

Extended Operational Space Formulation for Serial-to-Parallel Chain (Branching) Manipulators

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Abstract

This paper extends the Operational Space Formulation to the important class of serial-to-parallel chain (branching) manipulators. The various models and concepts developed in operational space, such as *Dynamically Consistent Force/Torque Decomposition* for control of redundant manipulators and the *Augmented Object Model* for cooperative manipulator systems, are shown to extend directly. Dynamic modeling and experimental results for a free-flying, two-arm space robot are presented to validate this extension.

1 Introduction

A growing body of work has emerged in the area of branching manipulators – systems in which several manipulators branch from a macro-manipulator or mobile base. Branching systems are common in applications such as mobile-based terrestrial, industrial, construction, underwater, and space vehicles with multiple manipulators, multi-legged walking robots, and fixed-base macro-mini systems with multiple end-effectors such as dexterous arms with fingers.

Initial work in dynamic modeling and control of branching systems has been reported for two-arm, free-flying space robots by Ullman [1] and Yoshida, et al. [2], and for multi-legged robots by Sunada, et al. [3] This initial work has drawn upon previous developments in control of redundant serial manipulators [4, 5] and cooperative manipulation by multiple fixed-base manipulators and redundant manipulators. [6, 7, 8, 9, 10]

The Operational Space Formulation [11, 12] is particularly useful for addressing the dynamic modeling and control of complex, redundant systems because it explicitly focuses upon the dynamic performance of the end-effector in the space in which manipulation

is performed. Complex internal dynamics for redundant systems are decoupled from the dynamics at the end-effector. Russakow and Khatib [13] have applied this formulation to free-flying, redundant robots with a single manipulator.

This paper presents an extension of the Operational Space Formulation to branching manipulator systems, a general class of manipulator systems in which multiple (sometimes cooperating) manipulators may be dynamically coupled. The basic structure of the formulation, as well as the concepts of *Dynamically Consistent Force/Torque Decomposition* of redundant systems and the *Augmented Object Model* for control, are preserved.

Section 2 briefly summarizes the Operational Space Formulation for serial manipulators and cooperating serial manipulators. Section 3 presents a more general formulation called Extended Operational Space, which addresses branching systems. Lastly, Section 4 features an experimental implementation of Extended Operational Space modeling and control for a free-flying, two-arm space robot.

2 Operational Space Review

The Operational Space Formulation projects the joint space dynamics of the system into the operational space, permitting both dynamic modeling and control directly in the end-effector/object space in which the manipulator(s) operate. This section offers a brief review of the Operational Space framework for individual and cooperating serial-chain manipulators. [11, 12]

2.1 End-Effector Dynamics

The operational space equations of motion are:

$$\Lambda(\mathbf{q})\ddot{\mathbf{x}} + \mu(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{p}(\mathbf{q}) = \mathbf{F}_{op} \quad (1)$$

$\Lambda(\mathbf{q})$ is the kinetic energy matrix of the system with respect to the operational point, \mathbf{x} , which defines the operational space coordinates of the end-effector (or object grasped by the end-effector). $\mu(\mathbf{q}, \dot{\mathbf{q}})$ represents the Coriolis and centrifugal forces acting at the same operational point, and $\mathbf{p}(\mathbf{q})$ depicts the gravitational forces also expressed at that point. \mathbf{F}_{op} is the generalized force vector expressed in the operational space. \mathbf{q} and $\dot{\mathbf{q}}$ are the vectors of generalized joint angles and velocities, respectively.

2.1.1 Serial-Manipulators

For a general redundant serial-chain manipulator:

$$\Lambda(\mathbf{q}) = (J(\mathbf{q})A^{-1}(\mathbf{q})J^T(\mathbf{q}))^{-1}, \quad (2)$$

$$\mu(\mathbf{q}, \dot{\mathbf{q}}) = \bar{J}^T(\mathbf{q})\mathbf{b}(\mathbf{q}) - \Lambda(\mathbf{q})\dot{J}(\mathbf{q})\dot{\mathbf{q}}, \quad (3)$$

$$\mathbf{p}(\mathbf{q}) = \bar{J}^T(\mathbf{q})\mathbf{g}(\mathbf{q}), \quad (4)$$

$$\bar{J}(\mathbf{q}) = A^{-1}(\mathbf{q})J^T(\mathbf{q})\Lambda(\mathbf{q}), \quad (5)$$

where $A(\mathbf{q})$ is the joint-space inertia matrix and $\mathbf{b}(\mathbf{q})$ and $\mathbf{g}(\mathbf{q})$ are the Coriolis/centrifugal and gravity force vectors in joint space. $J(\mathbf{q})$ is the Jacobian relating joint space velocities to the operational space velocity: $\dot{\mathbf{x}} = J(\mathbf{q})\dot{\mathbf{q}}$.

