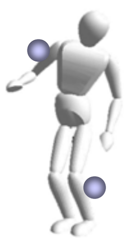
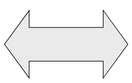
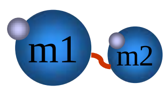
Inertial Properties

Efficient Dynamic Algorithms

Simulation, Contact Resolution, and Control







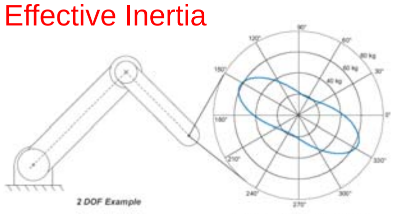
- Efficient algorithms for contact dynamics *O(n)*
- Avoid constraints elimination
- Cost-free effective mass at arbitrary contacts

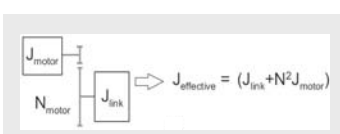
Human-Friendly Robots

- Safety
- Performance

Competing!

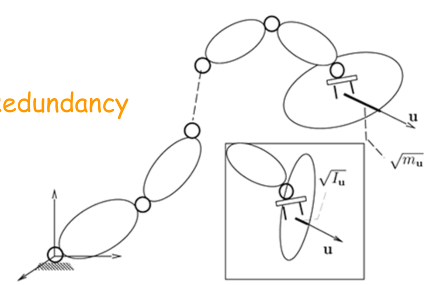
Effective Inertia





Effective Mass/Inertia

Task Redundancy



Redundancy

- Redundancy of a Manipulator

$$n > m_0$$

- Redundancy with respect to a Task

$$n > m_{Task(0)}$$

Task dynamics

$$\Lambda(q)\ddot{x} + \mu(q,\dot{q}) + p(q) = F$$

$$\Lambda = \left(JA^{-1}J^T\right)^{-1}$$

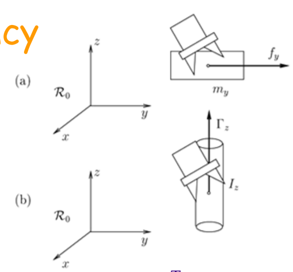
$$\mu(q,\dot{q}) = \bar{J}^T b(q,\dot{q}) - \Lambda(q)\dot{J}(q)\dot{q}$$

$$p(q) = \bar{J}^T g(q)$$

Task Redundancy

$$\dot{y} = \begin{pmatrix} \frac{\partial y}{\partial q_1} \cdots \frac{\partial y}{\partial q_n} \end{pmatrix} \dot{q}$$

$$\dot{y} = J_{v_y}\dot{q}$$



$$m_y = \frac{1}{J_{v_y} A^{-1} J_{v_y}^T}$$

$$J_{v_y} = y_0^T J_v$$

$$I_z = \frac{1}{J_{\omega_z} A^{-1} J_{\omega_z}{}^T}$$

$$J_{\omega_z} = z_0^T J_\omega$$

$$J_v = \begin{pmatrix} \frac{\partial x}{\partial q_1} & \cdots & \frac{\partial x}{\partial q_n} \\ \frac{\partial y}{\partial q_1} & \cdots & \frac{\partial y}{\partial q_n} \\ \frac{\partial z}{\partial q_1} & \cdots & \frac{\partial z}{\partial q_n} \end{pmatrix}$$

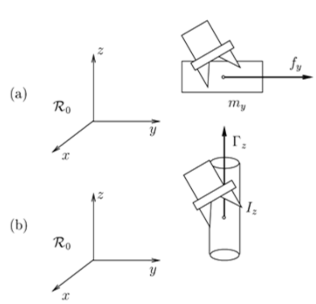
$$J_{v_y} = y_0^T J_v$$

$$J_{v_y} = (0\ 1\ 0)\, J_v = \begin{pmatrix} \frac{\partial y}{\partial q_1} & \cdots & \frac{\partial y}{\partial q_n} \end{pmatrix}$$

Effective Mass/Inertia

$$m_y = \frac{1}{y_0^T \Lambda_v^{-1} y_0}$$

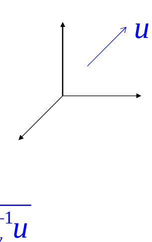
$$I_z = \frac{1}{z_0^T \Lambda_\omega^{-1} z_0}$$



