Problem Set 7

This problem set is due on **Wednesday, October 28, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. [40] Let

$$R := \{x : K(x) \geq |x|\}$$

   be the language of binary strings whose Kolmogorov complexity is at least their length. Prove that for every decidable subset $L \subseteq R$ it must be the case that L is finite.

2. [30] Let $\bar{R}$ be the complement of the above language, that is a binary string $x \in \{0,1\}^n$ is an element of $\bar{R}$ if and only if $K(x) \leq n-1$.

   Prove that $\bar{R}$ is *recognizable*.

3. [30] Prove that A_{TM} is NP-hard.