Some tips for doing CS223A problem sets:

- **Print this problem set and fill in your answers in the dedicated white space right below the question statement. Alternatively you may also submit a typewritten writeup.**
- Use abbreviations for trigonometric functions (e.g. \(c\theta\) for \(\cos(\theta)\), \(s_1\) or \(s_\theta_1\) for \(\sin(\theta_1)\)) in situations where it would be tedious to repeatedly write \(\sin, \cos\), etc.
- Unless instructed otherwise, leave square roots in symbolic form rather than writing out their decimal values.
- Use common sense for decimals –– if the question states \(a = 1.34\), then don’t give answers like \(2*a = 2.680001245735\).
- If you give a vector as an answer, make sure that you specify what frame it is given in (if it is not clear from context). The same rule applies to rotation and transformation matrices.

1. The mobile robot shown above can be represented as a PPRR manipulator with 4 links, as shown in the schematic.

   ![Mobile Robot Schematic](image)

   Luckily, you do not need to compute the forward kinematics, because they are given to you here:

   \[
   ^0T = \begin{bmatrix}
   1 & 0 & 0 & 0 \\
   0 & 1 & 0 & 0 \\
   0 & 0 & 1 & d_1 \\
   0 & 0 & 0 & 1 
   \end{bmatrix},
   ^0T = \begin{bmatrix}
   0 & -1 & 0 & 0 \\
   0 & 0 & 1 & d_2 \\
   -1 & 0 & 0 & d_1 \\
   0 & 0 & 0 & 1 
   \end{bmatrix},
   ^0T = \begin{bmatrix}
   0 & 0 & 1 & 3 \\
   s_3 & c_3 & 0 & d_2 \\
   -c_3 & s_3 & 0 & d_1 \\
   0 & 0 & 0 & 1 
   \end{bmatrix},
   \]

   \[
   ^0T = \begin{bmatrix}
   s_4 & c_4 & 0 & 3 + s_4 \\
   s_3c_4 & -s_3s_4 & -c_3 & d_2 + 2s_3 \\
   -c_3c_4 & c_3s_4 & -s_3 & d_1 - 2c_3 \\
   0 & 0 & 0 & 1 
   \end{bmatrix},
   \]

   \[
   ^{0E}T = \begin{bmatrix}
   s_4 & c_4 & 0 & 3 + s_4 \\
   s_3c_4 & -s_3s_4 & -c_3 & d_2 + 2s_3 + s_3c_4 \\
   -c_3c_4 & c_3s_4 & -s_3 & d_1 - 2c_3 - c_3c_4 \\
   0 & 0 & 0 & 1 
   \end{bmatrix}.
   \]
(a) Find the linear velocity and angular velocity of the end-effector in frame \{0\} as a function of the joint variables.

(b) Find the linear velocity of the end-effector with respect to frame \{2\} which is moving with the mobile base (please express this velocity in basis vectors of frame \{2\}).
(c) Assuming that the arm is straight out ($\theta_4 = 0^\circ$), find the joint velocities that will achieve an end-effector angular velocity of $\mathbf{w} = [1 \ 0 \ 0]^T$ in frame $\{0\}$ while keeping the end-effector’s position stationary (that is, linear velocities are zero!). Use the Jacobian and the information given to construct an equation of the form $\dot{\mathbf{q}} = \mathbf{f}(\mathbf{q})$. Use this result to find a relationship $g(\dot{q}_1, \dot{q}_2) = 0$ that must hold for any configuration $\mathbf{q}$ of the robot which satisfies the problem so far. Physically describe this constraint between $\dot{q}_1$ and $\dot{q}_2$.

(d) If the robot is stationary (i.e. $\dot{\mathbf{q}} = \mathbf{0}$ and $\ddot{\mathbf{q}} = \mathbf{0}$), and we apply a force (measured in frame $\{0\}$) of $\mathbf{0} = [F_x \ F_y \ F_z]^T$ on the end-effector, what are the resulting joint torques?
2. You are presented with the RRR manipulator below.

\[
\begin{bmatrix}
L_3 c_{23} + L_2 c_{12} \\
L_3 s_1 c_{23} + L_2 s_1 c_{2} \\
L_1 + L_2 s_2 + L_3 s_{23}
\end{bmatrix}
\]

The position of the end-effector is

\[
^0P_E = \begin{bmatrix}
L_3 c_1 c_{23} + L_2 c_1 c_2 \\
L_3 s_1 c_{23} + L_2 s_1 c_{2} \\
L_1 + L_2 s_2 + L_3 s_{23}
\end{bmatrix}
\]

where \(c_{23} = \cos(\theta_2 + \theta_3)\).

(a) Derive the Jacobian relating joint velocities to linear velocities of the end-effector.
(b) Find the singular configurations of this manipulator. For each singularity, draw the robot configuration and clearly state how movement is restricted in this configuration. (Hint: Try converting your Jacobian to frame \( \{1\} \) first.)