Effective Mass/Inertia

$$J_{v_u} = u^T J_v$$

$$J_{\omega_u} = u^T J_\omega$$



Effective Mass

$$m_u(\Lambda_v) = \frac{1}{u^T \Lambda_v^{-1} u}$$

Effective Inertia

$$I_u(\Lambda_\omega) = \frac{1}{u^T \Lambda_\omega^{-1} u}$$

Effective Mass/Inertia

$$\Lambda_0^{-1} = J_0 A^{-1} J_0^T$$

$$J_0 = \begin{pmatrix} J_v \\ J_\omega \end{pmatrix}$$

$$\Lambda_0^{-1} = \begin{pmatrix} J_v \\ J_\omega \end{pmatrix} A^{-1} \begin{pmatrix} J_v^T & J_\omega^T \end{pmatrix}$$

Effective Mass/Inertia

$$\Lambda_0^{-1} = \begin{bmatrix} J_v A^{-1} J_v^T & J_v A^{-1} J_\omega^T \\ J_\omega A^{-1} J_v^T & J_\omega A^{-1} J_\omega^T \end{bmatrix}$$

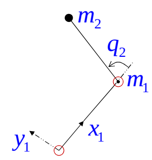
Λ_v^{-1} Λ_ω^{-1}

Effective Inertial Property

$$\sigma_w(\Lambda) = \frac{1}{w^T \Lambda^{-1} w}$$

w: A unit vector in the m_0-dimensional space

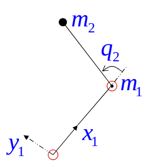
Example



$$J_{(1)} = \begin{pmatrix} -S2 & -S2 \\ 1+C2 & C2 \end{pmatrix}$$

$$A = \begin{pmatrix} m_1 + 2m_2(1+C2) & m_2(1+C2) \\ m_2(1+C2) & m_2 \end{pmatrix}$$

Redundancy



with respect to motions in the y_1 direction

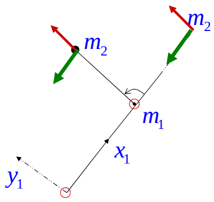
$$J_{y_1} = \begin{pmatrix} 1+C2 & C2 \end{pmatrix}$$

$$\Lambda_{y_1}^{-1} = J_{y_1} A^{-1} J_{y_1}^T$$

$$\Lambda_{y_1}^{-1} = \frac{1}{m_2\left(m_1 + m_2 S^2 2\right)} \begin{bmatrix} 1+C2 & C2 \end{bmatrix} \begin{bmatrix} m_2 & -m_2(1+C2) \\ -m_2(1+C2) & m_1 + 2m_2(1+C2) \end{bmatrix} \begin{bmatrix} 1+C2 \\ C2 \end{bmatrix}$$

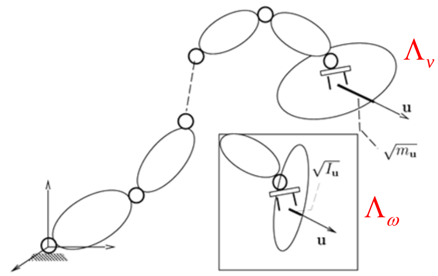
$$\frac{1}{m_{Y1}^*} = \frac{m_1 C^2 2 + m_2 S^2 2}{m_2\left(m_1 + m_2 S^2 2\right)}$$

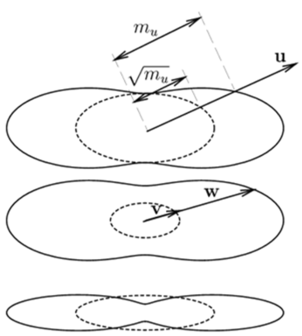
$$m_{y_1}^* = \frac{m_2\left(m_1 + m_2 S^2 2\right)}{m_1 C^2 2 + m_2 S^2 2}$$



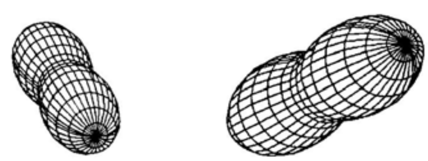
$$m_{x_1}^* = \frac{m_2\left(m_1 + m_2 S^2 2\right)}{\left(m_1 + m_2\right) S^2 2}$$