$\bar{J}(\mathbf{q})$ is the *dynamically consistent* generalized inverse of $J(\mathbf{q})$. For a non-redundant serial-chain manipulator, $J(\mathbf{q})$ is invertible and $\bar{J}(\mathbf{q}) = J^{-1}(\mathbf{q})$.

2.1.2 Augmented Object Control

When N manipulators (with the appropriate number of degrees-of-freedom) grasp the same object, Equation (1) still holds, and the operational space dynamics of the system simply become the sum of the operational space dynamics of the object and each manipulator, all expressed at the same operational point:

$$\Lambda(\mathbf{q}) = \sum_{i=1}^N \Lambda_i(\mathbf{q}_i) + \Lambda_{obj} \quad (6)$$

$$\mu(\mathbf{q}, \dot{\mathbf{q}}) = \sum_{i=1}^N \mu_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + \mu_{obj} \quad (7)$$

$$\mathbf{p}(\mathbf{q}) = \sum_{i=1}^N \mathbf{p}_i(\mathbf{q}_i) + \mathbf{p}_{obj} \quad (8)$$

$\Lambda_i(\mathbf{q}_i)$, $\mu_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$, and $\mathbf{p}_i(\mathbf{q}_i)$ represent the operational space dynamic components of the i^{th} manipulator. $J_i(\mathbf{q}_i)$ is the Jacobian of the i^{th} manipulator. Λ_{obj} , μ_{obj} , and \mathbf{p}_{obj} are the operational space dynamic components of the object.

2.2 Dynamic Decomposition and Control

End-effector motions are controlled by operational forces, \mathbf{F}_{op} , generated by the application of generalized joint torques, $\mathbf{\Gamma} = J^T(\mathbf{q})\mathbf{F}_{op}$. This relationship is incomplete for redundant manipulators that are in motion. For a given configuration of the manipulator, there are an infinite number of generalized joint torques that will produce a given force at the end-effector.

The dynamically consistent relationship between joint torques and operational forces for redundant manipulators is:

$$\mathbf{\Gamma} = J^T(\mathbf{q})\mathbf{F}_{op} + [I - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\mathbf{\Gamma}_o \quad (9)$$

This relationship provides a decomposition of joint torques into two dynamically decoupled control vectors: joint torques corresponding to forces acting at the end-effector, $J^T(\mathbf{q})\mathbf{F}_{op}$, and joint torques that only affect internal motions, $[I - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\mathbf{\Gamma}_o$.

3 Extended Operational Space

The Extended Operational Space Formulation expands the Operational Space framework to address branching manipulator systems. This formulation addresses two cases: 1) open-chain serial-to-parallel systems, in which each end-effector is independent; and 2) open/closed-chain serial-to-parallel systems, in which some end-effectors grasp common objects.

3.1 Open-Chain Systems

The Operational Space Formulation extends immediately to the case of a robot with multiple, independently moving arms. The operational space in this case is the superset of the domains of the operational space of each manipulator.

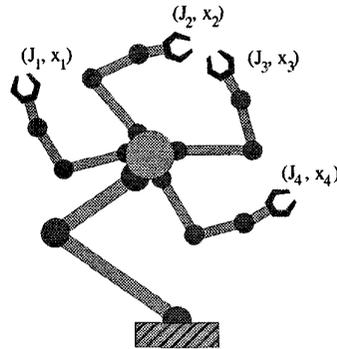


Figure 1: A Serial-to-Parallel Manipulator

For the system in Figure 1, for example, the operational space coordinates are

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \end{bmatrix}, \quad (10)$$

and the corresponding Jacobian will be a vertical concatenation of the Jacobians for each end-effector:

$$J(\mathbf{q}) = \begin{bmatrix} J_1(\mathbf{q}) \\ J_2(\mathbf{q}) \\ J_3(\mathbf{q}) \\ J_4(\mathbf{q}) \end{bmatrix}. \quad (11)$$

\mathbf{x}_i is the position and orientation of the i^{th} end-effector, and $J_i(\mathbf{q})$ is the basic Jacobian which yields the velocity of the i^{th} end-effector, given \mathbf{q} . \mathbf{q} is the vector of generalized joint coordinates for the entire branching manipulator system.

