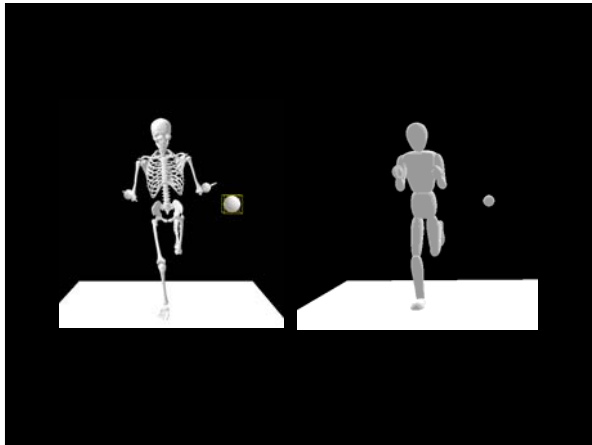
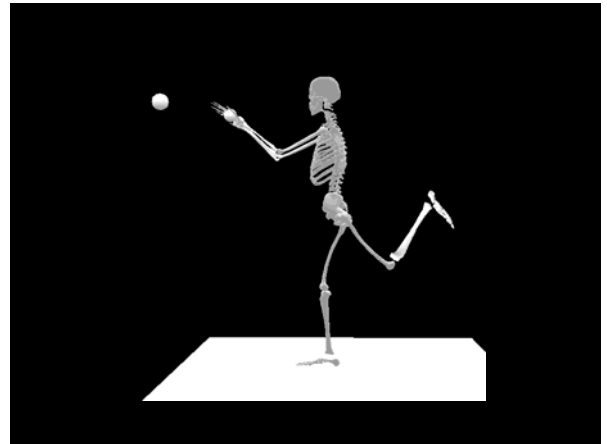




Redundancy



Equations of Motion

Joint Space

$$A(q)\ddot{q} + b(q, \dot{q}) + g(q) = \Gamma$$

Operational Space

$$\Lambda(x)\ddot{x} + \mu(x, \dot{x}) + p(x) = F$$

Relationships

$$\Gamma = J^T F$$

$$(A\ddot{q} + b + g) = J^T (\Lambda\ddot{x} + \mu + p)$$

$$(A\ddot{q} + b) = J^T (\Lambda\ddot{x} + \mu) \quad \text{Inertial forces}$$

Non Redundancy

$$A\ddot{q} + b + g = \Gamma \quad \text{(joint dynamics)}$$

$$J^{-T}$$

$$J^T$$

$$\Lambda\ddot{x} + \mu + p = F \quad \text{(Task dynamics)}$$

Redundancy

$$A\ddot{q} + b + g = \Gamma \quad \text{(joint dynamics)}$$

$$\text{projection } \bar{J}^T$$

$$J^T$$

$$\Lambda\ddot{x} + \mu + p = F \quad \text{(Task dynamics)}$$

where

$$\bar{J} = A^{-1} J^T \Lambda \quad \text{and} \quad \Lambda^{-1} = J A^{-1} J^T$$

\bar{J} : dynamically consistent generalized inverse

Redundancy

$\delta q = J^\# \delta x + [I - J^\# J] \delta q_0$
 $\Gamma = J^T F + \Gamma_{nullspace}$

Redundancy

Joint/Task Displacements
 $\delta x = J \delta q$
 $\delta q = J^\# \delta x + [I - J^\# J] \delta q_0$

Joint/Task Forces
 $\Gamma = J^T F$
 $F = ?$

$\Gamma = J^T F$

Given F , Γ is $(J^T F)$

Given Γ , what is F

$F = J^\# \Gamma ?$

However

different selections of $J^\#$ ($J = J J^\# J$)
 would lead
 to different solutions

Gravity Example

$p = ?$ $p = J^{+T} g$
 $J^+ = J^T (J J^T)^{-1}$

Gravity Example

$p = ?$ $p = \bar{J}^T g$
 $\bar{J} = A^{-1} J^T \Lambda$

Redundancy

$\Gamma^T \delta q \geq F^T \delta x$
 Assuming a Virtual Displacement
 $\delta q = J^\# \delta x + (I - J^\# J) \delta q_0$

Virtual Displacement $\delta q = J^\# \delta x + (I - J^\# J) \delta q_0$

Virtual Work $\delta w = \Gamma^T \delta q$

$$\delta w = \delta w_1 + \delta w_2$$

$$(J^{\#T} \Gamma)^T \delta x \quad \left[(I - J^\# J)^T \Gamma \right]^T \delta q_0$$

↓

$$\Gamma = J^T (J^{\#T} \Gamma) + (I - J^T J^{\#T}) \Gamma$$

Decomposition

$$\Gamma = J^T \left[J^{\#T} \Gamma \right] + \left[I - J^T J^{\#T} \right] \Gamma$$

Task Space
Forces
(F)