Effective Mass & Inertia



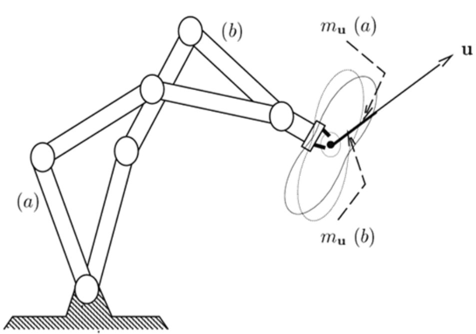


Belted Ellipsoid



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

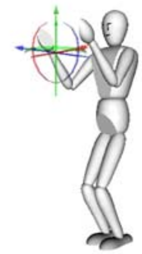
$$\Rightarrow \frac{x^2}{a^2\sqrt{x^2+y^2+z^2}} + \frac{y^2}{b^2\sqrt{x^2+y^2+z^2}} + \frac{z^2}{c^2\sqrt{x^2+y^2+z^2}} = 1$$



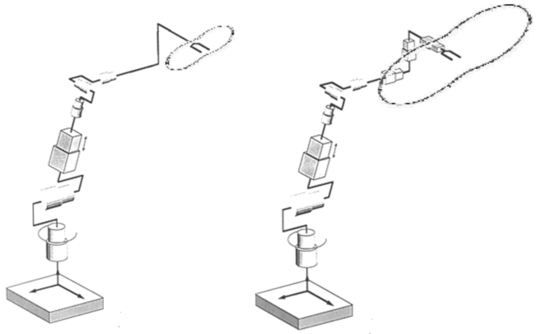
Macro/Mini Structures

Mobile Manipulators

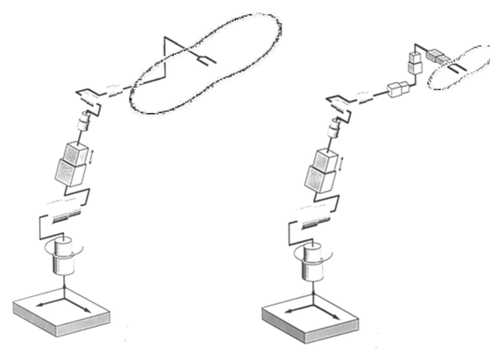
Humanoids



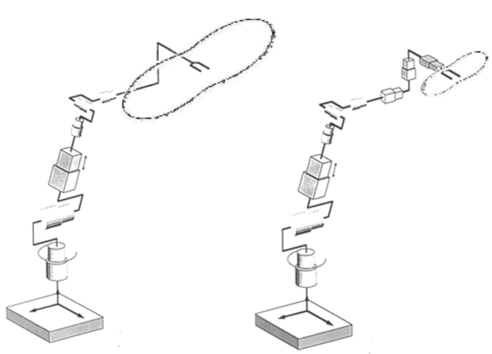
Inertial Properties



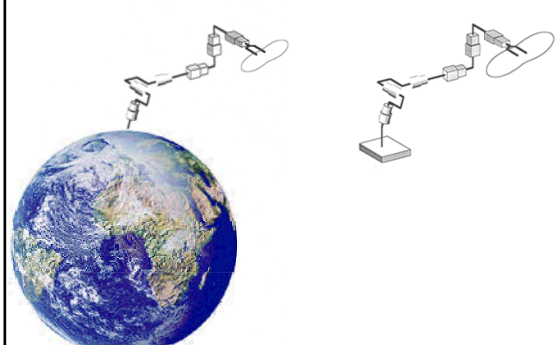
Inertial Properties



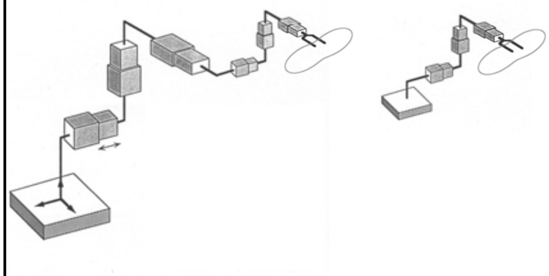
Inertial Properties



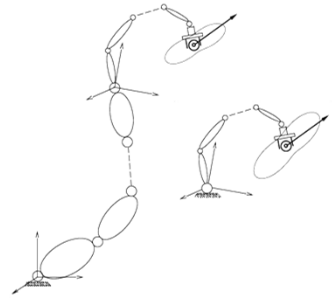
Reduced Effective Inertia



Effective Inertia: Equality



Reduced Effective Inertia



Inertia Property

Effective Inertia/mass perceived in a direction $\mathbf{w}$

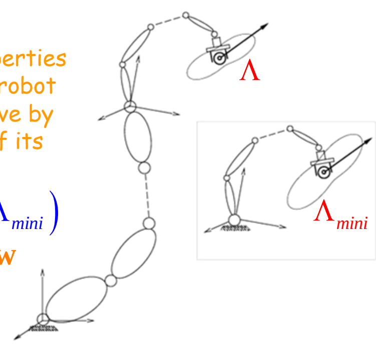
$$\sigma_w(\Lambda) = \frac{1}{\mathbf{w}^T \Lambda^{-1} \mathbf{w}}$$

