Problem Set 2

This problem set is due on **Wednesday, September 16, by 5:00pm.**

Use the CS172 drop box.

Write **your name and your student ID number** on your solution. Write legibly. The description of your proofs should be as *clear* as possible (which does not mean *long* – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage `www.cs.berkeley.edu/~luca/cs172`.

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1. [20/100] Construct a regular expression that accepts the language of C/Java comments, as in the last homework assignment.

2. [25/100] Fix an alphabet Σ. Let f be any function from Σ into Σ^*. In other words f assigns a string to each letter of the alphabet. For any string $w = w_1 w_2 \ldots w_n$, let

   $$f[w] = f(w_1) \circ f(w_2) \circ \ldots \circ f(w_n)$$

   I.e. $f[w]$ is made by replacing each letter of w with the string assigned by f. For any language L, define:

   $$^f L = \{y \mid y = f[x] \text{ for some } x \in L\}$$

   Show that if L is a regular language, then so is $^f L$.

3. [15/100] Consider the language L over $\Sigma = \{a, b\}$ consisting of all strings containing the pattern *abaa* twice. (The two occurrences can overlap.) For example, '*abaabaa*' and '*abaaababababaaaab*' are in the language, while '*abbaaabaa*' is not.

   Show that L is regular.

4. [40/100] A 2-NFA is like a standard NFA, but it is defined to accept an input x if and only if there are *exactly* two paths in the automaton that lead from the start state to an accept state on input x.

   (a) [15/100] Show that if L is regular, then there is a 2-NFA that accepts L.
   [Hint: start from a DFA that accepts L and transform it into a 2-NFA that accepts the same language]

   (b) [25/100] Show that if L is the language accepted by a 2-NFA M, then L is regular.
   [Hint: proceed similarly to the transformation of an NFA into a DFA, but add some extra information in each state.]