# Practice Midterm 2

1. Consider the following time-bounded variant of Kolmogorov complexity, written $K_L(x)$, and defined to be the shortest string $\langle M, w, t\rangle$ where $t$ is a positive integer written in binary, and $M$ is a $\mathsf{TM}$ that on input $w$ halts with $x$ on its tape within $t$ steps.

   (a) Show that $K_L(x)$ is computable (by describing an algorithm that on input $x$ outputs $K_L(x)$).

   (b) Prove that for all positive integers $n$, there exists a string $x$ of length $n$ such that $K(x) = O(\log n)$ and $K_L(x) \geq n$. (In fact, there is an algorithm that on input $n$ finds such a $x$.)

2. (Sipser 7.41) For a cnf-formula $\phi$ with $m$ variables and $c$ clauses (that is, $\phi$ is the AND of $c$ clauses, each of which is an OR of several variables), show that you can construct in polynomial time an $\mathsf{NFA}$ with $O(cm)$ states that accepts all nonsatisfying assignments, represented as Boolean strings of length $m$. Conclude that the problem of minimizing $\mathsf{NFA}$s (that is, on input a $\mathsf{NFA}$, find the $\mathsf{NFA}$ with the smallest number of states that recognizes the same language) cannot be done in polynomial time unless $\mathbf{P} = \mathbf{NP}$.

3. (Sipser 7.33) Prove that the following language is NP-hard

$$D = \{\langle p\rangle \mid p \text{ is a polynomial in several variables having an integral root}\}$$

   (The problem is in fact, undecidable. Turing first published the notion of a Turing machine and formalization of algorithms to prove the undecidability of this very problem.)