The kinetic energy matrix, $\Lambda(\mathbf{q})$, is still given by the form $(J(\mathbf{q})A^{-1}(\mathbf{q})J^T(\mathbf{q}))^{-1}$. For an open-chain system with N end-effectors, $\Lambda(\mathbf{q})$ takes the intuitively revealing form:

$$\Lambda = \begin{bmatrix} (J_1 A^{-1} J_1^T)^{-1} & (J_1 A^{-1} J_2^T)^{-1} & \dots & (J_1 A^{-1} J_N^T)^{-1} \\ (J_2 A^{-1} J_1^T)^{-1} & (J_2 A^{-1} J_2^T)^{-1} & & \\ \vdots & & \ddots & \\ (J_N A^{-1} J_1^T)^{-1} & \dots & & (J_N A^{-1} J_N^T)^{-1} \end{bmatrix}. \quad (12)$$

The inertial properties of each end-effector are not only dependent upon the end-effector's own configuration, but also upon the configuration of all other end-effectors in the system. The block diagonal matrices of Λ , $\Lambda_{ii} = (J_i A^{-1} J_i^T)^{-1}$, are each the inertia matrix that would occur if the i^{th} end-effector were the only end-effector present. The off-diagonal matrices, $\Lambda_{ij} = (J_i A^{-1} J_j^T)^{-1}$, may be regarded as cross-coupling inertias that are a measure of the inertial coupling to the i^{th} end-effector from the motion of the j^{th} end-effector via the common lower links of the robot.

With the Extended Operational Space coordinates defined as above, the dynamics of an open-chain branching system are now described by the *same* equations as for serial-chain systems, i.e. (1)-(5).

The dynamically consistent force/torque decomposition described in Equation (9) also applies directly. Under this decomposition, internal motions of the system are guaranteed not to introduce accelerations at *any* of the end-effectors.

3.2 Open/Closed-Chain Systems

When several end-effectors grasp a common object, a kinematic chain is closed and the dynamics of the system become constrained. In Augmented Object Control, where each manipulator has a stationary base fixed in a common inertial frame, each manipulator is an independent dynamic entity that only couples to other manipulators through the object. Hence if each manipulator has m degrees-of-freedom in operational space, the dynamic model for the resulting augmented system will also have m degrees-of-freedom in operational space.

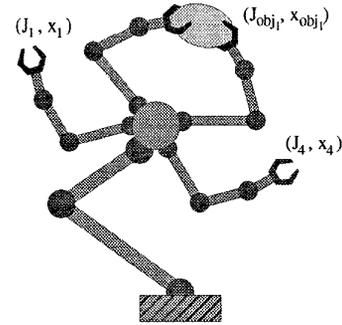


Figure 2: A Serial-to-Parallel Manipulator Grasping an Object

In contrast, all of the manipulators in branching systems are cross-coupled through their common links as part of a larger system. Their end-effectors are not dynamically independent before grasping the object. As discussed for the case of open-chain systems, the dimension of the operational space increases with the number of end-effectors. Similarly, as end-effectors grasp common objects, the dimension of the operational space decreases.

It therefore becomes necessary to consider a reduced, or constrained, set of dynamics:

$$A_c(\mathbf{q})\ddot{\mathbf{q}}_c + \mathbf{b}_c(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}_c(\mathbf{q}) = \Gamma_c \quad (13)$$

where

$$A_c(\mathbf{q}) = C^T(\mathbf{q})A(\mathbf{q})C(\mathbf{q}) \quad (14)$$

$$\mathbf{b}_c(\mathbf{q}, \dot{\mathbf{q}}) = C^T(\mathbf{q})\mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}) + C^T(\mathbf{q})A(\mathbf{q})\dot{C}(\mathbf{q})\dot{\mathbf{q}}_c \quad (15)$$

$$\mathbf{g}_c(\mathbf{q}) = C^T(\mathbf{q})\mathbf{g}(\mathbf{q}) \quad (16)$$

\mathbf{q}_c is a minimal set of generalized coordinates that describes the configuration of the full system. $C(\mathbf{q})$, the constraint matrix, is defined by $\dot{\mathbf{q}} = C(\mathbf{q})\dot{\mathbf{q}}_c$. Γ_c is the reduced set of generalized forces that corresponds to the choice of \mathbf{q}_c .

