Problem Set 8

This problem set is due on **Friday, November 6, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

Edited Nov. 4 to correct the example in Problem 1

In proofs of NP-completeness, you can use the fact that the following problems are NP-complete: Circuit-SAT, SAT, 3SAT, Independent Set, Vertex Cover, Clique, Subset Sum. In (2), you can also assume that the problem defined in (1) is NP-complete.

1. [45] If $G = (V, E)$ is an undirected graph and $S \subseteq V$, then define the "independence" of S as

   $$|S| - |\{\{u, v\} \in E : u \in S \wedge v \in S\}|$$

   the size of S minus the number of edges which connect pairs of vertices in S. For example if S is an independent set of size 5 then its independence is 5; if S is a clique of size 4 then its independence is -2; if the edges within S form a cycle of length S, then the independence of S is zero.

   Consider the *Independence* problem defined as follows: given a graph $G = (V, E)$ and an integer k, determine if there is a subset $S \subseteq V$ whose independence is at least k. Prove that this problem is NP-complete.

   [Hint: reduce from the Independent Set problem]

2. [35] The YourFace social networking site has a feature by which two users can "friend" each other, or "enemy" each other. (Two members who don't know each other will of course be neither friends nor enemies.) The "friendliness" of a subset S of users is the number of pairs of users who are friends minus the number of pairs of users who are enemies. As part of its targeted ad campaign, YourFace wants to find sets of users of high friendliness.

   Consider the *Friendliness* problem defined as follows: given the set of users U, the set of pairs of friends F, the set of pairs of enemies E, and an integer k, determine if there is a subset $S \subseteq U$ of users whose friendliness is at least k. Prove that this problem is NP-complete.

   [Hint: reduce from the Independence problem from the previous question. Try having one user who is friends with everybody else.]

   [Note: if you can't solve Problem 1, but you provide a correct reduction from Independence to Friendliness, then you receive full credit for Problem 2.]

3. [20] Prove that if P=NP then every language L in NP is NP-complete, except the languages $\emptyset$ and Σ^*.