Practice Final Exam

1. Let $L = \{wtw : w, t \in \Sigma^*\}$. Show that L is not regular.

2. Let $L = \{\langle r \rangle : r \text{ is a regular expression that accepts at least 2 strings}\}$. Show that L is decidable.

3. Let $L = \{\langle x, y, n \rangle : \max(K(x), K(y)) \geq n\}$. Show that L is not recognizable.

4. Let $L = \{\langle M_1, M_2 \rangle : L(M_1) = \overline{L(M_2)}\}$. Show that L is not recognizable.

5. We define the NEIGHBOR problem as follows. An instance consists of an undirected graph $G = (V, E)$ and an integer k. We must decide if there is a subset of nodes V' where every node in V is a distance of at most 1 from some node in V'. Additionally, we require $|V'| \leq k$.

   As an example, let $V = \{a, b, c\}$ and $E = \{\{a, b\}, \{b, c\}\}$ and $k = 1$. Then $V' = \{b\}$ satisfies the NEIGHBOR problem since a and c are adjacent to b.

   Show that NEIGHBOR is **NP**-complete.

6. Define $UCYCLE = \{\langle G \rangle : G \text{ is an undirected graph that contains a simple cycle}\}$. Show that $UCYCLE \in \mathbf{L}$. (Note: G may not be connected.)

   Hint. We can search through G by traversing the edges incident on a vertex in lexicographic order. That is, if we come in through the ith edge, we go out through the $(i+1)$th edge or the first if the degree is i. How does this algorithm behave on a tree? On a graph with a cycle?

7. Show that TIME($O(n^2)$) (the set of languages accepted by quadratic-time Turing machines) is distinct from **P**.