Again, the operational space equations of motion for the system are the same as in Equation (1). Only

now, the structure of \mathbf{x} and \mathbf{F}_{op} are determined by which end-effectors are grasping common objects and which are independent. For the example system in Figure 2, where the second and third end-effectors are grasping a common object, \mathbf{x} and \mathbf{F}_{op} will assume the form:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_{obj_1} \\ \mathbf{x}_4 \end{bmatrix}, \text{ and } \mathbf{F}_{op} = \begin{bmatrix} \mathbf{F}_{op_1} \\ \mathbf{F}_{obj_1} \\ \mathbf{F}_{op_4} \end{bmatrix}, \quad (17)$$

\mathbf{x}_i denotes the operational space coordinates of independent end-effectors, as before, and \mathbf{x}_{obj_k} denotes operational space coordinates of objects grasped by two or more end-effectors. The particular order of end-effector and object coordinates in \mathbf{x} follow directly from whichever manipulators happen to be grasping whichever objects. The notation convention for \mathbf{F}_{op} is identical.

The Jacobian for the constrained system, $J_c(\mathbf{q})$, will be the Jacobian relating the choice of $\dot{\mathbf{q}}_c$ to \mathbf{x} :

$$J_c(\mathbf{q}) = \begin{bmatrix} J_1(\mathbf{q}) \\ J_{obj_1}(\mathbf{q}) \\ J_4(\mathbf{q}) \end{bmatrix}. \quad (18)$$

$\Lambda(\mathbf{q})$, $\mu(\mathbf{q}, \dot{\mathbf{q}})$, $p(\mathbf{q})$, and $\bar{J}_c(\mathbf{q})$ are now defined by:

$$\Lambda(\mathbf{q}) = (J_c(\mathbf{q})A_c^{-1}(\mathbf{q})J_c^T(\mathbf{q}))^{-1}, \quad (19)$$

$$\mu(\mathbf{q}, \dot{\mathbf{q}}) = \bar{J}_c^T(\mathbf{q})\mathbf{b}_c(\mathbf{q}) - \Lambda(\mathbf{q})J_c(\mathbf{q})\dot{\mathbf{q}}_c, \quad (20)$$

$$\mathbf{p}(\mathbf{q}) = \bar{J}_c^T(\mathbf{q})\mathbf{g}_c(\mathbf{q}), \quad (21)$$

$$\text{and } \bar{J}_c(\mathbf{q}) = A_c^{-1}(\mathbf{q})J_c^T(\mathbf{q})\Lambda(\mathbf{q}). \quad (22)$$

Notice that the structure of these equations is identical to Equations (2)-(5). The operational space dynamics of this system are once again given by Equation (1).

The control of the robot system in this case will be similar to that of Equation (9) developed for the open-chain case. In this case, however, the manipulator system will have some closed chains which will have redundant actuation. The redundancy in the actuation may be used to control the internal forces of the grasped object or to minimize a performance criterion, e.g. minimizing actuator effort [12]. Williams and Khatib [14] have proposed a physically-based model, the Virtual Linkage Model, for the characterization and control of object internal forces.

Using the Virtual Linkage Model, the control equation for the system is

$$\Gamma = J^T(\mathbf{q})G^{-1} \begin{bmatrix} \mathbf{F}_{op} \\ \mathbf{F}_{int} \end{bmatrix} + [I - J^T(\mathbf{q})\bar{J}^T(\mathbf{q})]\Gamma_o, \quad (23)$$

$J(\mathbf{q})$ is the Jacobian for the full system described in Equation (11). \mathbf{F}_{int} is a vector associated with the control of internal forces on the grasped objects. G is the grasp matrix which relates end-effector forces to resultant forces and internal forces, \mathbf{F}_{op} and \mathbf{F}_{int} , exerted on the grasped objects.

4 Experiments

The Extended Operational Space Formulation has been implemented and validated experimentally at the Stanford University Aerospace Robotics Laboratory.

