1. Simple algorithmic tracing (5 points)

![Diagram of a tree structure with nodes labeled as follows:
- Talc
- Gypsum
- Calcite
- Orthoclase
- Apatite
- Fluorite
- Quartz
- Corundum
- Diamond
- Topaz]
2. Recursion (15 points)

```cpp
/*
 * Function: isReducible
 * Usage: if (isReducible(word, english)) . . .
 * --------------------------------------------------------
 * Determines whether a word is reducible, which means that (a) it
 * is a word and (b) it is possible to delete some letter and still
 * have a reducible word. To simplify the recursion, the empty
 * string is defined to be reducible.
 */

bool isReducible(string word, Lexicon & english) {
    if (word.length() == 0) {
        return true;
    } else if (!english.contains(word)) {
        return false;
    } else {
        for (int i = 0; i < word.length(); i++) {
            if (isReducible(removeLetter(word, i), english)) return true;
        }
        return false;
    }
}

/*
 * Function: removeLetter
 * Usage: string shorter = removeLetter(str, k);
 * --------------------------------------------------------
 * Returns the string of letters remaining after deleting the
 * character at index position k from the string str.
 */

string removeLetter(string str, int k) {
    if (k < 0 || k >= str.length()) {
        error("removeLetter: Index out of range");
    }
    str.erase(k, 1);
    return str;
}
```
3. Linear structures and hash tables (15 points)

```cpp
/*
 * Function: listMultiwordMatches
 * Usage: listMultiwordMatches(words, webMap);
 * ---------------------------------------------------------
 * Lists all of the matches for the sequence of words contained in the
 * first argument using the data for web matches stored in the webMap
 * table, as described in the problem.
 */

void listMultiwordMatches(Vector<string> & words,
                          Map<string, Vector<WebEntry> > & webMap) {
    foreach (WebEntry entry in webMap[words[0]]) {
        if (otherWordsMatch(words, entry, webMap)) {
            cout << entry.url << ":" << entry.index << endl;
        }
    }
}

/*
 * Function: otherWordsMatch
 * Usage: if (otherWordsMatch(words, entry, webMap)) . . .
 * ---------------------------------------------------------
 * Returns true if all the words (not counting the initial entry)
 * match the pattern specified by the entry structure. To do so,
 * those entries must have the same URL and an index that is
 * adjusted by the index of the word in the words list.
 */

bool otherWordsMatch(Vector<string> & words, WebEntry & entry,
                      Map<string, Vector<WebEntry> > & webMap) {
    WebEntry entryCopy = entry;
    for (int i = 1; i < words.size(); i++) {
        entryCopy.index++;
        if (!occursIn(entryCopy, webMap[words[i]])) return false;
    }
    return true;
}

/*
 * Function: occursIn
 * Usage: if (occursIn(entry, list)) . . .
 * ---------------------------------------------------------
 * Returns true if the entry appears somewhere in the list of matches.
 */

bool occursIn(WebEntry & entry, Vector<WebEntry> & list) {
    for (int i = 0; i < list.size(); i++) {
        if (entry.url == list[i].url & entry.index == list[i].index) {
            return true;
        }
    }
    return false;
}
```
4. Trees (15 points)

```c
/*
 * Implementation notes: unparse, unparseAtPrecedence
 * -------------------------------
 * The unparse function itself is a wrapper that uses unparseAtPrecedence
 * to do the real work. The prec argument indicates the current precedence.
 * Operators that have a lower precedence must be parenthesized. This
 * implementation ensures that operators associate to the left by
 * increasing the prevailing precedence for the right-hand operator
 * by 0.5; it would be equally effective to pass a boolean parameter
 * indicating whether this operand is a left or right child of its parent.
 */

string unparse(Expression *exp) {
    return unparseAtPrecedence(exp, 0);
}

string unparseAtPrecedence(Expression *exp, double prec) {
    ExpressionType type = exp->getType();
    if (type == CONSTANT || type == IDENTIFIER) {
        return exp->toString();
    } else {
        string op = exp->getOperator();
        Expression *lhs = exp->getLHS();
        Expression *rhs = exp->getRHS();
        int newPrec = precedence(op);
        string str = unparseAtPrecedence(lhs, newPrec);
        str += " " + op + " ";
        str += unparseAtPrecedence(rhs, newPrec + 0.5);
        if (newPrec < prec) str = "(" + str + ")";
        return str;
    }
}
```
5. Graphs (15 points)

```cpp
/*
 * Implementation notes: isBiconnected
 * -------------------------------
 * The only subtle piece of this implementation is the strategy
 * used in the inner foreach loop to select one node (first) and
 * put the other neighbors in a set (otherNeighbors). Once that
 * has been done, the solution to this problem is simply a matter
 * of checking that there is a path from first to each of the
 * other neighbors that doesn't require going through the node
 * you're removing from the graph.
 */

bool isBiconnected(Graph<Node, Arc> & g) {
    Set<Node> * visited;
    for (Node *node : g.getNodeSet()) {
        Set<Node> * neighbors = g.getNeighbors(node);
        Node *firstNeighbor = neighbors.first();
        for (Node *np : neighbors) {
            visited.clear();
            visited.add(node);
            if (!pathExistsVisited(firstNeighbor, np, visited)) {
                return false;
            }
        }
    }
    return true;
}

// The pathExistsVisited function appears on Handout 52A */

bool pathExistsVisited(Node *start, Node *finish, Set<Node> * & visited) {
    if (visited == start) return true;
    if (visited.contains(start)) return false;
    visited.add(start);
    for (Arc *arc : start->arcs) {
        if (pathExistsVisited(arc->finish, finish, visited)) return true;
    }
    return false;
}
```

6. Data structure design (15 points)

6a)

```cpp
/* Private section */

private:
    ValueType **elements; /* Two-level dynamic array of elements */
    int nRows; /* The number of rows in this grid */
    int nCols; /* The number of columns in this grid */
};
```
8. Short essay (10 points)

The critical point to make in this essay is that adopting a greedy strategy can miss an optimal solution. For full credit, answers must include at least something approaching an example, which is in line with my experience that students in the LaIR don’t usually understand this point until I give them an example. Here is one that fails for a stock length of 10 with either first-fit decreasing or first-fit increasing:

\[ [6, 6, 3, 3, 2, 2, 2, 2, 2, 2] \]
The following diagrams illustrate the problem:

Greedy algorithm requires 4

Optimal arrangement requires 3