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**DUE: Friday February 3rd, 5:00pm**

Suppose you are given a deterministic algorithm  $A$  that on input circuit  $C$  runs in time polynomial in the size of  $C$  and outputs a number  $a = A(C)$  such that

$$a - .1 \leq \Pr[C(x) = 1] \leq a + .1$$

that is, it outputs a number that is a 10% additive approximation of the fraction of inputs  $x$  that makes  $C$  accept.

Prove that, under the assumption that such an algorithm  $A$  exists:

1.  $P = BPP$
2. There is a deterministic algorithm  $A'$  that given a circuit  $C$  and a parameter  $\epsilon$  runs in time polynomial in the size of  $C$  and in  $1/\epsilon$  and returns a number  $a = A'(C, \epsilon)$  such that

$$a - \epsilon \leq \Pr[C(x) = 1] \leq a + \epsilon$$

*Hint: Show that for all  $\epsilon > 0$ , there exists a deterministic polynomial time algorithm  $Z$  such that*

$$Z(t, C) = \begin{cases} 1 & \text{if } \Pr[C(x) = 1] \geq t + \epsilon/2 \\ 0 & \text{if } \Pr[C(x) = 1] \leq t - \epsilon/2 \end{cases}$$