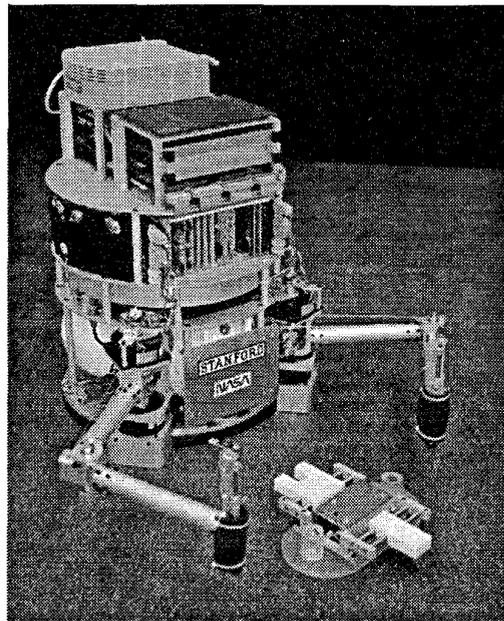


Figure 3: ARL Free-Flying Space Robot

The laboratory has three prototype free-flying space robots which float in two-dimensions on a frictionless air bearing over a granite surface plate. The robot shown in Figure 3 has thrusters and a momentum wheel for locomotion, two DC motor-driven manipulators with pneumatic grippers, force sensing, real-time vision, on-board computation and power, and wireless communications.

Figure 4 depicts the modeling for the free-flying space robot: a nine degree-of-freedom system that operates in a plane. A mobile-based robot can be modeled simply as a fixed-base robot in which the first two joints are prismatic and without joint limits, and the third joint is revolute.

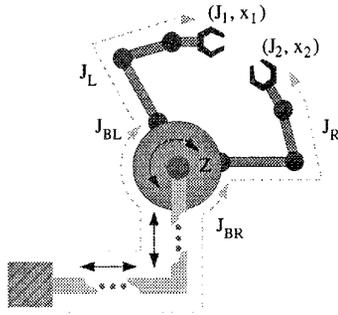


Figure 4: Model for a Free-Flying, Two-Arm Space Robot

4.1 Open-Chain Control

To apply the open-chain formulation of Section 3 to the space robot, let $\mathbf{x} = [\mathbf{x}_1^T \mathbf{x}_2^T]^T$ and $J(\mathbf{q}) = [J_1^T(\mathbf{q}) J_2^T(\mathbf{q})]^T$, where $J_1(\mathbf{q})$ and $J_2(\mathbf{q})$ are the basic Jacobians from the origin of the inertial frame to each end-effector. $J(\mathbf{q})$ is easily constructed from the Jacobians corresponding to the base and each arm:

$$J(\mathbf{q}) = \begin{bmatrix} J_1(\mathbf{q}) \\ J_2(\mathbf{q}) \end{bmatrix} = \begin{bmatrix} \hat{V}_R J_{BR} & Z J_R & \mathbf{0} \\ \hat{V}_L J_{BL} & \mathbf{0} & Z J_L \end{bmatrix} \quad (24)$$

As shown in Figure 4, J_L and J_R are the Jacobians for each manipulator extending from the base, as if they were fixed-based manipulators. Z is the rotation matrix that represents the rotation of the base relative to the inertial frame. J_{BL} and J_{BR} are the Jacobians from the origin of the inertial frame to each shoulder, where each branching manipulator begins. \hat{V}_L and \hat{V}_R are cross product operators that account for the effect of the rotation and translation of the base on the velocities at each end-effector.

Control of the robot is achieved by Equation (9). For simplicity, the first three terms of Γ_o are selected to be $-k_{p_i}(q_{i_{des}} - q_i) - k_{d_i}\dot{q}_i$, and the remaining six are $-k_{d_i}\dot{q}_i$. This choice will assert a PD controller on the base position and add extra damping within the dynamically consistent nullspace.

Figure 5a shows a photographic history of an independent arm slew, in which both manipulators execute step responses in operational space while the redundant degrees of freedom of the robot move within the dynamically consistent nullspace of the system to move the base to a new desired location. Figure 5b records the end-effector and base positions during the slew. While this system has limited performance due to finite, discrete thruster actuation at the base, these results demonstrate the effectiveness of the operational space approach in decoupling the end-effectors from the internal motions of the system.

