

CS 172 Spring 2007 — Discussion Handout 13

1. Getting started on hierarchies

Show that $\mathbf{NTIME}(n) \subsetneq \mathbf{PSPACE}$.

2. Diagonalization: hands-on

Prove that $\mathbf{NTIME}(n^2) \neq \mathbf{coNTIME}(n)$.

3. Scaling down or scaling up?

(Now a part of HW10)

Consider the function $pad : \Sigma^* \times \mathbb{N} \rightarrow \Sigma^* \#^*$ defined as $pad(s, l) = s\#^j$, where $j = \min(0, l - |s|)$. Thus, $pad(s, l)$ just adds enough copies of the new symbol $\#$ to the end of the string s so that the length of the new string is at least l . For a language A and a function $f : \mathbb{N} \rightarrow \mathbb{N}$, define the language $pad(A, f(n))$ to be

$$pad(A, f(n)) = \{pad(s, f(|s|)) \mid s \in A\}$$

(a) Prove that if $A \in \mathbf{TIME}(n^6)$, then $pad(A, n^2) \in \mathbf{TIME}(n^3)$

(b) Define $\mathbf{EXPTIME} = \mathbf{TIME}(2^{n^{O(1)}})$ and $\mathbf{NEXPTIME} = \mathbf{NTIME}(2^{n^{O(1)}})$. Use the function pad to prove that

$$\mathbf{NEXPTIME} \neq \mathbf{EXPTIME} \Rightarrow \mathbf{P} \neq \mathbf{NP}$$