Advanced Robotic Manipulation | Handout

CS327A (Spring 2017)

Problem Set #3 Due **Thurs, May** 4^{th}

Guidelines:

- This homework has both problem-solving and programming components. So please start early.
- In problems that require mathematical derivations, please try to be as detailed as possible so we can reward partially correct answers.
- For problems involving numerical answers, round off answers to two significant digits after the decimal point.
- We will be using the SAI2 (Simulation and Active Interfaces) software framework, developed in the Stanford Robotics Lab for this and future homeworks. Note that the SAI2-Simulation library is currently closed source. *So please do not forward or distribute.*
- Collaboration is allowed on this homework, but each student must submit their own original content (solution and code). In case you did collaborate, please note the names of other students you have worked with.

Submission Details:

We are using Gradescope for this class. So please submit your homework write-up through the Gradescope website directly. `https://gradescope.com/courses/7467`

For code submission, we'll be providing you a submission script with instructions soon.

Software update

The source code for this homework requires the latest version of SAI2-Common and the CS327A class repository.
To update the SAI2-Common repository, from within the `sai2-common` directory that you installed as part of homework 1, run

```
$ git pull
$ cd build && cmake -DCMAKE_BUILD_TYPE=Release .. && make && cd ../
```

To update the class repository, from within the `cs327a` directory, run

```
$ git stash
$ git pull --rebase
$ git stash pop
```

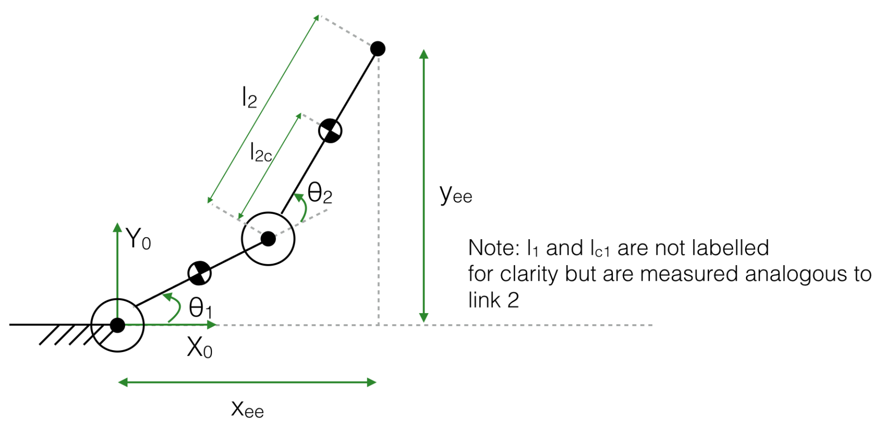
If you are asked to configure your computer for `git` with a username and email address, follow the instructions to do so, then run the above commands again.

Problem 1 - Inertial properties

In this problem, you will explore the inertial properties of a manipulator at its end-effector.

(a) Design optimization of RR manipulator

Suppose you have to design a RR planar manipulator as shown below to operate with an isotropic effective mass in the neighborhood of a given operating point. The only design parameters you are free to choose are the two link lengths, within some given range. Each link is modeled as a uniform cylinder with a fixed radius, $R = 0.1m$.



Answer the following questions:

i. For the given nominal position of the end-effector, $x_{ee,n} = 0.75m$, $y_{ee,n} = 0.0m$, find the inverse kinematics solution in the elbow up position. Your solution, q_n must be a function of l_1 and l_2.

ii. Given $A(q)$ and $J_v(q)$ at the end-effector, compute the minimum and maximum effective mass at the end-effector for the configuration found above, $q = q_n$. Hint: Assume you are provided a function `sigma = svd(L)` that returns an array of singular values of the matrix L, $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ such that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n \geq 0$. You can leave your answer in terms of the σ's.

iii. Over a range of $l_1 \in [0.1,\ 1.0]m$ and $l_2 \in [0.1,\ 1.0]m$, find a configuration of the manipulator at which the ratio of minimum to maximum effective mass is closest to unity. The MATLAB script under `hw3/p1.m` is meant to assist you in this. Implement your solutions from the parts above in the portions marked as `FILL ME IN`. Running the script generates two contour plots, one showing the ratio of minimum to maximum singular values, and the other showing the minimum singular values across the different possible combinations of l_1 and l_2. Attach the resulting plots. Explain the plots qualitatively.