Theorem: Reduced Effective Inertia

The inertial properties of a macro/mini robot are bounded above by the properties of its mini structure

$$\sigma_w(\Lambda) \le \sigma_w(\Lambda_{mini})$$

in any direction $\mathbf{w}$



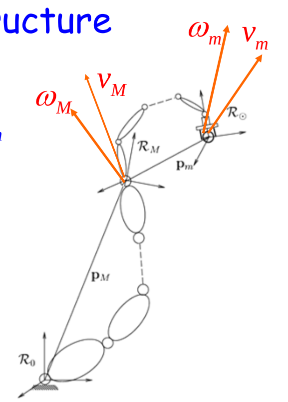
Macro/Mini Structure

$$v = v_M + v_m + \omega_M \times p_m$$

$$\omega = \omega_M + \omega_m$$

$$J_0 = [V\, J_M \mid J_m]$$

$$V = \begin{bmatrix} I_3 & -\hat{p}_m \\ 0 & I_3 \end{bmatrix}$$

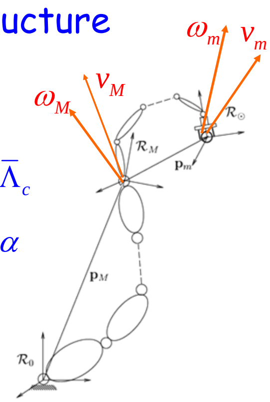


Macro/Mini Structure

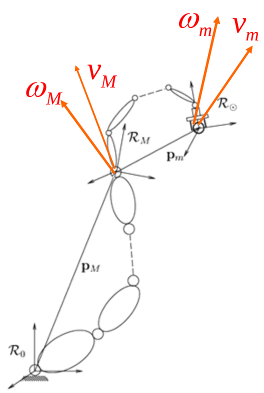
- $\Lambda^{-1} = \Lambda_m^{-1} + \bar{\Lambda}_c$

$$\frac{1}{\sigma_w(\Lambda)} = \frac{1}{\sigma_w(\Lambda_m)} + \alpha$$

- $\alpha = w^T \bar{\Lambda}_c w$



A and A_m



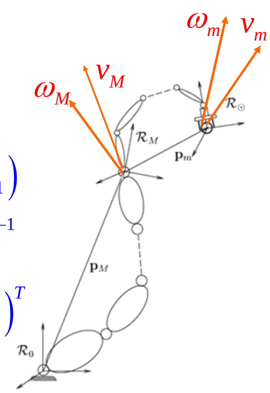
$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix}$$

$$A_m = A_{22}$$

Λ and Λ_m

$$\Lambda^{-1} = \Lambda_m^{-1} + \bar{\Lambda}_c$$

$$\bar{\Lambda}_c = \left(VJ_M - J_m A_m^{-1} A_{21}\right) \left(A_{11} - A_{21}^T A_m A_{21}\right)^{-1} \left(VJ_M - J_m A_m^{-1} A_{21}\right)^T$$



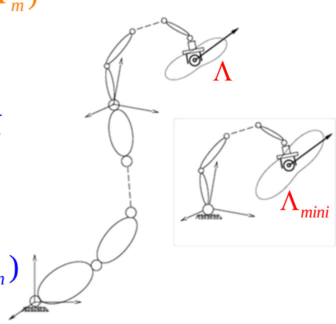
$\sigma_w(\Lambda)$ and $\sigma_w(\Lambda_m)$

$$\sigma(\Lambda) = \frac{1}{w^T \Lambda^{-1} w}$$

Theorem

$$\sigma_w(\Lambda) \le \sigma_w(\Lambda_m)$$

for any w



Corollary

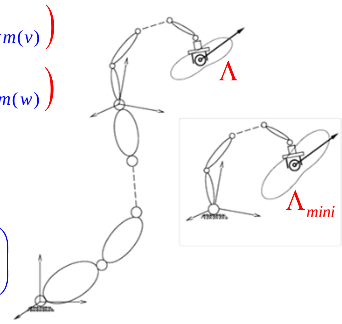
$$m_u\left(\Lambda_v\right) \le m_u\left(\Lambda_{m(v)}\right)$$

