1. The vector $A^P$ is rotated about $\hat{Y}_A$ by $\theta$ degrees and is subsequently rotated about $\hat{X}_A$ by $\phi$ degrees.

   (a) Give the rotation matrix, which accomplishes these rotations in the given order.

   (b) What is the modified vector if $\theta = 45^\circ$ and $\phi = 60^\circ$?
2. A frame \{B\} is initially coincident with a frame \{A\}. We rotate \{B\} about \(\hat{X}_B\) by \(\phi\). Next, we rotate the resulting frame \{B\} about \(\hat{Y}_B\) by \(\theta\) degrees. Finally, we rotate the resulting frame B about \(\hat{Z}_B\) by \(\phi + \theta\). Determine the \(3 \times 3\) rotation matrix, \(A_B^R\), which will change the description of a vector \(P\) in frame \{B\}, \(B_P\), to frame \{A\}, \(A_P\).

3. Frame \{A\} and frame \{B\} are fixed with respect to an inertial ground frame.

(a) Consider a velocity vector in frame \{A\}, \(A\mathbf{V}\). How will it change if we express it in frame \{B\}? Are \(A\mathbf{V}\) and \(B\mathbf{V}\) the same vector? Comment upon their magnitudes and directions. If they are different how can you transform one into the other?
(b) Given

\[ A \mathbf{v} = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}, \]

find \( B \mathbf{v} \) if transforming frame \( \{A\} \) into frame \( \{B\} \) requires translating \( \{A\} \) by \((1, 4, 3\sqrt{2})\) and then rotating it 45 degrees about the vector \((1,1,1)\). Use \((x,y,z)\) for the vector components.
4. Given the following transformation matrices:

\[
T_1 = \begin{bmatrix}
-\frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} & 1 \\
0 & 1 & 0 & -2 \\
-\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} & -1 \\
0 & 0 & 0 & 1
\end{bmatrix},
T_2 = \begin{bmatrix}
-\frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} & 0 \\
0 & 1 & 0 & 1 \\
\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
T_3 = \begin{bmatrix}
-\frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} & 1 \\
0 & 1 & 0 & 0 \\
\frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} & -1 \\
0 & 1 & 0 & 1
\end{bmatrix}
\]

(a) Are T1, T2 and T3 valid transformation matrices? Explain why or why not. (We define a transformation matrix as a rotation and a translation, i.e. a "rigid body" transformation)

(b) Find the Euler parameters that represent the rotations for the correct matrix (or matrices).

(c) Also find the unit vector that defines the axis of rotation, and the angle of rotation for the correct matrix (or matrices).