterize* the strategies that are *always* in Rat?
- Where does the Game Model come from?

Informative Actions



$$\mathcal{M}_0 \overset{\tau(\varphi_0)}{\Longrightarrow} \mathcal{M}_1 \overset{\tau(\varphi_1)}{\Longrightarrow} \cdots \overset{\tau(\varphi_n)}{\Longrightarrow} \mathcal{M}_{n+1} \Longrightarrow \cdots$$

- Does the process reach a fixed-point?
- Which model transformations? Where do the φ_k come from?

Categorizing Strategies: $S_i^+ \cup S_i^- \subset S_i$

Reasoning based expected utility [3]: $a \in S_i^+$ iff a maximizes EU for all probability measures, $a \in S_i^-$ iff a does not maximize EU for any probability measure, and S_i^+ receive positive probability, S_i^- receive probability zero.

	L	R
U	1,1	1,1
M_1	0,0	1,0
M_2	2,0	0,0
B	0,2	0,0

$S^+ = \{L\}$ $S^- = \{B\}$

	L	R
U	1,1	1,1
M_1	0,0	1,0
M_2	2,0	0,0
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$S^+ = \{L, R\}$ $S^- = \{B, M_1\}$

	L	R
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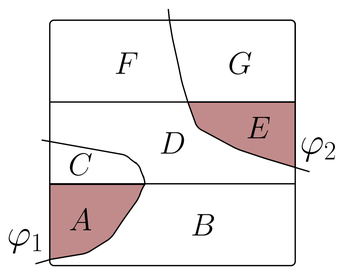
A generalization of "suspending judgement"

Upgrading with a *Set* of Formulas



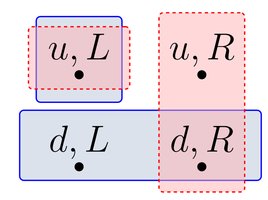
$$\uparrow\{\varphi_1, \varphi_2\} : A \cup E \prec B \prec C \cup D \prec F \cup G$$
$$\Uparrow\{\varphi_1, \varphi_2\} : A \prec E \prec B \prec C \cup D \prec F \cup G$$

Case Study: Iterated Admissibility

	L	R
u	1,1	1,0
d	1,0	0,1

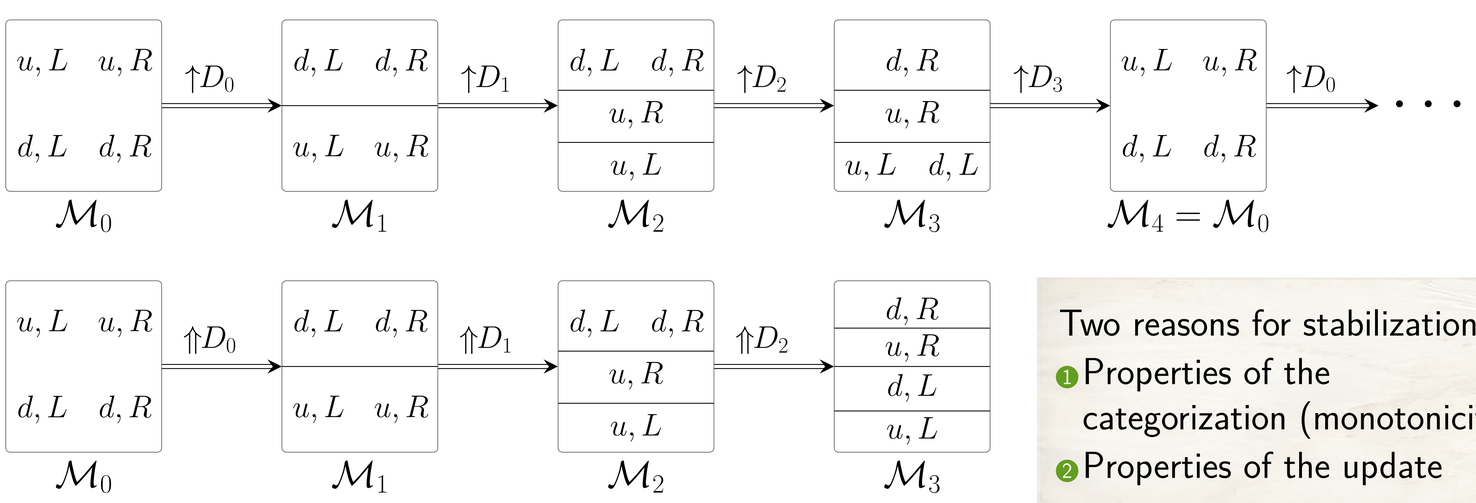
The IA set is $\{(u, L)\}$

- The "Undercut Problem": the reason for not playing d is undercut by a later reasoning step.
- L. . Samuelson [4] proved there is no (Aumann) model of this game satisfying *common knowledge of admissibility*.



What is wrong with this model?

Examples: A Dynamic Analysis of Iterated Admissibility



Two reasons for stabilization:

1. Properties of the categorization (monotonicity)
2. Properties of the update

Conclusions

Main Theorem: Under suitable conditions (eg., monotonicity of the categorization procedure), every rational deliberation process stabilizes.

Future Work

- Modeling *reasons* vs. *reasoning*: *explaining* vs. *describing* rational interaction
- Extensions to many agents
- Normative principles for categorizations

References

[1] J. van Benthem, "Rational dynamics and epistemic logic in games", *IGTR*, 9:1, 2007.

[2] A. Brandenburger, A. Friedenberg and H. J. Keisler, "Admissibility in Games", *Econometrica*, 76:2, 2008.

[3] R. Cubitt and R. Sugden , "The reasoning-based expected utility procedure", *GEB*, 2010.

[4] L. Samuelson, "Dominated Strategies and Common Knowledge", *GEB*, 1992.

[5] B. Skyrms, *Dynamic Deliberation*, HUP, 1990.