Joint Torques
acting in the
null space

$$\Gamma = J^T F + \left[I - J^T J^{\#T} \right] \Gamma_0$$

Dynamic Constraints

$$A(q) \ddot{q} + b(q, \dot{q}) + g(q) = \Gamma$$

$$\Gamma = J^T F + \left[I - J^T J^{\#T} \right] \Gamma_0$$

$$A \ddot{q} + (b + g) = J^T F + \left[I - J^T J^{\#T} \right] \Gamma_0$$

$$\ddot{q} + A^{-1}(b + g) = A^{-1} J^T F + A^{-1} \left[I - J^T J^{\#T} \right] \Gamma_0$$

$J \downarrow$

$$J \ddot{q} + JA^{-1}(b + g) = JA^{-1} J^T F + JA^{-1} \left[I - J^T J^{\#T} \right] \Gamma_0$$

$J \ddot{q} = \ddot{x} - \dot{J} \dot{q} \downarrow$

$$\ddot{x} + \left[JA^{-1}(b + g) - \dot{J} \dot{q} \right] = \underbrace{(JA^{-1} J^T)}_{\Lambda^{-1}} F + JA^{-1} \left[I - J^T J^{\#T} \right] \Gamma_0$$

$\ddot{x}_n = 0$

Dynamic Consistency

$\Gamma \longrightarrow JA^{-1} \Gamma$
 joint torques task acceleration

Relationship

$$\Gamma = J^T F + (I - J^T J^{\#T}) \Gamma_0$$

Dynamic Constraint

$$JA^{-1} (I - J^T J^{\#T}) \Gamma_0 \equiv 0$$

$$\Lambda \left\{ \begin{array}{l} JA^{-1} = (JA^{-1} J^T) J^{\#T} \\ (JA^{-1} J^T)^{-1} JA^{-1} = J^{\#T} \end{array} \right.$$

Dynamic Consistency

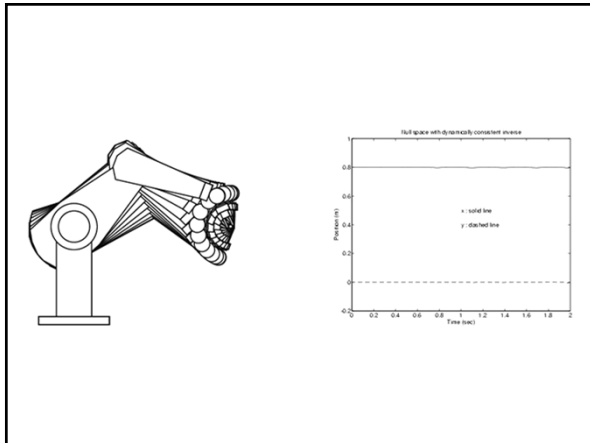
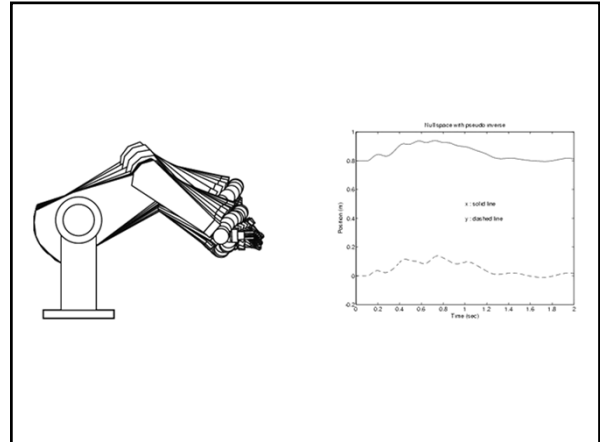
$\bar{J}(q)$ is the Dynamically Consistent Generalized Inverse

Theorem (Consistency)

$$\bar{J} \text{ is unique and } \bar{J} = A^{-1} J^T \Lambda$$

Non-redundant

$$\bar{J} = J^{-1}$$



Velocity Force Duality

	Velocity	Force
Non Red.	$\delta q = J^{-1} \delta x$	$\Gamma = J^T F$
Redundant	$\delta q = \bar{J} \delta x + [I - \bar{J} J] \delta q_0$	$\Gamma = J^T F + [I - J^T \bar{J}^T] \Gamma_0$