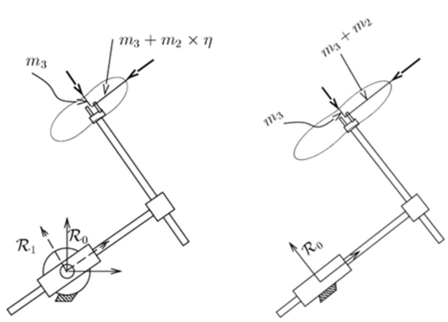
$$I_u\left(\Lambda_w\right) \le I_u\left(\Lambda_{m(w)}\right)$$

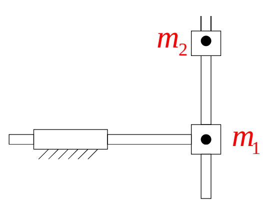
$$J_v = \begin{bmatrix} I & 0 \end{bmatrix} J$$

$$J_\omega = \begin{bmatrix} 0 & I \end{bmatrix} J$$

$$\bar{\Lambda}_{c(v)} = \begin{pmatrix} I & 0 \end{pmatrix} \bar{\Lambda}_c \begin{pmatrix} I \\ 0 \end{pmatrix}$$







$$\Lambda = \begin{pmatrix} m_1 + m_2 & 0 \\ 0 & m_2 \end{pmatrix}$$

$$\sigma_{y_0} = \frac{1}{y_0^T \Lambda^{-1} y_0}$$

$$\Lambda^{-1} = \begin{pmatrix} \frac{1}{m_1 + m_2} & 0 \\ 0 & \frac{1}{m_2} \end{pmatrix}$$

$$\frac{1}{\sigma_{Y_0}} = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{m_1 + m_2} & 0 \\ 0 & \frac{1}{m_2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{m_2}$$

Kinetic energy matrix, Λ_m associated with the 2-degree-of-freedom mini-manipulator

$$\Lambda_m = \begin{bmatrix} m_2 + m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$J_{(1)} = \begin{bmatrix} -q_3 & 1 & 0 \\ q_2 & 0 & 1 \end{bmatrix}$$

Joint space kinetic energy matrix

$$A(q) = \begin{bmatrix} I + m_1 q_2^2 + m_2\left(q_2^2 + q_3^2\right) & -q_3 m_2 & q_2 m_2 \\ -m_2 q_3 & m_1 + m_2 & 0 \\ m_2 q_2 & 0 & m_2 \end{bmatrix}$$

Kinetic energy matrix, $\Lambda_{0(1)}$ associated with the 3-degree-of-freedom macro-/mini-manipulator

$$\Lambda_{(1)} = \begin{bmatrix} m_2 + m_1 \times \eta & 0 \\ 0 & m_2 \end{bmatrix}$$

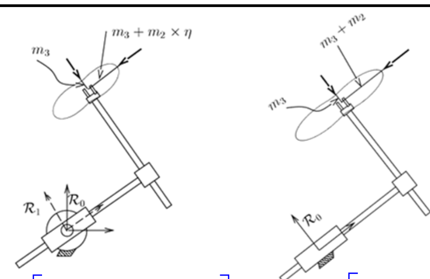
where $$\eta = \frac{I + m_1 q_2^2}{I + m_1\left(q_2^2 + q_3^2\right)}$$

Kinetic energy matrix, Λ_0

$$\Lambda = \Omega \; \Lambda_{(1)} \; \Omega^T$$

where $$\Omega = \begin{bmatrix} C1 & -S1 \\ S1 & C1 \end{bmatrix}$$

with $S1 \doteq \sin(q_1)$ and $C1 \doteq \cos(q_1)$



$$\Lambda_{\{1\}} = \begin{bmatrix} m_3 + m_2 \times \eta & 0 \\ 0 & m_3 \end{bmatrix} \qquad \Lambda_m = \begin{bmatrix} m_2 + m_3 & 0 \\ 0 & m_3 \end{bmatrix}$$

$$o \le \eta \le 1; \qquad \eta = \frac{I + m_2 q_2^2}{I + m_2\left(q_2^2 + q_3^2\right)}$$