Problem Set 9

This problem set is due on **Wednesday, November 18, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

When asked to prove that a problem is NL-complete, make sure to include a proof that the problem is in NL. When specifying an NL algorithm, preferably give pseudo-code for it, and use terminology like "guess $x \in S$" to specify a non-deterministic step in which variable x can take any of the values in the set S.

1. [35] Consider the *acceptance problem for NFA*, that is the language

$$A_{NFA} := \{(M, w) : M \text{ is an NFA } \wedge M \text{ accepts } w\}$$

   Show that A_{NFA} is NL-complete under $\leq_m^{\log}$ reductions.

   [You can use the fact that the reachability problem for directed graphs is NL-complete]

2. [35] Consider the DAG-Reach problem defined as follows

$$DAG - Reach := \{(G, s, t) : G \text{ G is a DAG} \wedge t \text{ is reachable from } s\}$$

   (where DAG stands for directed acyclic graph). Show that DAG-Reach is NL-complete. (This shows that the reachability problem in directed graphs remains NL-complete even when restricted to acyclic graphs.)

   [Hint: create multiple copies of each vertex/configuration, and arrange them in layers.]

3. [30] Consider the language

$$StrongConn := \{G : G \text{ is a strongly connected directed graph}\}$$

   Recall that a graph $G = (V, E)$ is strongly connected if for every two vertices u, v there is a part from u to v and also a path from v to u.

   Prove that StrongConn is NL-complete

   [Hint: reduce from DAG-Reach]