(b) Belted ellipsoid of effective mass

For the RR manipulator above, now assume $l_1 = 0.6m$ and $l_2 = 0.5m$. For these link lengths, the masses, centers of mass and moment of inertia of the links are given by $m_1 = 11.3kg$, $m_2 = 9.4kg$, $l_{c1} = 0.3m$, $l_{c2} = 0.25m$, $I_1 = 0.3676kg.m^2$ and $I_2 = 0.2199kg.m^2$.

i. Evaluate the general inverse kinematic solution you found in part (a) for the given l_1 and l_2 to find q_n.

ii. Draw the belted ellipsoid of effective mass at the end effector in frame $\{0\}$ for the configuration q_n. The linear velocity Jacobian for the end effector in frame $\{0\}$ and the generalized mass matrix are as follows:

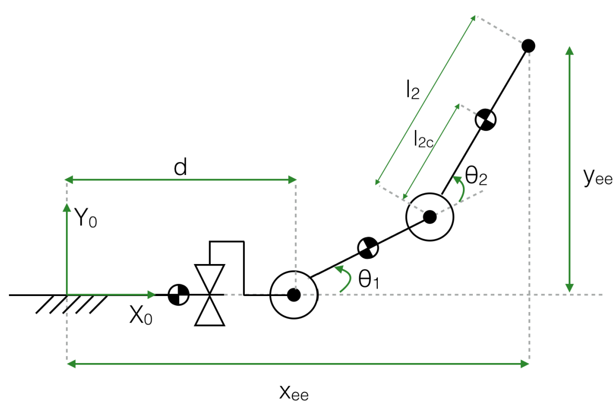
$$J_v = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}$$

$$A = \begin{bmatrix} I_1 + I_2 + m_1 l_{c1}^2 + m_2 \left(l_{c2}^2 + l_1^2 + 2 l_1 l_{c2} c_2\right) & I_2 + m_2 \left(l_{c2}^2 + l_1 l_{c2} c_2\right) \\ I_2 + m_2 \left(l_{c2}^2 + l_1 l_{c2} c_2\right) & I_2 + m_2 l_{c2}^2 \end{bmatrix}$$

where $s_1 = \sin\theta_1$, $s_{12} = \sin(\theta_1 + \theta_2)$, $c_1 = \cos\theta_1$, $c_{12} = \cos(\theta_1 + \theta_2)$.

(c) Macro-mini inertial properties

Now, we will explore the effect of mounting the RR manipulator on a moving base, as shown below.



Once again assume that the revolute joints are at the configuration q_n you found above for link lengths equal to the $l_1 = 0.6m$ and $l_2 = 0.5m$.

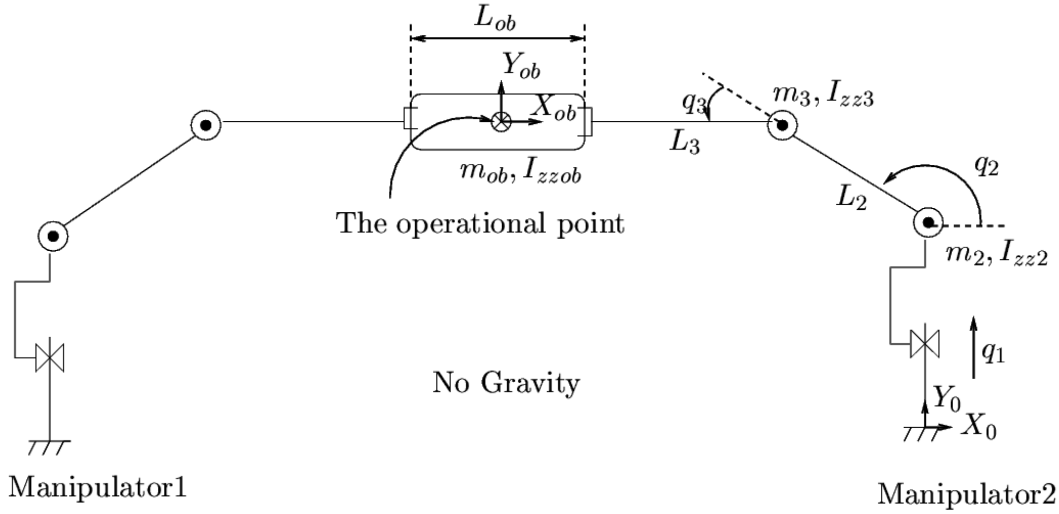
Assume that the base has a mass $m_0 = 10kg$ and is at position $d = 0m$ for the current analysis.

