Problem Set 5

This problem set is due on **Wednesday, October 14, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/∼luca/cs172`.

1. [25/100] A string $x = x_1 \cdots x_n$ is a *proper prefix* of a string $y = y_1 \cdots y_m$ if $m > n$ and $x_1 \cdots x_n = y_1 \cdots y_n$. For example *art* is a proper prefix of *artist*. A language L is *prefix free* if there exist no two strings $x, y \in L$ in the language such that x is proper prefix of y. For example, the language 0*1 is prefix-free but the language $(0 \cup 11)$*0 is not prefix-free.

   Prove that the language

   $$PFD := \{\langle D \rangle : D \text{ is a DFA and } L(D) \text{ is prefix free }\}$$

   is decidable. That is, provide an algorithm that, on input a DFA D determines whether or not the language recognized by D is prefix-free.

2. [45/100] Consider the language

   $$PFTM := \{\langle M \rangle : M \text{ is a Turing machine and } L(M) \text{ is prefix free }\}$$

   where $L(M)$ is the set of strings accepted by M.

   (a) [25] Prove that PFTM is undecidable.
   (b) [10] Is PFTM recognizable?
   (c) [10] Is the complement of PFTM recognizable?

3. [30] Prove that a language L is recognizable if and only if there exists a decidable language D with the property that for every x

   $$x \in L \Leftrightarrow \exists y. (x, y) \in D$$