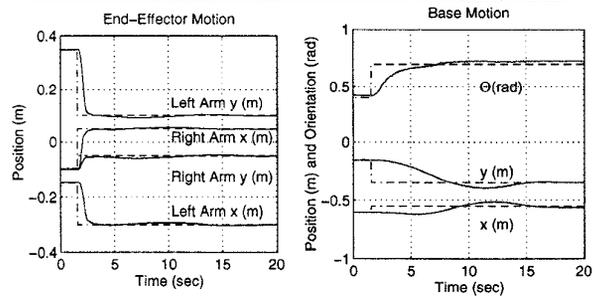
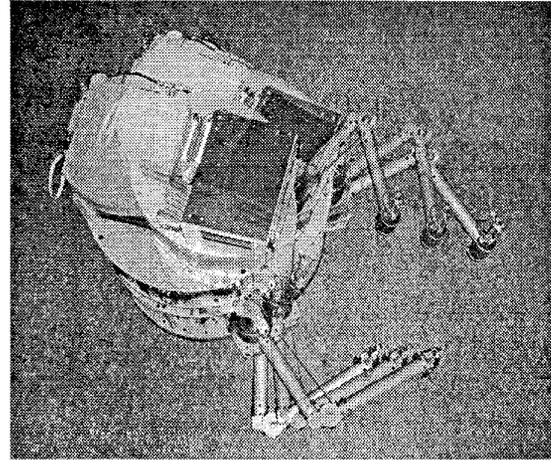


Figure 5: Slew Under Extended Operational Space Control

4.2 Open/Closed-Chain Control

When the two manipulators of the robot grasp a common object, \mathbf{x} reduces to $\mathbf{x} = \mathbf{x}_{obj}$. A convenient choice for the reduced Jacobian for this system is the Jacobian from the base to the right end-effector, $J_c = [\hat{V}_R J_{BR} Z J_R]$. \mathbf{q}_c will then be the six coordinates corresponding to the base and the right manipulator. The constraint matrix is:

$$C(\mathbf{q}) = \begin{bmatrix} I & \mathbf{0} \\ (Z J_L)^{-1} (\hat{V}_R J_{BR} - \hat{V}_L J_{BL}) & (Z J_L)^{-1} Z J_R \end{bmatrix} \quad (25)$$

Γ_o is the same as before, \mathbf{F}_{int} is chosen to be zero internal forces/moments on the object, and the grasp matrix is defined by:

$$G = \begin{bmatrix} I & I \\ I & -I \end{bmatrix}. \quad (26)$$

Figure 6a shows a photographic history of an augmented arm slew, in which both manipulators cooperatively execute a step response on an object in translation and orientation while the base moves to a new location.

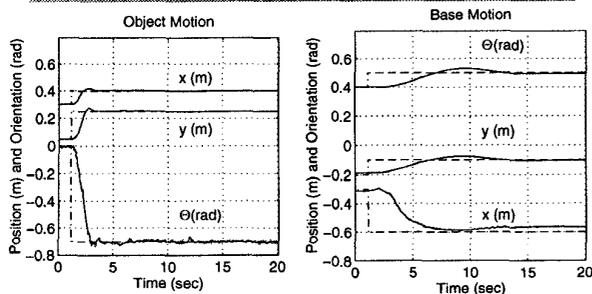
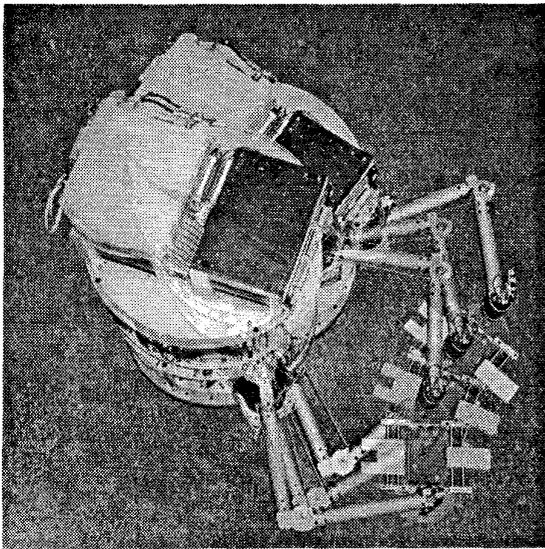


Figure 6: Slew Under Extended Augmented Object Control

Figure 6b records the object and base time history. Notice that long after the object has been moved to the new desired location, the base of the robot continues to move to its desired position and dynamically settle. This motion of the base *does not* introduce unwanted coupling forces to the object because the control of the redundant degrees of the freedom are selected from the *dynamically consistent nullspace* of the system.

5 Conclusions

We have presented in this paper an extension of the Operational Space Formulation to address an important class of serial-to-parallel (branching) manipulators. The concepts of Augmented Object and Dynamically Consistent Force/Torque Decomposition for redundant manipulators have been shown to extend directly to this larger, more general class. The experimental results shown for a free-flying space robot illustrate the effectiveness of this new formulation.

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