Problem Set 10

This problem set is due on **Wednesday, December 2, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

You can use the fact that NL=coNL in your solutions

1. [35] Consider the problem

    $$\text{ShortestPath} = \{(G, s, t, k) : \text{ the shortest path from } s \text{ to } t \text{ in } G \text{ has length exactly } k\}$$

    where $G = (V, E)$ is a directed graph, s, t are vertices,and the length of a path is the number of edges in the path.

    (a) [15] Show that ShortestPath is in NL
    (b) [20] Show that ShortestPath is NL-hard

2. [35] A DFA D is *minimal* if the language $L(D)$ accepted by D is not accepted by any other DFA having fewer states than D. Define the problem

    $$\text{MinDFA} := \{\langle D \rangle : D \text{ is minimal }\}$$

    Prove that MinDFA is in NL.

3. [30] An undirected graph $G = (V, E)$ is 2-colorable if there is a coloring $c : V \to \{\text{Blue}, \text{Red}\}$ that assigns a color to each vertex such that for every edge (u, v) we have $c(u) \neq c(v)$.

    (a) [15] Prove that G is 2-colorable if and only if G contains no cycle of odd length.
    (b) [15] Prove that the language

    $$2Col := \{G : G \text{ is 2-colorable}\}$$

    is in NL