i. Find the modified linear velocity Jacobian in frame $\{0\}$ and the modified A matrix.

ii. Find the belted ellipsoid of effective mass at the end effector, again in frame $\{0\}$, and overlay on top, the belted ellipsoid you found in part (b). Explain your observation.

Problem 2 - Augmented Object Model/Virtual Linkage

Let's consider two planar PRR-manipulators with an object. The task is to *position* and *orient* the object. Both PRR-manipulators are the same: $m_{object} = 1kg$, $I_{zz,object} = 1kgm^2$ and $L_{object} = 1m$.



The physical properties of each PRR-manipulator are $m_2 = m_3 = 1kg, I_{zz2} = I_{zz3} = 1kgm^2$, and $L_2 = L_3 = 1m$ (you may assume the prismatic joint is massless). The corresponding mass matrix and Jacobian for the end-effector are:

$$J = \begin{bmatrix} 0 & -L_2 sin(q_2) - L_3 sin(q_2+q_3) & -L_3 sin(q_2+q_3) \\ 1 & L_2 cos(q_2) + L_3 cos(q_2+q_3) & L_3 cos(q_2+q_3) \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -s_2 - s_{23} & -s_{23} \\ 1 & c_2 + c_{23} & c_{23} \\ 0 & 1 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} m_2 + m_3 & m_3 L_2 c_2 & 0 \\ m_3 L_2 c_2 & m_3 {L_2}^2 + I_{zz2} + I_{zz3} & I_{zz3} \\ 0 & I_{zz3} & I_{zz3} \end{bmatrix} = \begin{bmatrix} 2 & c_2 & 0 \\ c_2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

The Jacobian for the operational point is

$$J_{ob} = \begin{bmatrix} 0 & -L_2 sin(q_2) - (L_3 + \frac{L_{ob}}{2}) sin(q_2+q_3) & -(L_3 + \frac{L_{ob}}{2}) sin(q_2+q_3) \\ 1 & L_2 cos(q_2) + (L_3 + \frac{L_{ob}}{2}) cos(q_2+q_3) & (L_3 + \frac{L_{ob}}{2}) cos(q_2+q_3) \\ 0 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & -s_2 - 1.5 s_{23} & -1.5 s_{23} \\ 1 & c_2 + 1.5 c_{23} & 1.5 c_{23} \\ 0 & 1 & 1 \end{bmatrix}$$

Let's consider the configuration where $q_1 = 1m$, $q_2 = 30^o$, $q_3 = -30^o$ for the manipulator1 and $q_1 = 1m$, $q_2 = 150^o$, $q_3 = 30^o$ for the manipulator2.

i. Calculate the pseudo kinetic energy matrix $\Lambda_\oplus$ for this augmented object.

ii. Find the W matrix, which relates resultant forces/moments and the applied forces/moments at the grasp points in the local frame of the object, i.e.

$$\begin{bmatrix} f_r \\ m_r \end{bmatrix} = \begin{bmatrix} f_{r,x} \\ f_{r,y} \\ m_r \end{bmatrix} = W \begin{bmatrix} f_{1,x} \\ f_{1,y} \\ f_{2,x} \\ f_{2,y} \\ m_1 \\ m_2 \end{bmatrix}, \text{where} \quad W = [W_f \; W_m].$$

iii. Find E matrix which relates the applied forces at the grasp points with internal forces, i.e.

$$\begin{bmatrix} f_{1,x} \\ f_{1,y} \\ f_{2,x} \\ f_{2,y} \end{bmatrix} = Et, \text{where } t \text{ is tension between two grasping points.}$$

iv. Compute the *Grasp Description Matrix*, G.