Task dynamics

$$\Lambda(q) \ddot{x} + \mu(q, \dot{q}) + p(q) = F$$

$$\Lambda = (J A^{-1} J^T)^{-1}$$

$$\mu(q, \dot{q}) = \bar{J}^T b(q, \dot{q}) - \Lambda(q) \dot{J}(q) \dot{q}$$

$$p(q) = \bar{J}^T g(q)$$

Redundant Robot Control

Task Space: J^T

Null Space: N^T

where $N = I - \bar{J} J$

Robot Control

$$\Gamma = J^T F + N^T \Gamma_0$$

$$\Gamma_1 \quad \Gamma_2$$

dynamically decoupled

Stability

$$\Gamma_{dis}^T \dot{q} \leq 0 ; \text{ for } \dot{q} \neq 0$$

$$\Gamma_{dis} = -k_v J^T \dot{x} = -k_v J^T J \dot{q}$$

$$\dot{q}^T D(q) \dot{q} \geq 0 ; \quad \dot{q} \neq 0$$

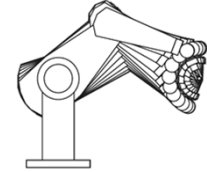
$$D(q) = k_v (J^T J)$$



$J^T J$: is a $n \times n$ matrix of rank m_0
it is Positive Semi-definite

The System is Stable, but not asymptotically stable

$$\dot{q}^T D(q) \dot{q} = 0$$



Asymptotic Stability

$$\Gamma_{dis}^T \dot{q} < 0 ; \text{ for } \dot{q} \neq 0$$

$$\Gamma_{dis} = -k_v J^T J \dot{q} - k_v N^T \dot{q}$$



$$D(q) = k_v (J^T J + N^T)$$

Positive definite

$$\dot{q}^T D(q) \dot{q} > 0 \quad \text{for } \dot{q} \neq 0$$

Asymptotic Stability

$$\Gamma_{dis}^T \dot{q} < 0 ; \text{ for } \dot{q} \neq 0$$

$$\Gamma_{dis} = -k_v J^T J \dot{q} - k_{vq} \dot{q}$$



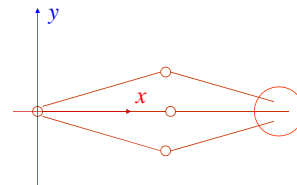
$$D(q) = k_v J^T J + k_{vq} I_n$$

Positive definite

$$\dot{q}^T D(q) \dot{q} > 0 \quad \text{for } \dot{q} \neq 0$$



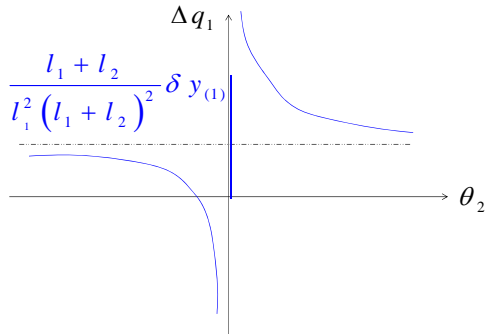
Kinematic Singularities



Joint Space Formulation

Find a pseudo-inverse J^+

Pseudo Inverse Solution



Kinematic Singularities

The end-effector mobility locally decreases

Singularities

$$S(q) = \det[J(q)] = S_1(q) \cdot S_2(q) \cdots S_{n_s}(q)$$

Singular direction

$S_i = 0$; ζ_i $\begin{cases} \rightarrow$ Infinite effective mass \\ \rightarrow infinite effective inertia \end{cases}

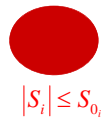
Kinematic Singularities

Singularity Neighborhood

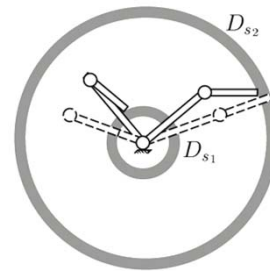
$$S(q) = S_1(q) \cdot S_2(q) \cdot S_3(q) \cdots S_{n_s}(q)$$

Singularity S_i

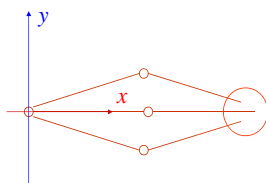
$$D_{S_i} = \{q \mid |S_i(q)| \leq S_{0_i}\}$$



Singularity Neighborhood



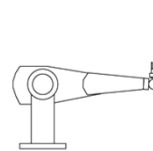
Approach



In D_S , the robot is treated as redundant w.r.t. motions in the subspace \perp to the singular direction

- Along Singular Directions:
Control in Null Space $\Gamma_{null-space}$
- In subspace \perp to singular direction
Control in sub-O-Space F_{sub-os}

Types of Singularities



Elbow Lock

Type 1



Wrist Lock



Overhead Lock

Type 2

Types of Singularities

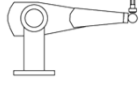
Control Strategy

Type 1

Motion in Null Space

⇒ Motion along/about ζ_i

Control S_i



Type 2

Motion in Null Space

⇒ Only changes of ζ_i

Control ζ_i



Singularity Control

$$\Gamma = J_{sub}^T F_{sub} + N_{sub}^T \Gamma_{S_i}$$

where

$$N_{sub} = I - \bar{J}_{sub} J_{sub} \quad \text{and} \quad \Gamma_{S_i} = -\nabla V_i(S_i)$$

Moving to a singularity

Control $S_i(q)$ to reach $S_i = 0$

Moving out of a singularity

Control \dot{S}_i from zero to the desired Velocity at the singularity boundary

