Practice Midterm II

1. Define the language

$$L := \{ \ \langle M \rangle : \text{for all strings } x,\ M(x) \text{ halts within } |x|^2 \text{ steps } \}.$$

   Show that L is not recognizable but $\overline{L}$ is recognizable.

2. Define the language

$$L := \{ \ \langle x, y \rangle : K(x) > K(y) \ \}.$$

   Show that L is not recognizable.

3. Prove that the class of NP-complete languages is not closed under union and intersection. That is

   - Show that there are languages A, B which are NP complete but such at $A \cup B$ is not NP-complete.
   - Show that there are languages A, B which are NP-complete but such that $A \cap B$ is not NP-complete.

   [Hint: recall that $\emptyset$ and Σ^* cannot be NP-complete]

4. Define the problem

$$CLIQUE_{\frac{1}{2}} := \{ \ \langle G, k \rangle : G \text{ is an undirected graph with a clique of size at least } |V|/2 \ \}.$$

   Show that this problem is NP-complete.