Problem Set 3

This problem set is due on **Wednesday, September 23, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. [45/100] Let Σ be a finite language and $f : \Sigma \to \Sigma^*$ be a function mapping symbols of the alphabet to strings. For a string $w = w_1 \cdots w_n$, define as in the last homework

$$f[w] := f(w_1) \circ f(w_2) \circ \cdots \circ f(w_n)$$

For example, if $\Sigma = \{a, b\}$, $f(a) = b$ and $f(b) = aab$, then $f[\epsilon] = \epsilon$ and $f(aba) = baabb$.

If L is a language over Σ and $f : \Sigma \to \Sigma^*$ is a function, define the languages

$$A_f(L) := \{w \in \Sigma^* : f[w] \in L\}$$

$$B_f(L) := \{w \in \Sigma^* : \exists x \in L. f[x] = w\}$$

In a previous problem, we proved that if L is regular then $B_f(L)$ is regular. For each of the following statements, either provide a proof that it is true, or provide a counterexample and a proof that the counterexample contradicts the statement:

   (a) [20/100] For every alphabet Σ, function f, and regular language L, the language $A_f(L)$ is regular.

   (b) [25/100] For every alphabet Σ and function f, if the language $B_f(L)$ is regular, then the language L must also be regular.

2. [25] Let L be a language over the alphabet $\{0, 1\}$. Define

$$C(L) := \{x : \exists n \geq 0 \exists y \in L. x = y^n\}$$

Either provide a proof of the following statement, or provide a counterexample and a proof that the counterexample contradicts the statement:

If L is regular, then $C(L)$ is also regular.

3. [30] For a binary string w, let $NUM(w)$ be the interpretation of w as a binary number with the most significant bit first. For example $NUM(110) = 6$, $NUM(00001) = 1$ and $NUM(\epsilon) = 0$.

Consider the language

$$L := \{w : NUM(w) \text{ is divisible by } 5\ \}$$

Give a DFA for L and prove it is minimal.